Failure Mechanism and Ultimate Strength of Headed Bar Anchored in Deep Beam Using Truss Models

트러스 모델을 이용한 춤이 깊은 보에 정착된 헤드 철근의 파괴 메커니즘과 극한 내력 해석

Chun, Sung Chul* Hong, Sung Gul**
천 성 철 홍 성 걸

ABSTRACT

최근 들어 90도 표준갈고리의 대안으로 정착판을 지니는 헤드 철근(headed bar)에 대한 관심이 높아지고 있다. 헤드 철근의 정착내력은, 정착판의 지압력과 위험단면에서 헤드까지 정착길이의 부착력으로 발현된다. 실제 구조물에서는 정착되는 부재의 재료 및 기하학적 물성에 의해 다양한 파괴가 발생된다. 따라서 헤드 철근의 정착내력은 단순히 지압력과 부착력의 합으로 산정될 수 없으며, 발생 가능한 모든 파괴양상을 고려한 최소 내력으로 결정되어야 한다.

헤드 철근의 정착내력을 산정하기 위한 기본적인 해석모델로, CCT 절점에 정착된 헤드 철근의 트러스 모델을 제안하였다. 제안된 트러스 모델의 파괴는 부착파괴와 콘크리트의 압축파괴로 구분되며, 재료 및 기하학적 물성에 의해 파괴 양상이 결정된다. 이러한 트러스 모델은 외부 보-기둥 접합부와 같이 보다 복잡한 부위에 정착된 헤드철근의 정착 기구를 설명하는데 활용될 수 있다.

1. Introduction

Hooks are generally used to provide anchorage when there is insufficient length available to develop a bar. For last decade use of headed bar shown in Fig. 1 has provided viable option for hooks.[1] DeVries, et al.[2] investigated additional anchorage strength provided by available bonded length of headed bars in case of side-blowout capacity of deeply embedded headed bars. At failure the bonded length carried approximately 33% of the total load based on experimental observation. ACI 352-02 specifies that the development length of a headed bar should be taken as 3/4 of the value of a hooked bar. The applicability of the design formulas in the provision of ACI 352-02 statically derived from the test data is usually restricted to the prediction of behavior with the same test condition from which formulas were derived.

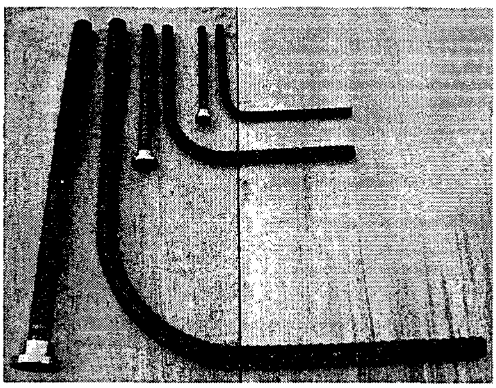

Fig. 1 Headed bars and hooked bars

* Senior researcher, Daewoo Institute of Construciton Technology
** Associate professor, Department of Architecture, Seoul National University

This paper presents strut-and-tie models for a headed bar development with consideration of bonded lengths of bars depending on surrounding structural configuration. Force transfer by headed bars develops disturbed stress regions adjacent to bar and therefore, strut-and-tie model may be one of promising design tools for detailing of headed bar in concrete members.

2. Truss model for headed bar

2.1 Idealized deep beam and assumptions

A deep beam shown in Fig. 2 is considered as an idealized structure to study the anchorage capacity and failure mechanism of a headed bar development in C-C-T node. The plastic idealization of the material properties of concrete and reinforcing ensures a rigorous analysis within the theory of plasticity.[3]

It is assumed first that a strut-and-tie model for the headed bar consists of uniform diagonal compression fields (*ST1*) with bi-axially compressed nodal zones at head and supports and fan-shaped stress fields (*ST2*) transferring the bond stress along the bonded length to the supports as shown in Fig. 3. Possible failure modes for the deep beam with headed bar involve the yielding of headed bar, the crushing of struts, and the crushing failure combined with the bond failure of headed bar.

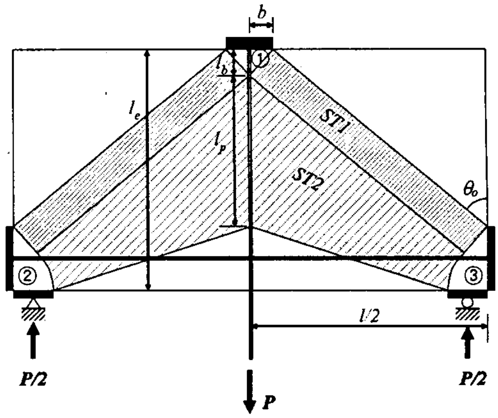

Fig. 2 Truss model of headed bar in CCT node

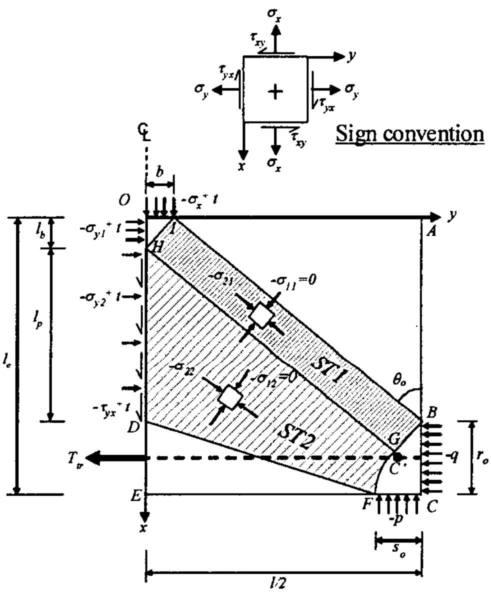

Fig. 3 Free-body diagram for half of the truss model excluding vertical bar

2.2 Force equilibrium

Half of the structure is necessary in the following discussion because of its symmetry with respect to the vertical center line. Fig. 3 shows the free-body diagram for the right half of the deep beam. The stresses of the biaxial compression node OHI are assumed to be equal to effective compressive strength of concrete ($\sigma_x^+=\sigma_y^+=f_{cu}$) to simulate stress concentrations in front of the head. The value of a net head area A_{nh} divided by a thickness of the beam t is defined as an equivalent effective width of a head $2b$, and then the width of node OHI is b. From the force equilibrium of node OHI and the strut *ST1*, the stress of the strut *ST1* should be equal to an effective compressive strength of concrete ($\sigma_{21}=f_{cu}$) and the width of *ST1* should be uniform.

Region HGFD is assumed to be uniaxially stressed with a principal compressive stress σ_{22}. r_o and s_o denote the size of the bearing and anchor plates, respectively. In the following it is assumed that minimum size bearing and anchor plates are used which results in bearing stresses and, hence, principal stresses in the biaxially compressed nodal zone equal to the effective compressive strength of the concrete ($p=q=f_{cu}$).

The failure modes of the system are divided into two kinds depending on the stress states of the compression field *ST2*. If the shear stress τ_{vx}^{+} along the vertical headed bar reaches its ultimate strength $U_p/(2t)$ in Segment DH, *bond failure* occurs. The other failure is *concrete diagonal crushing failure* when the principal stress σ_{22} of the compression field *ST2* reaches the effective compressive strength of concrete f_{cu} before the bond stress along the vertical headed bar does not reach its yield strength.

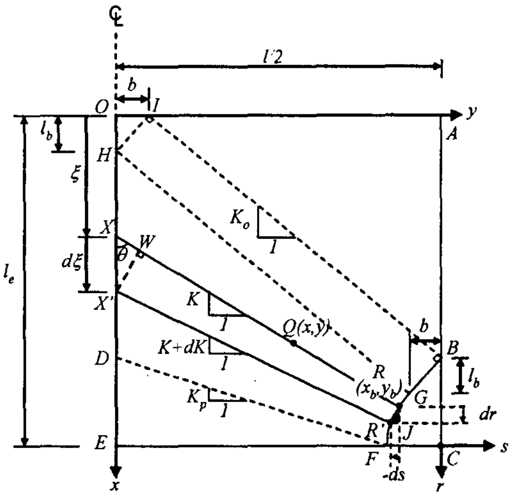

Fig. 4 Geometry of half of the structure

2.3 Bond failure with the yielding of the horizontal bar

In the bond failure, the bond stress resultant U along the vertical headed bar reaches its ultimate bond strength U_p in Segment DH while the concrete of the compression field *ST2* does not crush. For the failure of the system, another structural component of the truss model needs to be in an ultimate stress state, so the tensile force of the reinforcing bar is assumed to reach its yielding strength.

From yield conditions, equilibrium conditions for infinitesimal elements XX'W and XX'R'JR, boundary conditions of nodes and fan-shaped stress field *ST2*, and material properties, the governing equations are determined as

$$\xi(s) = -\frac{2}{\gamma}s + l_b - \frac{2}{\gamma}b \tag{1}$$

$$\ln\left(\frac{(K_{\max}-K)(K-K_{\min})}{(K_{\max}-K_o)(K_o-K_{\min})}\right) + \frac{1}{\sqrt{1-\gamma^2}}\ln\left(\frac{(K_{\max}-K)(K_o-K_{\min})}{(K-K_{\min})(K_{\max}-K_o)}\right) + ln\left(\frac{1+\frac{s}{l/2}}{1-\frac{\beta}{\alpha}}\right)^2 = 0 \tag{2}$$

where $\alpha=(l/2)/l_e$, $\beta=b/l_e$, $\gamma=U_p/(f_{cu}t)$, $\omega=(A_{tr}f_p)/(f_{cu}tl_e)$, A_{tr}=area of horizontal reinforcing bar, f_p=effective yield strength of reinforcing bar, $K_o=(1-\omega)/(\alpha-\beta)$, $K_{\max} = (1+\sqrt{1-\gamma^2})/\gamma$, and $K_{\min} = (1-\sqrt{1-\gamma^2})/\gamma$

The effective bond length l_p is determined form the slope K_p which can be expressed with the geometry of the line DF in Fig. 4 (Eq. (3). s_o can be expressed in terms of α, β, γ, ω, and K_p. Substituting $K=K_p$ and $s=s_o$, in Eq. (2) yields an equation which involves an unknown slope K_p as Eq. (4).

$$K_p = \frac{1-\beta/K_o - l_p/l_e}{\alpha-\beta-(\gamma/2)(l_p/l_e)} \tag{3}$$

$$\left(\frac{K_{\max}-K_p}{K_{\max}-K_o}\right)^{\frac{1}{\sqrt{1-\gamma^2}}+1}\left(\frac{K_o-K_{\min}}{K_p-K_{\min}}\right)^{\frac{1}{\sqrt{1-\gamma^2}}-1}\left(\frac{1-\left(\frac{1}{\alpha}-\frac{(\alpha-\beta)}{(1-\omega)}\frac{\beta}{\alpha}-\frac{(\alpha-\beta)}{\alpha}K_p\right)\Big/\left(\frac{2}{\gamma}-K_p\right)-\frac{\beta}{\alpha}}{1-\frac{\beta}{\alpha}}\right)^2 = 1 \tag{4}$$

Solving Eq. (4) for K_p and substituting K_p into Eq. (3) yield the effective bond length l_p. The ultimate load P_u can be obtained from Eq. (5).

$$P_u = U_p l_p + 2tbf_{cu} = 2s_o t f_{cu} \tag{5}$$

2.4 Concrete diagonal crushing failure

While the bond stress resultant along the vertical headed bar and the tensile force of the headed bar are less than their yield strengths, the principal stress σ_{22} of the compression field *ST2* reaches the effective compressive strength of concrete f_{cu} and the tensile force of the reinforcing bar reaches its yielding strength in the concrete diagonal crushing failure.

The governing equations are obtained from yield conditions, equilibrium conditions for infinitesimal elements XX'W and XX'R'IR, boundary conditions of nodes and fan-shaped stress field *ST2*, and material properties.

$$\xi(s) = -\left(K_o + \frac{1}{K_o}\right)s, \quad r(s) = -\frac{1}{K_o}s + \omega l_e \tag{6}$$

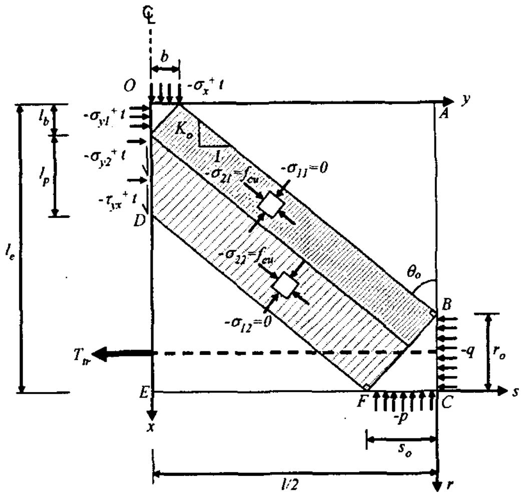

Fig. 5 Truss model and ultimate stress state for concrete diaonal crushing failure mode

The slope K is constant as K_o for all s. The size of bearing plate s_o is determined from Eq. (6) by setting r=0 and $s=s_o$. The ultimate load can be determined from the equilibrium condition in the x-direction.

$$P_u = 2f_{cu}s_o t = \frac{2\omega(1-\omega)l_e t f_{cu}}{\alpha - \beta} \tag{7}$$

3. Conclusion

1. The presented truss model for headed bar development in C-C-T nodes explains two type of load transfers of headed bars from the headed bar to the supports. One is a uniform strut *ST1* formed from the front of the head by bearing and the other is a fan-shaped compression field *ST2* along bar/concrete interfaces by bond.
2. The stress state of the compression field *ST2* determines the controlling failure mode of deep beam with a headed bar. If the shear stress at the headed bar/concrete interfaces reaches its yield strength, ***bond failure*** occurs. When the principal stress of the compressive field *ST2* reaches an effective compressive strength of concrete, *concrete diagonal crushing failure* occurs.
3. The failure mode and the ultimate load for a headed bar in C-C-T nodes are dependent on the head size, material strengths, and the surrounding structural configuration including geometry and reinforcement details.

References

1. Wallace, J. W., "Headed Reinforcement A Viable Option," *Concrete International*, V. 19, No. 12, Dec. 1997, pp. 47-53.
2. DeVries, R. A.; Jirsa, J. O.; and Bashandy, T. "Project Effects of Transverse Reinforcement and Bonded Length on the Side-Blowout Capacity of Headed Reinforcement," *Bond and Development of Reinforcement-A Tribute to Dr. Peter Gergely*, edited by Leon, R., SP-180, ACI, 1998, pp. 367-38
3. Hong, S.-G., "Consistent Truss Modeling for Bar Development and Anchorage," PhD dissertation, Department of Civil Engineering, Lehigh University, Bethlehem, Pennsylvania, May 1994, 399 pp.