Image Reconstruction Techniques for Radioactive Waste Assay by Tomographic Gamma Scanning Method

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Abstract

The tomographic gamma scanner (TGS) method, a further of extension of segmented gamma scanner (SGS), is most accurate and precise for assaying heterogeneous drummed nuclear radioactive waste; it is widely used in nuclear power plants and radioactive waste storages and disposal sites. The transmission and emission images are reconstructed by image reconstruction techniques. In the paper, the principle of TGS is introduced; image reconstruction techniques are discussed as well; finally, it is demonstrated that TGS method performance.

Key words: Tomographic Gamma Scanner (TGS), Image reconstruction, radioactive waste

1. Introduction

Over the years, there are a number of radioactive wastes produced by different nuclear facilities in Korea, mainly from nuclear power plants. These radioactive wastes are packed into waste drums to await further treatment, storage, and final disposal. In Korea, as in the entire world, there are very strict regulations and the national waste acceptance criteria for management of nuclear waste and their sites. Contents of these drums must be characterized before they are transported for permanent disposal. The problems of radioactive waste analysis and disposal present an enormous global challenge.

To characterize waste means that the determination of the physical, chemical and radiological properties of the waste to establish the need for further adjustment, treatment, conditioning, or its suitability for further handling, processing, storage or disposal. Waste items in inventory have different matrix densities and compositions, different radionuclide and isotopic compositions, different physical, chemical and distributional forms of radioactivity. Opening the sealed drums for analyzing is time-consuming and prohibitively expensive, so, the traditional preferred methods for waste assay are non-destructive assay (NDA). Tomographic gamma scanner (TGS) method, one the gamma-based spectroscopy NDA methods, improves on the well-established segmented gamma scanner (SGS) by performing 3-dimensional transmission and emission scanning, the content matrix within the waste drum is determined form transmission measurement data by using tomographic image reconstruction technique, similar to the transmission image reconstruction, the emission image, radionuclides distribution within the waste drum, is obtained from emission measurement data by using tomographic image reconstruction technique.

Tomography technique has been used for radioactive waste characterization for many years. Los Alamos National laboratory (LLNL) [1] and Lawrence Livermore National Laboratory (LLNL) [2] have developed this technique for use in safeguards and radwaste assay, and transferred into companies to manufacture TGS instruments successfully which have been played an important role in a number of nuclear radwaste sites.

In this paper, the principle of TGS is introduced, image reconstruction techniques for the radwaste image within the sealed drum are discussed in detail, finally, the images within the proposed drum model are reconstructed by TGS method with we developed software under MATLAB platform.

2. Mathematical description of TGS principle

TGS method includes transmission and emission measurements. The former is used to develop a three-dimensional spatial map of the attenuation coefficient of radwaste matrices by using an external transmission source. By interpolating and extrapolating the transmission data, spatial maps of the attenuation coefficient can be obtained at any energy. The latter uses transmission-corrected, single-photon emission, computerized tomography (SPECT) to determine the spatial distribution and quantity of radionuclides within a drum.

High-resolution gamma-ray spectroscopy is used to make accurate measurements of individual gamma rays in complex spectra.

In a TGS scan, the sample is divided into several axial sections (or layers), depending on the height of the sample (e.g. 16 layers for 208L drum). Within each layer, the sample is rotated and translated. Transmission and emission data are stored and are used to reconstruct the attenuation coefficient image and emission distribution.

From the point of tomography view, the determination of the contents within the radwaste drum from measurement data is the reconstruction from projections. These two processes, transmission tomography and emission tomography, are referred to as reconstruction. Reconstruction may be used to locate, identify, and quantify all detectable radioisotopes in the drum. These assay data are used to calculate a few parameters, for example, the thermal power, total alpha activity, necessary for the characterization of the radwaste. In the following sections, the mathematical bases of transmission and emission are given.

2.1. Transmission equation

For simplicity, we use voxel model as a basis for describing the transmission and emission equations. Conventional transmission computed tomography (CT) scanner measure the effects of an object on an incident ray that travels in a straight path. TGS transmission measurement is shown in Fig.1. I_0 is the photon count of incident gamma ray from transmission source, I_i is the photon count in the i'th transmission measurement.

By using the Beer's law, the equation maybe written as

$$I_i = I_0 \exp[-\sum_{j=1}^{N_v} T_{ij}\mu_j] \qquad (1)$$

Let $p_i = \frac{I_i}{I_0}$ and define the logarithmic transmission V_i by the following relation

$$V_i = -\ln(p_i) \qquad (2)$$

Combing (1) and (2), transmission equation is formulated as follows

$$V_i = \sum_{j=1}^{N_v} T_{ij}\mu_j \text{ , i=1, 2, ..., } N_d \qquad (3)$$

Where T is thickness matrix, each element T_{ij} is the linear thickness of the j'th voxel along a ray connecting the transmission source and the detector in the i'th measurement position. μ_j is the linear attenuation coefficient of the j'th voxel. V_i is transmission measurement data in the i'th measurement position. N_d is the number of detector positions and N_v is the number of voxels in the drum.

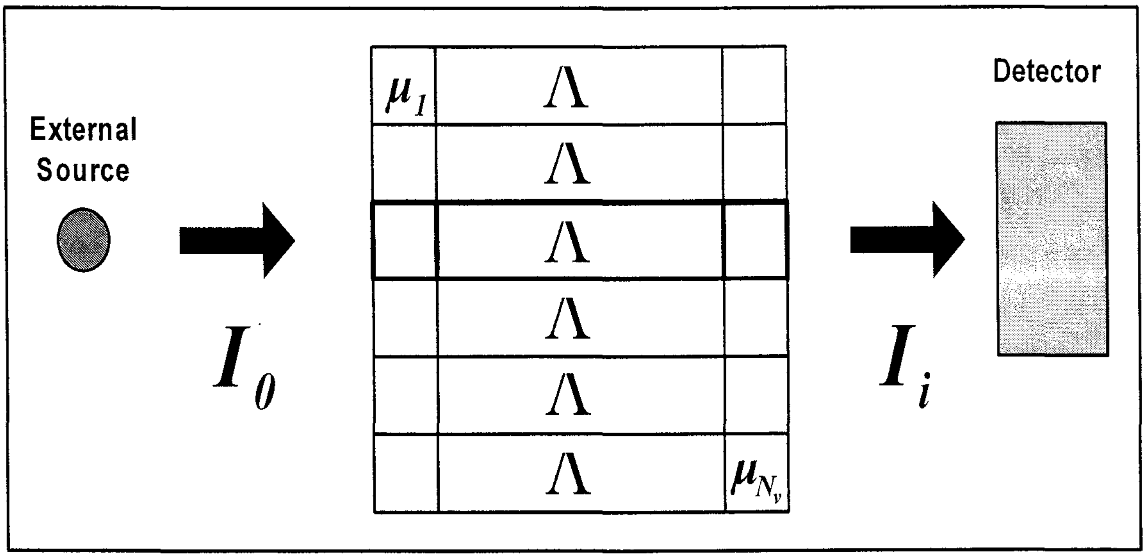

Fig.1. Schematic Principle of Transmission TGS

From all the transmission data, complete transmission projections can be determined. The problem of finding the μ_j from transmission projections is solved by image reconstruction methods.

2.2. Emission equation

As described earlier, the transmission image is to build gamma-ray attenuation corrections for the emission imaging problem. The TGS emission measures the identity and quantity (uncorrected for attenuation) of the gamma-ray emitting radionuclides within radwaste drum. As shown in Fig.2. The TGS transmission equation can be written as

$$D_i = \sum_{j=1}^{N_v} F_{ij} s_j \qquad (4)$$

Where s_j is the radioactivity in the j'th voxel of the drum, D_i is the counting rate in the i'th emission measurement position, F_{ij} is the system matrix element, it corresponds to the counting rate in the j'th emission measurement position for the unit radioactivity in the i'th voxel and is given by the relation

$$F_{ij} = E_{ij} \bullet A_{ij} \qquad (5)$$

Where E_{ij} is proportional to the probability that a photon (of the correct energy) emitted from the j'th voxel will be detected in the i'th measurement. A_{ij} is the fractional attenuation of photons emitted from the j'th voxel in the i'th emission measurement.

The values of A_{ij} are estimated from the transmission image by using Beer's law

$$A_{ij} = \prod_k \exp(-t_{ijk}\mu_k) \qquad (6)$$

Where the t_{ijk} is the linear thickness of the k'th absorbing voxel along a ray connecting the j'th emitting voxel and the detector in the i'th measurement position. The μ_j is determined from transmission TGS.

The emission image is found as the solution of the transmission equation by image reconstruction methods.

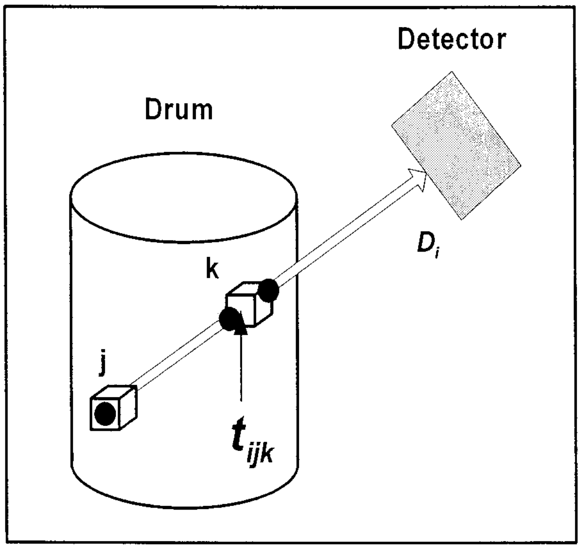

Fig.2. Schematic Principle of Emission TGS

3. Image reconstruction algorithms [3]

Image reconstruction is the process of estimating an image from a set of projections. It is well-known that there are two main types of tomographic algorithms: analytic and iterative. The analytic tomographic algorithms are based on the Radon transform introduced by Radon in 1917, but the applicability for tomography was not discovered until the 1960's. The algorithm that is currently being used in almost all applications of straight ray tomography is the filtered backprojection algorithm (FBP). The iterative reconstruction technique, also known as algebraic reconstruction technique-ART, was the original reconstruction method used to solve the tomographic problem, currently, it is getting more and more utilization, especially, in tomographic gamming scanning system of nuclear radwaste. In this part, we introduce and discuss FBP and ART algorithms.

3.1. FBP algorithm

Radon transform and Fourier slice theorem are the bases of FBP algorithm, they are described below.

3.1.1. Radon transform

The Radon transform is known as the mathematical basis for tomographic imaging from projection data. A line integral represents the integral of some parameter of the object along a line. In transmission TGS, the parameter of the drum is linear attenuation coefficient. In this case the object is modeled as a two-dimensional distribution of the gamma-ray attenuation constant and a line integral represents the total attenuation suffered by a beam of gamma-rays as it travels in a straight line through the object.

We will use the coordinate system defined in Fig.3. to describe line integrals and projections. In this example the object is represented by a two-dimensional function $f(x,y)$ and each line integral by the (θ, t) parameters. Function $f(x,y)$, its line integral, or Radon transform, denoted as $P_\theta(t)$.

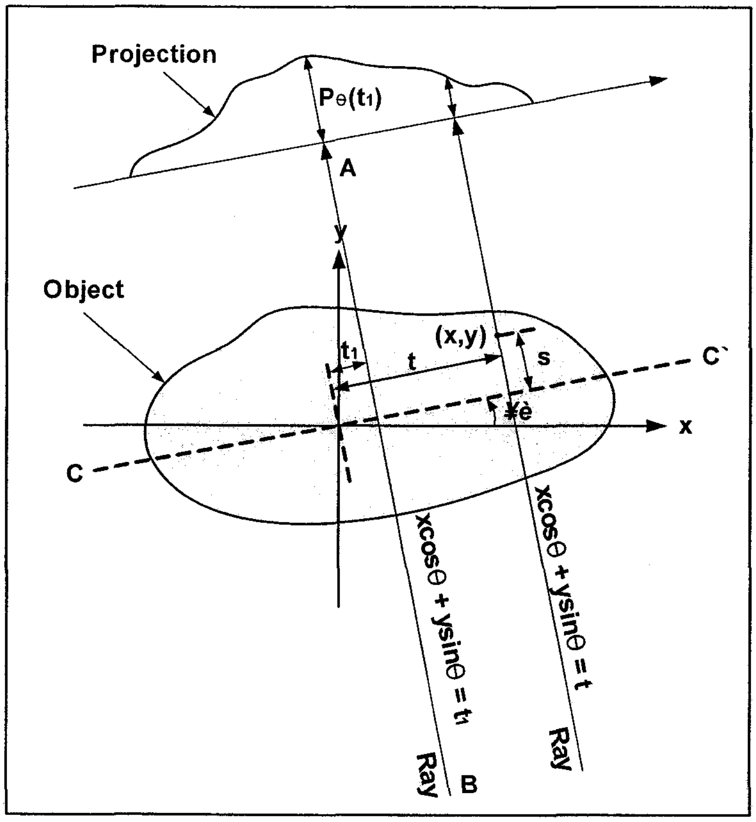

Fig.3. Coordinate System of Tomography

The equation of line L is defined by

$$x\cos\theta + y\sin\theta = t \qquad (7)$$

The line integral of $P_\theta(t)$ defined as

$$P_\theta(t) = \int_L f(x,y)ds \qquad (8)$$

This can be rewritten as follows by using a delta function

$$P_\theta(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(x\cos\theta + y\sin\theta - t)dxdy \qquad (9)$$

The function $P_\theta(t)$ is known as the Radon transform of the function $f(x,y)$.

A projection is formed by combining a set of line integrals, that is to say, the determination of $P_\theta(t)$ for a fixed value of θ is called a projection. A complete set of projections at all θ is called a sinogram.

3.1.2. The Fourier Slice Theorem

The Fourier Slice Theorem is stated as:

The Fourier transform of a parallel projection of an image $f(x,y)$ taken at angle θ gives a slice of the two-dimensional transform, $F(u,v)$, subtending an angle θ with the u-axis. In other words, the Fourier transform of $P_\theta(t)$ gives the values of $F(u,v)$ along line AA in Fig.4. formula expression as

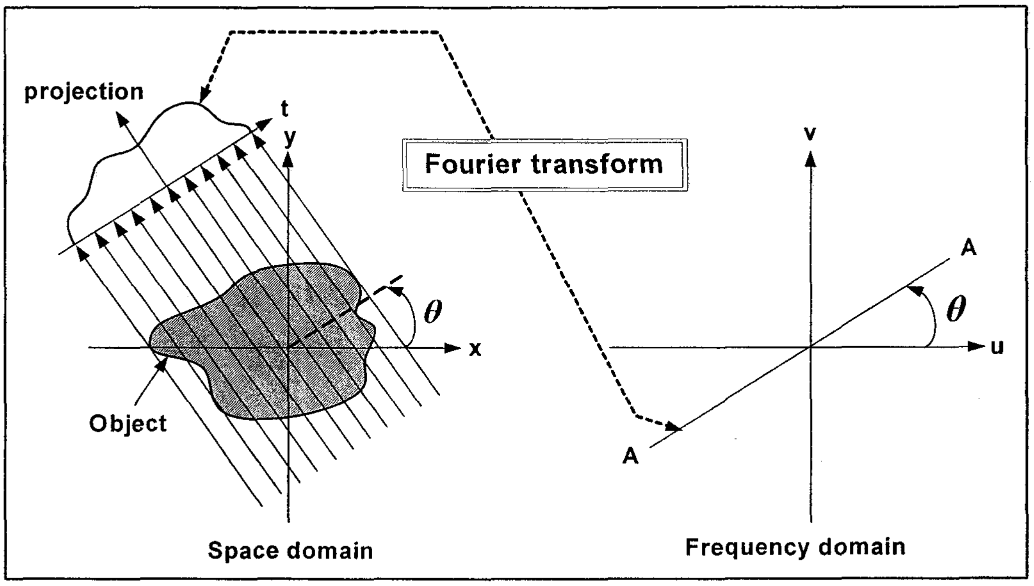

Fig.4.The Fourier Slice Theorem relates the Fourier transform of a projection to the Fourier transform of the object along a radial line.

$$F_1 P_\theta(t) = F(u,v) \qquad (10)$$

The Fourier Slice Theorem establishes quantitatively relationship between the representation of an object in the projection domain (sinogram domain) and its representation in a spatial frequency domain.

3.1.3. FBP algorithm

Due to the Fourier Slice Theorem, measuring the projections all around the object is equivalent to measuring the 2D Fourier Transform of the object using a polar coordinate system. Thus, the representation of the object in direct space can be obtained by inverting this frequency space representation, being aware that the inverse Fourier Transform has to be applied in a Cartesian coordinate system. Therefore, a Jacobean must be introduced to hold for the change of variables in the frequency space from polar coordinates to Cartesian coordinates. This ends up in multiplying the frequency space representation of the object obtained from the 1D Fourier Transform of the measured projections by the absolute value of the frequency. In other words, a ramp filter is applied to the measured projections, and the representation of the object in the direct space is obtained after backprojection of these filtered projections onto the lines of projections. This is so called filtered backprojection (FBP) algorithm, it is implement in three steps: finding the Fourier transform in 1D of the projections; finding the filtered projections; finding the back projections.

3.2. ART algorithm

The method was published by Gordon, Bender and Herman in 1970 for the first time and is called Algebraic Reconstruction Technique (ART). ART is essentially the application of Kaczmarz's method to the specific problems of computer tomography. Various variations of the algorithm were developed.

This algorithm is entirely differ transform-based algorithms (e.g. FBP). The algorithm is for imaging consists of assuming that the cross section consists of an array of unknowns, and then setting up algebraic equations for the unknowns in terms of the measured projection data, finally, obtaining the solution of the algebraic equations.

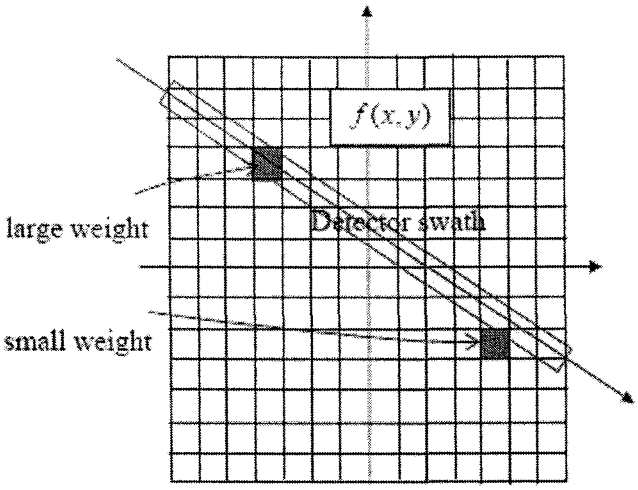

Fig.5.The Schematic Discretize Object in ART Method

In Fig.5. the object image $f(x,y)$ has been divided into a number of voxel (square grid), it is assumed that in each voxel the function is constant denoted as f_j in the j'th voxel. Let p_i be the projection measured in i'th measurement. The relationship between f_j's and p_i's may be expressed as

$$\sum_{j=1}^{N} w_{ij} f_j = p_i \qquad \mathbf{i=1, 2, \ldots, M} \qquad (11)$$

where N is the total number of voxels, M is the total number of measurements data and w_{ij} is the weighting factor that represents the contribution of the j'th voxel to the i'th ray integral.

The ART is to find a solution by successive estimates. The projections corresponding to the current estimate are compared with the measured projections. The result of the comparison is used to modify the current estimate, thereby creating a new estimate. The algorithms differ in the way the measured and estimated projections are compared and the kind of correction applied to the current estimate. The process is initiated by arbitrarily creating a first estimate.

For illustration purpose, one iterative process is given by:

$$f_j^{(k)} = f_j^{(k-1)} - R_p \frac{w_{ij}}{\sum_j (w_{ij})^2}(-p_i + \sum_j w_j w_{ij}) \qquad (12)$$

The computational steps of ART are expressed as:

(1) initialize a solution
(2) calculate the projection for initial solution
(3) compute the difference between the calculated and measured projection data
(4) upgrade the image by iterative equation (12)

4. Simulation results and validation [4]

In the absence of experimental conditions, simulation research is an appropriate method to research TGS. An essential goal of this study is to obtain preliminary experience in the field of tomography algorithms for TGS in order to obtain the conceptional knowledge for further development of TGS techniques in practical for the radwaste characterization.

We have developed the simulation platform based on the MATLAB environment with three functions, the first one is for simulating transmission and emission TGS measurements, that is to create database of transmission and emission TGS measurements data for arbitrary matrix (represented as linear attenuation coefficient) and radioactive intensity distributions within the simulated models; the second one is to reconstruct the transmission and emission images form generated TGS database or real experimental data by image reconstruction techniques that we described former sections; the third is to validate and verify the TGS simulation methodology through comparing the differences between the reconstructed TGS images and references values.

We list some simulation results and verification results partially. The preliminary results are to show the feasibility and reliability of simulation methodology.

4.1. Simulation results

In our transmission simulation, a mathematical model of object is created, an external transmission source is generated, and corresponding measurement date of a detector is calculated by the simulation platform.

The simulation sample model is proposed under considering some certain real conditions of radwaste drums. 3 by 3 by 3 model as TGS object model is divided into three layers that each layer has nine voxels, each voxel of 5cm×5cm ×5cm, external source is located left side of sample, detector is in the opposite of external source. It is 26.2cm from external source to the center of sample, the distance of detector to the center of sample is 43.9cm. Fig.6. is the schematic illustration of transmission TGS system.

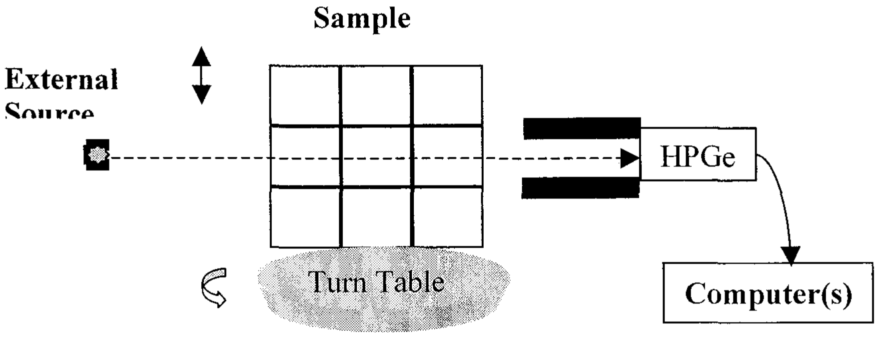

Fig.6. Schematic Illustration of Transmission TGS

The linear attenuation coefficients μ of five medium (iron, aluminum, plastic, air, and lead) for one certain gamma-ray energy are pre-set randomly in the model sample voxels. TGS transmission measurement data was performed to obtain one group transmission rate which corresponded to one group μ distribution by MC simulation in established simulation platform. In each layer, scanning pattern is 3 translation positions with four rotation angles, 0^0, 45^0, 90^0, 135^0, in each position. The ART algorithm is used to reconstruct the μ of the matrices in the voxels. The reconstructed images of the three layers are listed in Figs.7. 8. 9 respectively. The relative errors are found to be less than 2%.

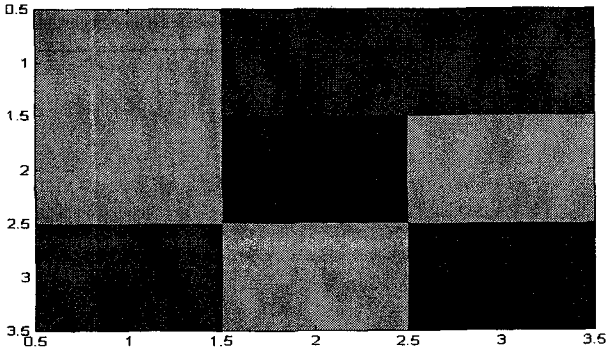

Fig.7.The Reconstructed Image of the First Layer

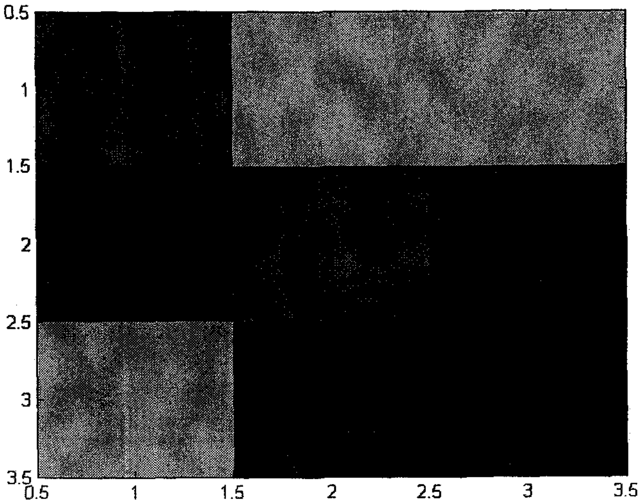

Fig.8.The Reconstructed Image of the Second Layer

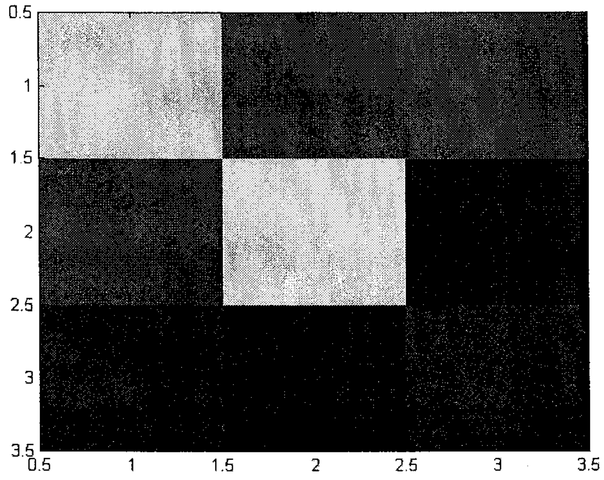

Fig.9.The Reconstructed Image of the Third Layer

4.2. Experimental verification results

Four components constitute a TGS scanner. A radioactive source is used to acquire transmission attenuation data. Both the transmission and emission TGS measurements require an energy discriminating detector; we used a HPGe detector that type No. is GR3019. A hand-controlled (will be changed into computer-controlled) stage translates and rotates a drum; and elevates either the drum, or the source and detector together, to enable data acquisition in a CT fashion. A computer carries out data acquisition, image processing, reconstruction and assay calculations.

External transmission source used to acquire the attenuation map; HPGe detector used in both the transmission and emission modes; staging system is for manipulating the drum or source; computer system used to acquire data, process data, reconstruct images, and control TGS scanning.

The detailed description about experimental verification is in reference [4]. Some of results are list as follows.

ART algorithm was used to perform the reconstruction calculation for the experimentally obtained transmission rates and to calculate the linear attenuation coefficients of the different voxel media. The experimentally reconstructed values of linear attenuation coefficients were compared to the reference values. It is shown that, for the attenuation coefficients of iron and aluminum in the second layer and lead in the third layer, the relative errors between reconstructed values and the reference values were less than 4%.

The experimentally measured data of linear attenuation coefficients obtained above for transmission measurement media. The corrected experimental results were compared to the standard values and the relative deviation was found to be 10%.

The preliminary results we obtained for the simple TGS model show that TGS technology can locates, identifies and quantifies gamma-ray emitting isotopes in the sealed container. The image reconstruction algorithms for complicated and larger radwaste drum are developing. The hardware and software for TGS system are under consideration and development.

Acknowledgements

The author (Zhang) thanks CIAE TGS group members for their contributions during the verification experiments.

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