Effect of Diagonal Cracking on the Strength of Concrete Strut in RC Members

철근콘크리트 부재에서 대각선 균열이 압축스트럿의 강도에 미치는 영향

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ABSTRACT

철근콘크리트 부재가 하중을 받을 때, 응력교란구역에서의 힘의 흐름은 스트럿-타이 모델을 이용하여 효과적으로 표현할 수 있다. 그러나 스트럿-타이 모델을 이용하여 철근콘크리트 부재의 해석과 설계를 하기 위해서는 콘크리트 압축스트럿이 가지는 유효강도를 정확히 산정하여야 한다. 본 연구는 철근콘크리트 부재에 휨과 전단력이 동시에 작용할 때 발생하는 대각선 균열이 콘크리트 압축스트럿에 미치는 영향에 대해 설명하고 있다. 대각선 균열의 발생 메커니즘과 이로 인한 콘크리트 압축스트럿의 강도 저하를 이론적으로 설명하였으며, 그 결과를 철근콘크리트 보의 강도 산정에 적용하였다. 최종적으로 철근콘크리트 보의 강도 예측값을 기존 연구자들의 실험결과와 비교하여 제안된 이론의 합리성을 검증하였다.

1. Introduction

Shear-related problem of reinforced concrete members has always been an ever-challenging issue during the last century and still there is no one generally consented theory which is capable of predicting the essence of member behavior failing in shear. Especially when the shear force is combined with flexural bending moment, the problem becomes more complex due to a number of indeterminate parameters related with diagonal cracking. The "strut-and-tie model" is a representative design instrument of the plasticity theory for reinforced concrete members. Although the application of the plasticity theory to the non-plastic materials like concrete may cause some contradiction, the strut-and-tie model successfully simulates the ultimate strength of reinforced concrete members like beams and columns in an intuitive manner by adopting the concept of "effective strength of concrete". There are many theories explaining the effective strength of the concrete but most of them adopted empirical methods, which lacks in the theoretical background. Current study finds the appropriate reasoning in the mechanism of diagonal cracking of reinforced concrete members. The reduced strength of the concrete strut in the strut-and-tie model is first derived as a function of material properties and member geometries considering the diagonal cracking. Proposed model is then used to find the shear strength of reinforced concrete beams without shear reinforcement and verified by comparing it with experimental results from published researches.

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2. Effect of Diagonal Cracking on the Strength of Concrete Strut

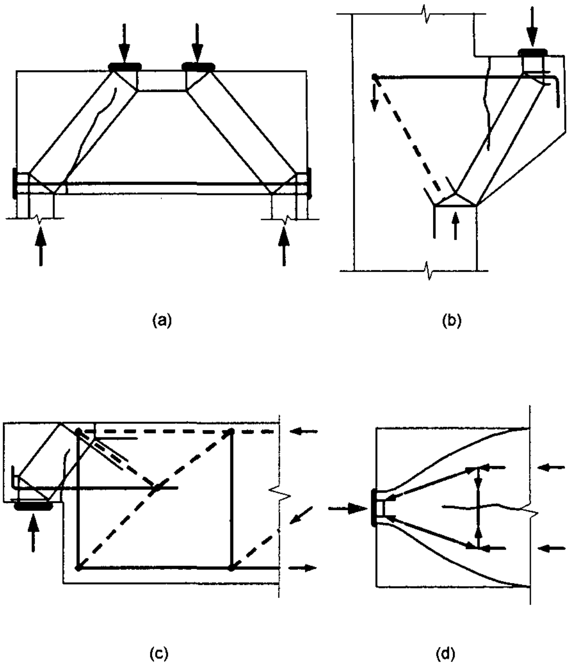

Figure 1 Strut-and-tie models with their compressive struts being disturbed by cracks from outside of strut.

Four instances of strut-and-tie models for so-called D-regions are shown in Figure 1 with traces of diagonal cracks for each member. Structural cracks in reinforced concrete members usually appear in tension, not in compression. The tension in the concrete is induced by flexural stress or bond stress between concrete and reinforcing bars. The capacity of the concrete strut is undermined since the strut may be completely penetrated by or partially interfered with flexural cracks. The undermined capacity of the concrete strut can be visually represented as the reduced width of strut. According to the above reasoning, deep beams are typical examples where the reduced width of compressive strut can be applied (Figure 1a). Corbels and dapped ends are also considered to have reduced-width strut due to the flexural cracks as shown in Figure 1b and c. Bottle shaped strut in Figure 1d with cental cracks can be equilibrated with two inclined struts with reduced width. Any other strut-and-tie models can be simulated by strut with reduced width if there is no restraining force against the flexural cracks which penetrate into the strut. Figure 2a shows an ideal strut-and-tie model with an inclined concrete strut and a horizontal steel reinforcing bar with an uniform tension. When the reinforcing bar is buried in the concrete as in Figure 2b, the tension in the reinforcing bar is transferred to the surrounding concrete with possible crack development and the concrete strut is affected by this neighboring stress condition. Neither the exact distribution of the steel tension and bond stress between cracks nor the affected shape of the concrete strut is not known. Concrete strut with reduced width as shown in Figure 2c is assumed here to consider the effect of bond cracks without much preventing the original simplicity of the strut-and-tie model.

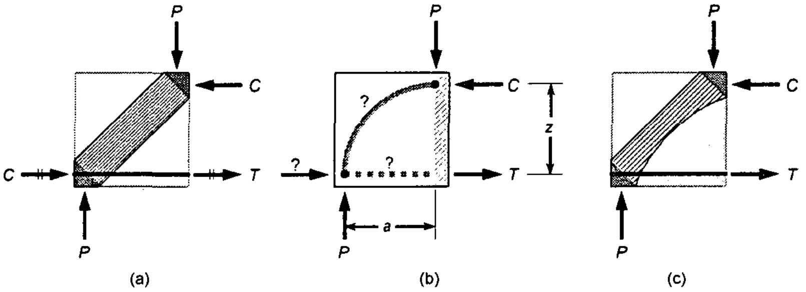

Figure 2 Inclined strut: (a) Ideal strut-and-tie model; (b) Influence of flexure and bond; (c) Reduced strut width.

3. Effective Strength of Concrete Strut

A series of straight crack segments is shown overlapped on a simple strut-and-tie model in Figure 3. The crack segments represent the trajectory of a diagonal crack. It is divided into three segments according to the process of crack development. The first segment starts vertically from point *A* in Figure 3a until it reaches the location of horizontal reinforcing steel. Only flexural tension is exerted on the concrete during this stage. Once the reinforcing steel is engaged in the flexural resistance, the state of stress in the concrete gets more complicated. Successive progress of crack segment is developed in a somewhat deviated direction under combined flexural tension and bond stress between the concrete and reinforcing steel. The second crack segment meets the horizontal neutral axis at point *B*, above which the flexural compression exists in the area of the concrete strut. Remaining segment of diagonal crack, *BC*, follows the original yield line of the concrete sturt. While the first and the second segments of crack represent the diagonal cracking process, the third segment of crack is developed due to the direct load-resisting mechanism of the concrete strut. Therefore, the effective strength of the concrete strut can be explained by the reduced width of the concrete strut, which is denoted as w'_s in Figure 3b. Detailed procedure for deriving the equation for the reduced width of the concrete strut is explained elsewhere.[1] The effective width of concrete strut is expressed as normalized form as follows:

$$\frac{w'_s}{h} = 1.3\zeta^2\sqrt{1+\left(\frac{1-\zeta}{\xi}\right)^2} \tag{1}$$

where $\xi = a/h$, $\zeta = c_n/h = 1.15\sqrt[3]{\rho}$. These represent the aspect ratio of a member containing the concrete strut and normalized vertical dimension of the node, respectively; see Figure 3b. Note that ζ is expressed as a only function of the horizontal reinforcement ratio.

4. Model Application and Verification

An example application of the effective strength of concrete strut is presented for the shear strength of a reinforced concrete beam without shear reinforcement. The shear strength, *P*, of the beam in Figure 3, which contains the concrete strut, can be derived as the vertical component of a strut force.

$$P = f'_c\, w'_s\, b \sin\theta \tag{2}$$

Normalizing the result with respect to the area of beam section yield the following equation for shear stress.

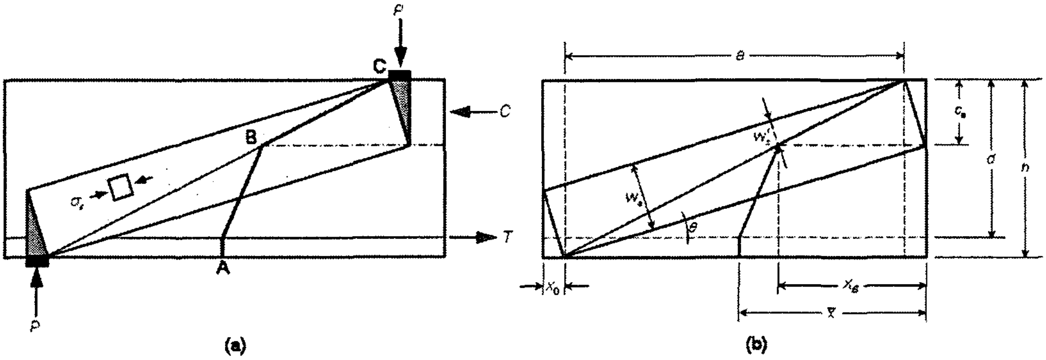

Figure 3 Strut-and-tie model with traces of diagonal cracking (a) and its notation (b).

$$\tau = \frac{P}{bh} = 1.3 f'_c \frac{\zeta^2(1-\zeta)}{\xi} \qquad (3)$$

The derived equation for the shear strength is expressed in terms of the strength of concrete, ratio of longitudinal reinforcement, and shear span-to-depth ratio. In Figure 4, Eq.(3) is compared with 176 reinforced concrete beams without shear reinforcement which have failed by shear. The details of these beams are found in the first reference. The predicted values generally provide a safe lower bound solution. Mean value and standard deviation of τ/τ_t (predicted/measured) are 0.739 and 0.186, respectively and the value of coefficient of correlation is 0.95, which means an excellent correlation between the two values. The test results of Kani's fifty beams without shear reinforcement are compared with Eq.(3) in Figure 5 to find the effect of shear span-to-depth ratio on the shear strength.[2] The strength of the concrete strut closely simulates the test results for members with $2.0 \le \xi \le 4.0$ but generally underestimates the shear strength of members with $\xi \le 2.0$ or $4.0 \le \xi$. Original strut-and-tie model with intact strut width seems more suitable for members with $\xi \le 2.0$ and diagonal cracking strength, which was explained earlier, successfully reproduces the shear strength for more slender members with $4.0 \le \xi$.

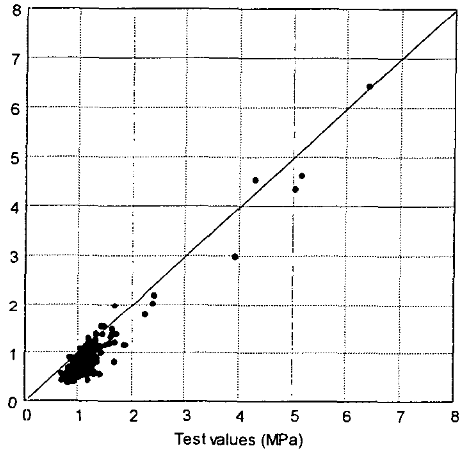

Figure 4 Test values vs. predicted values for shear strength of reinforced concrete beams.

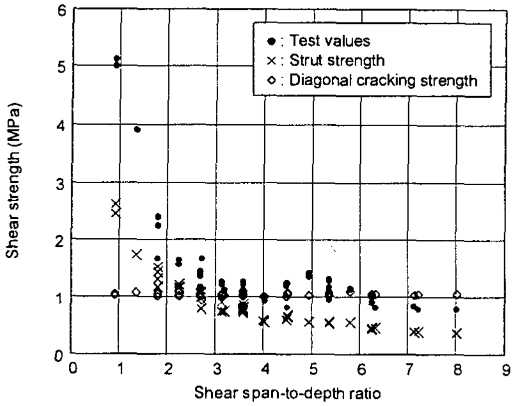

Figure 5 Comparison of predicted shear strength with test results by Kani.

5. Conclusion

Diagonal cracking mechanism is initiated by the bond stress between concrete and longitudinal reinforcement. The development of the diagonal crack into concrete strut impaired the capacity of the concrete strut when compared with original strut-and-tie model. The reduced strength of concrete strut can be expressed by reduced width of the strut. The proposed model for reduced strut width is used to calculate the shear strength of non-shear reinforced beams, which predicts the test results well for members with shear span-to-depth ratio between $2.0 \le \xi \le 4.0$.

References

1) Ha, T. (2004). "Effective strength of compressive strut in reinforced concrete members," Ph.D. Thesis, Seoul National University, Seoul.

2) Kani, M. W., Huggins, M. W., and Wittkopp, R. R. (1979). *Kani on shear reinforced concrete*, University of Toronto Dept. of Civil Engineering, Toronto.