Decorrelation of Variance-Covariance Matrix

Samsung Lim

School of Surveying & Spatial Information Systems
The University of New South Wales, Australia

Outlines

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- Numerical Example
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- Concluding Remarks

Introduction

- Float estimate for initial ambiguity of GPS carrier phase measurements
- To fix the integer value in order for high precision DGPS positioning
- Problem:

$$\min S(a) = (\hat{a} - a)^T Q_{\hat{a}}^{-1} (\hat{a} - a) \text{ with } a \in Z^n$$

Introduction (cont.)

- *Ambiguity search space*

$$(\hat{a} - a)^T Q_{\hat{a}}^{-1} (\hat{a} - a) \leq \chi^2$$

- Original objective function is equivalent to:

$$\min S(z) = (\hat{z} - z)^T Q_{\hat{z}}^{-1} (\hat{z} - z)$$

$$\text{with } \hat{z} = Z^T \hat{a},\ z = Z^T a,\ Q_{\hat{z}} = Z^T Q_{\hat{a}} Z$$

Introduction (cont.)

- *In order to preserve the nature of a in z, the transformation matrix Z must satisfy two conditions:*

✓ *C1: Elements of Z must be integers;*

✓ *C2: Elements of* Z^T *be integers too.*

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S. Lim

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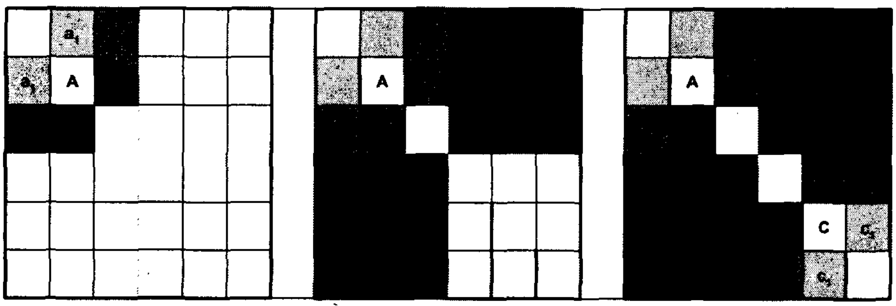
Block Correlation Method



Block A Block B Block C

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Block Correlation Method (Cont.)

$$Z_k^T Q_{\hat{a}} Z_k = \begin{bmatrix} I^k & 0 & 0 \\ x_k^T & 1 & 0 \\ 0 & 0 & I^{n-k-1} \end{bmatrix} * \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{12}^T & q_{22} & q_{23} \\ q_{13}^T & q_{23}^T & q_{33} \end{bmatrix} * \begin{bmatrix} I^k & x_k & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I^{n-k-1} \end{bmatrix} = \begin{bmatrix} q_{11} & s_k & q_{13} \\ s_k^T & p_k & t_k^T \\ q_{13}^T & t_k & q_{33} \end{bmatrix}$$

$$s_k = q_{11} x_k + q_{12}$$

$$t_k = q_{13}^T x + q_{23}^T$$

$$p_k = (x_k^T q_{11} + q_{12}^T) x_k + x_k^T q_{12} + q_{22} = s_k^T x_k + x_k^T q_{12} + q_{22}$$

$$s_h = q_{33} \hat{x}_h + q_{23}^T = 0 \text{ or } \hat{x}_h = -q_{33}^{-1} q_{23}^T$$

$$x_h = [\hat{x}_h]^{\text{int}}$$

Block A

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Block Correlation Method (Cont.)

$$Z_g^T Q_{\hat{a}} Z_g = \begin{bmatrix} I^g & 0 \\ y_g^T & I^{n-g} \end{bmatrix} * \begin{bmatrix} q_{11} & q_{12} \\ q_{12}^T & q_{22} \end{bmatrix} * \begin{bmatrix} I^g & y_g \\ 0 & I^{n-g} \end{bmatrix} = \begin{bmatrix} q_{11} & s_g \\ s_g^T & p_g \end{bmatrix}$$

$$s_g = q_{11} y_g + q_{12}$$

$$p_g = (y_g^T q_{11} + q_{12}^T) y_g + y_g^T q_{12} + q_{22} = s_g^T y_g + y_g^T q_{12} + q_{22}$$

$$q_{11} \hat{y}_g + q_{12} = 0 \text{ or } \hat{y}_g = -q_{11}^{-1} q_{12}$$

$$y_g = [\hat{y}]^{\text{int}}$$

Block B

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Block Correlation Method (Cont.)

$$Z_h^T Q_{\hat{a}} Z_h = \begin{bmatrix} I^h & 0 & 0 \\ 0 & 1 & x_h^T \\ 0 & 0 & I^{n-h-1} \end{bmatrix} * \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{12}^T & q_{22} & q_{23} \\ q_{13}^T & q_{23}^T & q_{33} \end{bmatrix} * \begin{bmatrix} I^h & 0 & 0 \\ 0 & 1 & 0 \\ 0 & x_h & I^{n-h-1} \end{bmatrix} = \begin{bmatrix} q_{11} & t_h & q_{13} \\ t_h^T & p_h & s_h^T \\ q_{13}^T & s_h & q_{33} \end{bmatrix}$$

$$s_h = q_{33} x_h + q_{23}^T$$

$$t_h = q_{13} x_h + q_{12}$$

$$p_h = (x_h^T q_{33} + q_{23}) x_h + x_h^T q_{23}^T + q_{22} = s_h^T x_h + x_h^T q_{23}^T + q_{22}$$

Block C

$$s_k = q_{11} \hat{x}_k + q_{12} = 0 \text{ or } \hat{x}_k = -q_{11}^{-1} q_{12}$$

$$x_k = [\hat{x}_k]^{\text{int}}$$

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Block Correlation Method (cont.)

- ***Step 1:*** Permute matrix $Q_{\hat{a}}$ to obtain $\vec{Q}_{\hat{a}}$ so that its first m diagonal elements are minimal and stay in increasing order.
- ***Step 2:*** Apply the decorrelation process to the upper-left block **A** of $\vec{Q}_{\hat{a}}$.
- ***Step 3:*** Decorrelate blocks **B** and its transpose of Q_A.

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Block Correlation Method (cont.)

- ***Step 4:*** Permute Q_{AB} to yield $\overleftarrow{Q}_{AB}$ so that the diagonal elements of its lower-right block **C** stays in decreasing order
- ***Step 5:*** Decorrelate the block **C** in $\overleftarrow{Q}_{AB}$.

Numerical Example

- To quantify the decorrelation, two measures are used:

✓The correlation coefficients;

✓The condition number ***c*** which is the ratio of the largest and the smallest singular value of variance-covariance matrix.

Numerical Example (cont.)

NN	Description	$Q_{\hat{a}}$		$Q_{\hat{z}}$		number of iteration
		c	ρ_{min}	c	ρ_{max}	
1	6 ambiguities, Block decorrelation method	$2.2*10^7$	0.8442	12.2	0.3990	6
2	6 ambiguities, United decorrelation method	$2.2*10^7$	0.8442	12.2	0.3990	6
3	6 ambiguities, Gauss transformation	$2.2*10^7$	0.8442	11.7	0.4275	6
4	12 ambiguities, Block decorrelation method	$2.1*10^5$	0.9448	24.8	0.5056	9
5	12 ambiguities, Gauss transformation	$2.1*10^5$	0.9448	54.5	0.4778	10

Test Results

- Highly correlated ambiguities are significantly decorrelated
- Condition number and the corresponding elongation of the search ellipsoid drastically reduced from 10^5-10^7 to less than 100
- Average correlation coefficients diminished more than 2 times
- BDM is 70-120% faster than Gauss transformation depending on the original matrix in the cases of 12 ambiguities

Concluding Remarks

- Decorrelation process plays an important role in resolving integer ambiguities of GPS carrier phase measurements
- New method for the decorrelation is introduced
- The method is based on dividing the variance-covariance matrix into 4 small blocks and decorrelating them simultaneously

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Concluding Remarks (cont.)

- The algorithm reduces the dimension of the original variance-covariance matrix and therefore increases the speed of the decorrelation process
- The proposed algorithm provides comparable or better results than the existing algorithms

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