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Stochastic Modeling for Precise GPS/Glonass Positioning

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Outline

- What are important aspects in satellite positioning?
- Why is the stochastic model critical?
- Stochastic modeling for static positioning
- Stochastic modeling for kinematic positioning

What Are Important Aspects in GPS Positioning?

- **Methods/Instruments for positioning – SPP or RTK?**
- **Measurements/Observations – geometry**
- **Processing the measurements for positioning**
  - **Modeling** the measurements (Knowledge about the methods/measurements is essential)
  - **Estimation** theory, algorithms and implementation
  - **Quality control (QC):** uncertainty, reliability, etc.

  Data processing has been commonly based on the least-squares/Kalman filtering.

Least Squares

Functional Model: $l + v = Ax$

Stochastic Model: $D = \sigma_0^2 Q = \sigma_0^2 P^{-1}$

Least Squares Estimates:

$$\hat{x} = \left(A^T PA\right)^{-1} A^T Pl$$

$$C_{\hat{x}} = \hat{s}_0^2 \left(A^T PA\right)^{-1}$$

$$\hat{s}_0^2 = \frac{V^T PV}{f}$$

What are important in the LS ?
A!, D!, L! - modelling/QC...

Kalman Filtering

Mathematical Models

Functional Models

- dynamic model $x_k = \Phi_{k,k-1} x_{k-1} + \tau_k$
- measurement model $z_k = H_k x_k + \varepsilon_k$
- stochastic model

$$E(\tau_k \tau_i^T) = \begin{cases} Q_k & i = k \\ 0 & i \neq k \end{cases} \qquad E(\varepsilon_k \varepsilon_i^T) = \begin{cases} R_k & i = k \\ 0 & i \neq k \end{cases}$$

$$E(\varepsilon_k \tau_i^T) = 0$$

Estimators

$$\hat{x}_k = \bar{x}_k + G_k d_k$$

$$Q_{\hat{x}_k} = Q_{\bar{x}_k} - G_k Q_{d_k}$$

$\bar{x}_k$ is the predicted state values with covariance $Q_{\bar{x}_k}$

G_k is the gain matrix

Innovation Vector

$$d_k = z_k - H_k \bar{x}_k$$

$$Q_{d_k} = R_k + H_k Q_{\bar{x}_k} H_k^T$$

Filtering Residuals

$$v_{z_k} = H_k \hat{x}_k - z_k$$

Why Is the Stochastic Model Critical?

- **Any deficiency in stochastic models may lead to:**
  - — **inefficient statistic testing results;**
  - — **unreliable estimates of the unknown parameters;**
  - — **unrealistic evaluation of the quality of positioning results.**

Widely Used Method: Error Propagation

Suppose two GPS receivers (A, B) simultaneously track 4 satellites (1,2,3,4) at epoch t. We then get 3 DD phase observations :

$$l_t = \begin{bmatrix} 1 & -1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Phi_A^1 \\ \Phi_A^2 \\ \Phi_A^3 \\ \Phi_A^4 \\ \Phi_B^1 \\ \Phi_B^2 \\ \Phi_B^3 \\ \Phi_B^4 \end{bmatrix} = B\Phi$$

Using error propagation law, we get Covariance matrix :

$$C_{l_t} = BC_\Phi B^T = \sigma_\Phi^2 BIB^T = \sigma_\Phi^2 \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

Stochastic Modelling: Assumptions

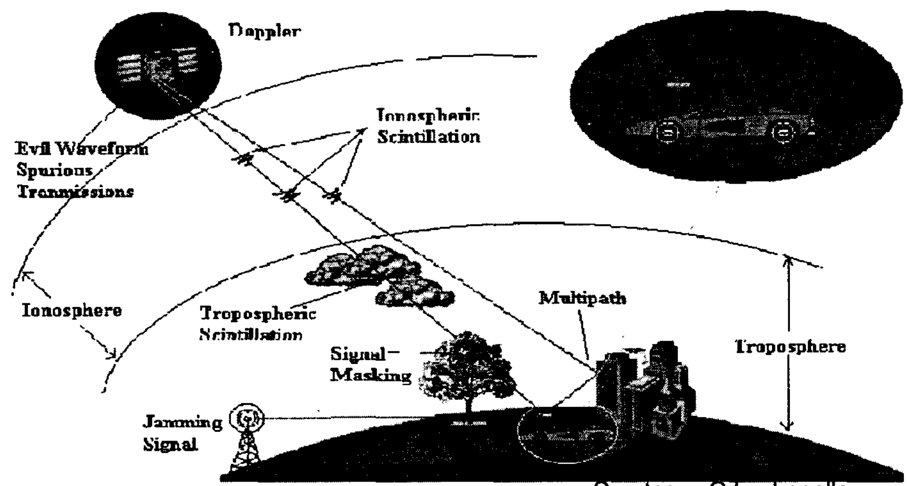
Stochastic Modelling for GPS/GNSS Positioning

The assumptions used in the existing stochastic models:

- **The measurements from DIFFERENT satellites have the SAME accuracy.**
- **All the measurements are statistically independent in time and in space.**

<u>These assumptions are not realistic!</u>

<u>Why not?</u>



Courtesy: G Lachapelle

The measurements from different satellites contain different errors, and therefore, will not have the same accuracy

How to Estimate the Parameters in Stochastic Models

Functional model : $l + v = Ax$

Decomposition of Covariance Matrix : $C_l = \sum_{1}^{m} \theta_i T_i$, where θ_i will be estimated.

For instance, in case of 3 DD observations in each epoch. The covariance matrix in each epoch can be decomposed as the following:

$$\mathbf{C} = \sigma_{11}^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \sigma_{22}^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \sigma_{33}^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$+ \sigma_{12}^2 \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \sigma_{13}^2 \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} + \sigma_{23}^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

MINQUE (Minimum Norm Quadratic Unbiased Estimation) can be used to estimate variance covariance components.

The estimated covariance matrix (mm*mm) for the one GPS baseline data set:

DD obs. :	18-4	18-14	18-19	18-24	18-27	18-29
	6.861	-0.718	2.391	-1.132	1.601	4.801
		33.599	-0.046	-5.252	-4.207	-20.480
			5.405	3.350	1.485	0.952
				29.400	5.684	14.085
	(symmetric)				9.832	8.940
						34.951

Var(18-29)/Var(18-19) = **5** !!

Correlation coefficient (18-14, 18-29) = **-0.60** !!

Effects of the Estimated Covariance Matrix on AR

Satellite Pairs :	18-4	18-14	18-19	18-24	18-29	18-29	
True Ambiguities :	4	0	-50	8	-6	1	
***With commonly used method*:**							
Amb. Estimates :	3.79	0.45	-49.89	7.55	-6.03	1.20	(cycles)
Std. Deviations :	0.13	0.18	0.08	0.21	0.10	0.10	(cycles)
F-test Ratio :	5.70						
***With the proposed method*:**							
Amb. Estimates :	3.97	0.17	-50.02	7.97	-5.99	1.12	(cycles)
Std. Deviations :	0.06	0.08	0.04	0.11	0.06	0.05	(cycles)
F-test Ratio :	12.64						

But no temporal correlation has been taken into account so far!

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The new modelling method for static positioning is based on a novel error analysis framework

proposed by Wang, Satirapod and Rizos (2002), which takes into account:

Diversity in accuracy - hetero-scedastic
Correlation in time - temporal
Correlation in space - spatial

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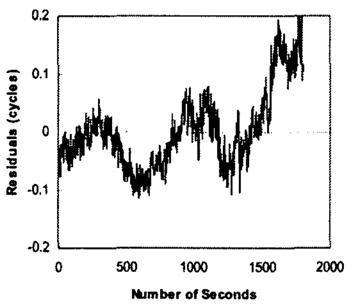
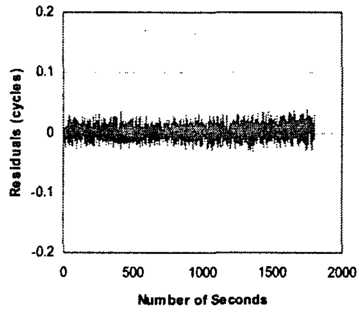
Stochastic Modeling Results



Residuals using the existing modelling method



Residuals using the proposed modelling method

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Impact of MP Errors on AR (with a normal stochastic model)

MP Errors	Accepted /Incorrect				
	F-ratio	Waratio		Wsratio	
		95%	99%	95%	99%
Free	0/0	0/0	0/0	0/0	0/0
SV03	**25/59**	**23/59**	**10/59**	**24/59**	**11/59**
SV06	0/3	0/3	0/3	0/3	0/3
SV17	0/4	0/4	0/4	0/4	0/4
SV21	0/3	0/3	0/3	0/3	0/3
SV23	**0/31**	**2/31**	**0/31**	**1/31**	**0/31**

(for single epoch solutions with baseline length of 5km and data interval : 1second)

Impact of MP Errors on AR– Improved Results (with the new stochastic modelling method)

MP Errors	Accepted / Incorrect					
	F-ratio	Wa (95% / 99%)		Ws (95% / 99%)		
SV 03	**14/46**	**14/46**	**6/46**	**8/46**	**3/46**	11/59 !
SV 06	0/0	0/0	0/0	0/0	0/0	
SV 17	0/0	0/0	0/0	0/0	0/0	
SV 21	0/0	0/0	0/0	0/0	0/0	
SV 23	**0/2**	**0/2**	**0/2**	**0/2**	**0/2**	1/31 !

Stochastic Modelling for Kinematic Positioning

- **An adaptive procedure for estimating the covariance matrix of measurement models has been proposed.**

  – Basic idea

  The filtering residuals (not innovations) from the previous epochs to estimate the covariance matrix for the current epoch.

  – Elements

  a) Derive a suitable formula for constructing the covariance matrix;

  b) Select an optimal window size (application dependent).

Online Stochastic Modelling Procedure

The covariance matrix may be constructed using the 'covariance matching' method

$$v_{z_k} = z_k - H_k \hat{x}_k$$

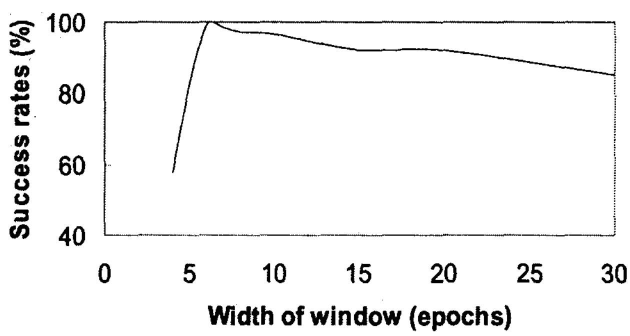
$$Q_{v_{z_k}} = \frac{1}{m}\sum_{i=0}^{m-1} v_{z_{k-i}} v_{z_{k-i}}^T \cong R_k - H_k Q_{\hat{x}_k} H_k^T$$

$$\hat{R}_k = H_k Q_{\hat{x}_k} H_k^T + \frac{1}{m}\sum_{i=0}^{m-1} v_{z_{k-i}} v_{z_{k-i}}^T$$

Experiment: GPS/GLONASS Integration

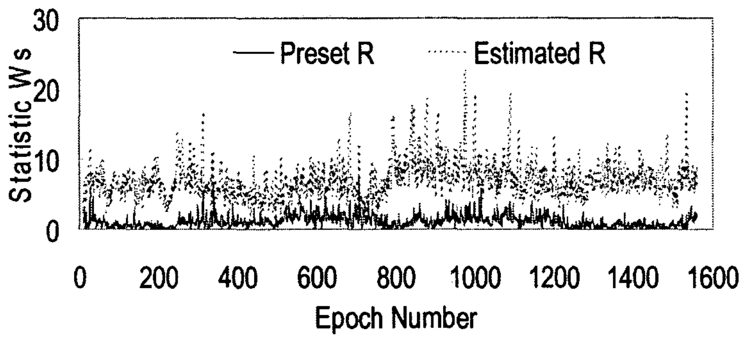
- **Optimal window size**

  (selection criterion: success rate of ambiguity resolution)



Experiment: GPS/GLONASS Integration

➢ **Ambiguity separability test results: W-ratio values**



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Challenging Problems

Practical Problems

- ➢ **Modelling temporal correlations on-line;**
- ➢ **Simultaneously estimating both Q and R covariance matrices;**
- ➢ **On-line determination of the optimal window size.**

Theoretical Problems

- ➢ **Statistical properties of the estimators for the covariance matrices**

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