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A Note on Intuitionistic Fuzzy Subgroups

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Abstract

In this paper, We discuss various types of sublattice of the lattice of intuitionistic fuzzy subgroups of a given group. We prove that a special class of intuitionistic fuzzy normal subgroups constitutes a modular sublattice of the lattice of intuitionistic fuzzy subgroups. Moreover, we exhibit the relationship of the sublattices of the lattice of intuitionistic fuzzy subgroups.

1. Introduction

In 1965, L. A. Zadeh[21] introduced the concept of a fuzzy set. After that time, it has been a tremendous interest in the subject due to its diverse application ranging from engineering and computer science to social behavior studies. In particular, several researchers have applied he notion of fuzzy sets to group theory [1],[8],[19],[20].

In 1986, Atanassov[2] introduced the concept of intuitionistic fuzzy sets as the generalization of fuzzy sets. Since then, Coker *et al.*[6],[7],[10] lee *et al.*[18], and Hur *et al.*[13] applied the notion of intuitionistic fuzzy sets to topology. In particular, Hur *et al.*[15] applied one to topological group. Moreover, many researchers[3],[4],[11],[12],[14],[16],[17] applied the concept of intuitionistic fuzzy sets to algebra.

In this paper, we discuss some interesting sublattices of the lattice of intuotionistic fuzzy subgroups of a group. *As a main result of our paper, we prove the set of all* intuitionistic fuzzy normal subgroups with finite range and attaining the same value at the identity element of the group forms a modular sublattice of the lattice of intuitionistic fuzzy subgroups. In fact, it is an intuitionistic fuzzy version of a well-known result of classical lattice theory. Finally, though a lattice diagram we exhibit the interrelationship of the sublattices of the lattice of intuitionistic fuzzy subgroups discussed here.

2. Preliminaries

We will list some concepts and some results needed in the later sections.

For sets X, Y and Z, $f=(f_1, f_2): X \to Y \times Z$ is called a *complex mapping* if $f_1 : X \to Y$ and $f_2 : X \to Z$ are mapping. Throughout this paper, we will denote the unit interval [0,1] as I. And for an ordinary subset A of a set X, we will denote the characteristic function of A as χ_A.

Definition 1[2],[6]. Let X be a non-empty set. A complex mapping $A=(\mu_A, \nu_A): X \to I \times I$ is called an *intuitionistic fuzzy set*(in short, IFS) in X if $\mu_A(x)+\nu_A(x) \leq 1$ for each $x \in X$, where the mapping $\mu_A: X \to I$ and $\nu_A: X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership(namely $\nu_A(x)$) of each $x \in X$ to A, respectively, In particular, $0_\sim$ and $1_\sim$ denote the intuitionistic fuzzy empty set and the intuitionistic fuzzy hole set in a set X defined by $0_\sim(x)=(0,1)$ and $1_{SIM}(x)=(1,0)$ for each $x \in X$, respectively. We will denote the set of all IFSs in X as IFS(X).

Definition 2[2]. Let X be non-empty sets and $A=(\mu_A, \nu_A)$ and $B=(\mu_B, \nu_B)$ be an IFSs in X. Then

(1) $A \subset B$ if and only if $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$.

(2) $A=B$ if and only if $A \subset B$ and $B \subset A$.

(3) $A^c=(\nu_A, \mu_A)$.

(4) $A \cap B=(\mu_A \wedge \mu_B, \nu_A \vee \nu_B)$.

(5) $A \cup B=(\mu_A \vee \mu_B, \nu_A \wedge \nu_B)$.

(6) $[]A=(\mu_A, 1-\mu_A)$, $< \; > A=(1-\nu_A, \nu_A)$.

Definition 3[6]. Let $\{A_i\}_{i \in J}$ be and arbitrary family of IFSs in X, where $A_i=(\mu_{A_i}, \nu_{A_i})$ for each $i \in J$. Then

(1) $\cap A_i=(\wedge \mu_{A_i}, \vee \nu_{A_i})$

(2) $\cup A_i=(\vee \mu_{A_i}, \wedge \nu_{A_i})$

Definition 4[11]. Let $(X, \cdot)$ be a groupoid and let $A \in IFS(X)$. Then A is called an *intuitionistic fuzzy subgroup*(in short, IFGP) of X if for any $x, y \in X$, $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$

Definition 5[12]. Let G be a group and let $A \in IFS(G)$ of G if ti satisfies the following conditions:

(1) A is an IFGP of G

(2) $\mu_A(x^{-1}) \geq \mu_A(x)$ and $\nu_A(x^{-1}) \geq \nu_A(x)$ for each $x \in G$

We will denote the set of all IFGs of G as IFGG.

Result 1[12, Proposition 2.3]. Let G be a group and let $\{A_\alpha\}_{\alpha\in\Gamma}\subset IFG(G)$. Then $\cap_{\alpha\in\Gamma}A_\alpha\in IFG(G)$.

Result 2[12, Proposition 2.6]. Let G be a group and let $A,B\in IFG(G)$. Then $A(x^{-1})=A(x)$, $\mu_A(x)\le\mu_A(e)$ and $\nu_A(x)\ge\nu_A(e)$ for each $x\in G$, where e is identity element of G.

Result 3[12, Proposition 2.2]. Let G be a group and let $A\subset G$. Then A is a subgroup of G if and only if $(\chi_A,\chi_{A^c})\in IFG(G)$.

Definition 6[11]. Let A be an IFS in a set X and let $(\lambda,\mu)\in I\times I$ with $\lambda+\mu\le1$. Then the set $(A^{\lambda,\mu})=\{x\in X:\ \mu_A(x)\ge\lambda$ and $\nu_A(x)\le\mu\}$ is called a (λ,μ) *-level subset* of A.

3.Lattices of Intuitionistic Fuzzy Subgroups

In this section, we study the lattice the structure of the set of intuitionistic fuzzy subgroups of a given group. From Definition 1, 2 and 3, we can see that for a set X , IFS(S) forms a complete lattice under the usual ordering of intuitionistic fuzzy inclusion $\subset$, where the inf and the sup are the intersection and the union of intuitionistic fuzzy sets, respectively. To construct the lattice of intuitionistic fuzzy subgroups, we define the inf of a family A_α of intuitionistic fuzzy subgroups to be intersection $\cap A_\alpha$. However, the sup is defined as the intuitionistic fuzzy subgroup generated by the union $\cup A_\alpha$ and denoted by $(\cup A_\alpha)$. Thus we have the following result.

Proposition 1. Let G be a group. Then IFG(G) forms a complete lattice under the usual ordering of intuitionistic fuzzy set inclusion $\subset$.

Proof. Let $\{A_\alpha\}_{\alpha\in\Gamma}$ be any subset of IFG(G). Then, by Result 1, $\cap_{\alpha\in\Gamma}\in IFG(G)$. Moreover, it is clear that $\cap_{\alpha\in\Gamma}A_\alpha$ is the largest intuitionistic fuzzy subgroup contained in A_α for each $\alpha\in\Gamma$. So $\wedge_{\alpha\in\Gamma}A_\alpha$. On the other hand, $(\cup_{\alpha\in\Gamma}A_\alpha)$ is the least intuitionistic fuzzy subgroup containing A_α for each $\alpha\in\Gamma$. So $\wedge_{\alpha\in\Gamma}A_\alpha=(\cap_{\alpha\in\Gamma}A_\alpha)$. Hence IFG)G) is a complete lattice.

Next we construct certain sublattice of the lattice IFG(G). In fact, these sublattices reflect ceratin peculiarities of the intuitionistic fuzzy setting. For a group G, Let $IFG_f(G)=\{A\in IFG(G)\ :\ Im\ A\ is\ finite\}$ and let $IFG_{(s,\ t)}(G)=\{A\in IFG(G)\ :\ A(e)=(s,\ t)\}$, where e is identity of G. Then it is clear that $IFG_{(s,\ t)}(G)[resp.IFG_{(s,\ t)}(G)]$ is sublattice of IFG(G). Moreover, $IFG_f(G)\cap IFG_{(s,t)}(G)$ is also a sublattice of IFG(G).

Now to obtain our main results, we start with following lemma.

Lemma 1. Let G be a group and let $A,B\in IFG(G)$. Then for each $(\lambda,\mu)\in I\times I$ with $\lambda+\mu\le1$, $A^{(\lambda,\mu)}\cdot B^{(\lambda,\mu)}\subset(A\circ B)^{(\lambda,\mu)}$.

Proof. Let $z\in A^{(\lambda,\mu)}\cdot B^{(\lambda,\mu)}$. Then there exist $x_0,y_0\in G$ Then for each $(\lambda,\mu)\in I\times I$ with $\lambda+\mu\le1$, $A^{(\lambda,\mu)}\cdot B^{(\lambda,\mu)}$. Then there exist $x_0,y_0\in G$ such that $z=x_0y_0$. Thus $\mu_A(x_0)\ge\lambda$, $\nu_A(x_0)\le\mu$ and $\mu_A(y_0)\ge\lambda$, $\nu_A(y_0)\le\mu$. So
$\mu_{A\circ B}(z)=\vee_{z=xy}[\mu_A(x)\wedge\mu_B(y)]\ge\mu_A(x_0)\wedge\mu_B(y_0)\ge\lambda$
and
$\nu_{A\cdot B}(z)=\wedge_{z=xy}[\nu_{A(x)}\vee\nu_{B(y)}]\le\nu_{A(x_0)}\vee\nu_B(y_0)\le\mu$.
Thus $z\in(A\circ B)^{(\lambda,\mu)}$ .Hence $A^{(\lambda,\mu)}\cdot B^{(\lambda,\mu)}\subset(A\circ B)^{(\lambda,\mu)}$.
The following is the converse of Lemma 1.

Lemma 2. Let G be a group and let $A,B\in IFG(G)$. If Im A and Im B are finite, then for each $(\lambda,\mu)\in I\times I$ with $\lambda+\mu\le1$, $(A\circ B)^{(\lambda,\mu)}\subset A^{(\lambda,\mu)}\cdot B^{(\lambda,\mu)}$.

Proof. Let $z\in(A\cdot B)^{(\lambda,\mu)}$. Then

$$\mu_{(A\cdot B)}(z)=\vee_{z=xy}[\mu_A(x)\wedge\mu_B(y)]\ge\lambda$$

and

$$\mu_{(A\cdot B)}(z)=\wedge_{z=xy}[\mu_A(x)\vee\mu_B(y)]\le\mu$$

Since Im A and Im B are finite, there exist $x_0,y_0\in G$ with $z=x_0y_0$ such that

$$\vee_{z=xy}[\mu_A(x)\wedge\mu_B(y)]=\mu_A(x_0)\wedge\mu_B(y_0)\ge\lambda$$

and

$$\wedge_{z=xy}[\mu_A(x)\vee\mu_B(y)]=\mu_A(x_0)\vee\mu_B(y_0)\le\mu$$

Thus $\mu_A(x_0)\ge\lambda,\nu_A(x_0)\le\mu$ and $\mu_B(y_0)\ge\lambda,\nu_B(y_0)\le\mu$. So $x_0\in A^{(\lambda,\mu)}$ and $y_0\in B^{(\lambda,\mu)}$, i.e., $z=x_0y_0\in A^{(\lambda,\mu)}\cdot B^{(\lambda,\mu)}$. Hence $(A\circ B)^{(\lambda,\mu)}\subset A^{(\lambda,\mu)}\cdot B^{(\lambda,\mu)}$
This completes the proof.

The following is the immediate result of Lemmas 1 and 2.

Proposition 2. Let G be a group and let $A,B\in IFG(G)$. IF Im A and Im B are finite, then for each $(\lambda,\mu)\in I\times I$ with $\lambda+\mu\le1$, $(A\circ B)^{(\lambda,\mu)}=A^{(\lambda,\mu)}\cdot B^{(\lambda,\mu)}$.

Definition 7[12]. Let G be a group and let $A\in IFG(G)$. Then A called an *intuitionistic fuzzy normal subgroup* (in short, IFNG) of G if $A(xy)=A(yx)$ for any $x,y\in G$.

We will denote the set of all IFNGs of G as IFNG(G). It is clear that if G is abelian, then every IFG of G is an IFNG of G.

Result 4[16, Proposition 2.13]. Let G be a group, let $A\in IFG(G)$ and let $(\lambda,\mu)\in I\times I$ with $\lambda+\mu\le1$. Then $A^{(\lambda,\mu)}G$, where $A^{(\lambda,\mu)}G$ means that $A^{(\lambda,\mu)}G$ is a normal subgroup of G.

Result 5[16, Proposition 2.18]. Let G be a group and let $A\in IFG(G)$. if $A^{(\lambda,\mu)}G$ for each $(\lambda,\mu)\in Im\ A$, Then $A\in IFNG(G)$.

The following is the immediate result of Result 4 and 5.

Theorem 1. Let G be a group and let $A\in IFG(G)$. Then $A\in IFNG(G)$ if and only if for each $(\lambda,\mu)\in ImA$, $A^{(\lambda,\mu)}G$.

Result 6[12, Proposition 3.3]. Let G be a group and let $A \in IFNG(G)$. If $B \in IFG(G)$, then $B \circ A \in IFG(G)$

The following is the immediate result of Result 1 and Definition 7.

Proposition 3. Let G be a group and let $A, B \in IFNG(G)$. Then $A \cap B \in IFNG(G)$.

It is well-known that the set of all normal subgroups of a group forms a sublattice of the lattice of its subgroups. As an intuitionistic fuzzy analog of this classical result we obtain the following result.

Theorem 2. Let G be a group and let $IFN_{f(s,t)}(G) = \{A \in IFNG(G)$: ImA is finite and $A(e) = (s,t)$ $\}$. Then $IFN_{f(s,t)}(G)$ is a sublattice of $IFN_f(G) \cap IFG_{(s,t)}(G)$. Hence $IFN_{f(s,t)}(G)$ is a sublattice of IFG(G).

Proof. Let $A, B \in IFN_{f(s,t)}(G)$. Then, by Result 6, $A \circ B \in IFG(G)$. Let $z \in G$.

Then

$$\begin{aligned}\mu_{A \circ B}(z) &= \vee_{z=xy}[\mu_A(x) \wedge \mu_B(y)] \geq \mu_A(z) \wedge \mu_B(e) \\ &= \mu_A(z) \wedge \mu_A(e) \qquad (Since\ A(e) = (s,t) = B(e)) \\ &= \mu_A(z) \qquad (By\ Result 2)\end{aligned}$$

and

$$\begin{aligned}\nu_{A \circ B}(z) &= \wedge_{z=xy}[\nu_A(x) \wedge \nu_B(y)] \leq \mu_A(z) \vee \mu_B(e) \\ &= \mu_A(z) \vee \mu_A(e) = \mu_A(z)\end{aligned}$$

Thus $A \subset A \circ B$. Similarly, we have $B \subset A \circ B$. Let $C \in IFG(G)$ such that $A \subset C$ and $B \subset C$. Let $z \in G$. Then

$$\begin{aligned}\mu_{A \circ B}(z) &= \vee_{z=xy}[\mu_A(x) \wedge \mu_B(y)] \\ &\leq \vee_{z=xy}[\mu_C(z) \wedge \mu_C(y)] \\ &\leq \mu_C(xy) \qquad (Since\ A(e) = (s,t) = B(e)) \\ &= \mu_C(z) \qquad (By\ Result 2)\end{aligned}$$

and

$$\begin{aligned}\nu_{A \circ B}(z) &= \wedge_{z=xy}[\nu_A(x) \wedge \nu_B(y)] \geq \wedge_{z=xy}[\mu_C(x) \vee \mu_C(y)] \\ &\geq \mu_C(xy) = \mu_C(z)\end{aligned}$$

Thus $A \circ B \subset C$. So $A \circ B \subset A \vee B$.

Now let $(\lambda, \mu) \in I \times I$ with $\lambda + \mu \leq 1$. Since $A, B \in IFNG(G)$, $A^{(\lambda,\mu)}G$ and $B^{(\lambda,\mu)}G$. Then $A^{(\lambda,\mu)} \circ B^{(\lambda,\mu)}G$. By Proposition 2, $(A \circ B)^{(\lambda,\mu)}G$. Thus, by Theorem 1, $A \circ B \in IFNG(G)$. So $A \vee B \in IFN_{f(s,t)}(G)$. From Proposition 3, it is clear that $A \wedge B \in IFNG(G)$. Thus $A \wedge B \in IFN_{f(s,t)}(G)$. Hence $IFN_{f(s,t)}(G)$ is a sublattice of $IFN_f \cap IFN_{(s,t)}(G)$, and therefore of IFG(G). This complete the proof.

The relationship of intuitionistic fuzzy sub-group discussed herein can be visualized by the lattice diagram in Figure 1.

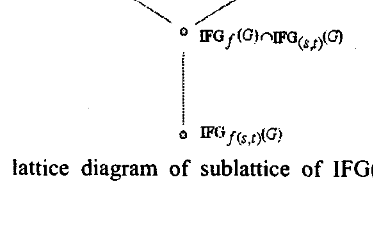

Fig. 1. The lattice diagram of sublattice of IFG(G)

It is also well-known[5, Theorem I.11] that the sublattice of normal subgroups of a group is modular. As an intutionistic fuzzy version to the classical theoretic result, we prove that $IFN_{(s,t)}(G)$ forms a modular lattice.

Result 7[17, Lemma 2.2]. Let G be a group and let $A \in$ IFG(G). If for any $x, y \in G, \mu_A(x) < \mu_A(y)$ and $\nu_A(x) > \nu_A(y)$, then $A(xy) = A(x) = A(yx)$.

Definition 8[5],[9]. A lattice $(L, \wedge, \vee)$ is said to be *modular* if for any $x, y, z \in L$ with $x \leq z$ [resp. $x \geq z$] $x \vee (y \wedge z) = (x \vee y) \wedge z$ [resp.. $x \wedge (y \vee z) = (x \wedge y) \vee z$].

In any lattice L, it is well-known [9, Lemma I.4.9] that for any $x, y, z \in L$ if $x \leq z$ [resp. $x \geq z$] ,then $x \vee (y \wedge z) = (x \vee y) \wedge z$ [resp.. $x \wedge (y \vee z) = (x \wedge y) \vee z$].

The inequality is called the *modular equality*.

Theorem 3. The lattice IFN$_{f(s,t)}(G)$ is modular.

Proof. Let $A, B, C \in$ IFN$_{f(s,t)}(G)$ such that $A \supset C$.

Then, by the modular inequality, $(A \wedge B) \vee C \subset A \wedge (B \vee C)$.

Assume that $(A \wedge B) \vee C \not\subset A \wedge (B \vee C)$, i.e., there exists $z \in G$ such that

$\mu_{A \wedge (B \vee C)}(z) > \mu_{(A \wedge B) \vee C}(z)$ and $\nu_{A \wedge (B \vee C)}(z) > \nu_{(A \wedge B) \vee C}(z)$.

Since Im B and C are finite, there exist $x_0, y_0 \in G$ with $z = x_0 y_0$ such that

$(B \vee C)(z) = (B \circ C)(z)$ (By the process of the proof of Theorem 2)

$$\begin{aligned}&= (\vee_{z=xy}[\mu_B(x) \wedge \mu_C(y)], \wedge_{z=xy}[\nu_B(x) \vee \nu_C(y)]) \\ &= \mu_B(x_0) \wedge \mu_C(y_0), \nu_B(x_0) \vee \nu_C(y_0))\end{aligned}$$

Thus

$$\begin{aligned}[A \wedge (B \vee C)](z) = (&\mu_A(z) \wedge (\mu_B(x_0) \wedge \mu_C(y_0)), \\ &\nu_A(z) \vee (\nu_B(x_0) \vee \nu_C(y_0))).(*)\end{aligned}$$

On the other hand,

$$\begin{aligned}&\nu_{(A \wedge B) \wedge}(z) = \wedge_{z=xy}[\nu_{A \wedge B}(x) \vee \nu_C(y)] \\ &\leq \mu_{A \wedge B}(x_0) \vee \mu_C(y_0) = \mu_A(x_0) \vee \mu_B(x_0) \vee \mu_C(y_0) \ (**')\end{aligned}$$

and

By (*),(**) and (**'),

$$\mu_A(z) \wedge \mu_B(x_0) \wedge \mu_C(y_0) > \mu_A(x_0) \wedge \mu_B(x_0) \wedge \mu_C(y_0)$$

and

$$\nu_A(z) \vee \nu_B(x_0) \vee \nu_C(y_0) < \nu_A(x_0) \wedge \nu_B(x_0) \wedge \nu_C(y_0)$$

Then

$$\mu_A(z), \mu_B(x_0), \mu_C(y_0) > \mu_A(x_0) \wedge \mu_B(x_0) \wedge \mu_C(y_0)$$

and

$$\nu_A(z), \nu_B(x_0), \nu_C(y_0) < \nu_A(x_0) \wedge \nu_B(x_0) \wedge \nu_C(y_0)$$

Thus

$$\begin{aligned}\mu_A(x_0) \wedge \mu_B(x_0) \wedge \mu_C(y_0) = \mu_A(x_0) \text{ and } &\nu_A(x_0) \vee \nu_B(x_0) \vee \nu_C(y_0) \\ &= \mu_A(x_0)\end{aligned}$$

So

$\mu_A(z) > \mu_A(x_0), \nu_A(z) < \nu_A(x_0)$, $\mu_C(z) > \mu_A(x_0), \nu_C(z) < \nu_A(x_0)$

By Result 2, $\mu_A(x_0^{-1}) = \mu_A(x_0) < \mu_A(x_0 y_0)$ and $\nu_A(x_0^{-1}) = \nu_A(x_0) > \nu_A(x_0 y_0)$.

By Result 7, $A(x_0) = A(y_0 x_0 x_0^{-1}) = A(y_0)$.

Thus $\mu_C(y_0) > \mu_A(y_0)$ and $\nu_C(y_0) < \nu_A(y_0)$. This contradicts the fact that $A \supset C$. So $A \wedge (B \vee C) \subset (A \wedge B) \vee C$.

Hence $A \wedge (B \vee C) = (A \wedge B) \vee C$. Therefore IFN$_{f(s,t)}(G)$ is modular. This complete the proof.

We discuss some interesting facts concerning a special class of intuitionistic fuzzy subgroups that attain value(1,0) at the identity element of G.

Lemma 3. Let A be a subset of a group G. Then

$$((\chi_A, \chi_{A^c})) = (\chi_{<A>}, \chi_{<A>^c})$$

Thus $\mu_B(x) = 1$ and $\nu_B(x) = 0$. Since $B \in \text{IFG}(G)$, $B = 1_{\sim}$ for any composite of element of A. So $(\chi_{<A>}, \chi_{<A>^c}) \subset B$ Hence $(\chi_{<A>}, \chi_{<A>^c}) \subset \cap B$.

By result 3. $(\chi_{<A>}, \chi_{<A>^c}) \subset \text{IFG}(G)$. Moreover, $(\chi_{<A>}, \chi_{<A>^c}) \in B$. Threrfore $(\chi_{<A>}, \chi_{<A>^c}) = \cap B = ((\chi_A, \chi_{A^c}))$. The following can be easily seen.

Lemma 4. Let A and B subgroups of a group G. Then

(1)AG if and only if $(\chi_A, \chi_{A^c}) \in \text{IFNG}(G)$

(2)$(\chi_A, \chi_{A^c}) \circ (\chi_B, \chi_{B^c}) = (\chi_{A \cdot B}, \chi_{(A \cdot B)^c})$

Proposition 4. Let $S(G)$ be the set of all subgroup of a group G and let $\text{IFG}(S(G)) = (\chi_A, \chi_{A^c}) : A \in S(G)$. Then $\text{IFG}(S(G))$ forms a sublattice of $\text{IFG}_f(G) \cap \text{IFG}_{(1,0)}(G)$ and hence of $\text{IFG}(G)$.

Proof. Let $A, B \in S(G)$. Then it is clear that $(\chi_A, \chi_{A^c}) \cap (\chi_B, \chi_{B^c}) = (\chi_{A \cap B}, \chi_{(A \cup B)^c}) \in IFG(SG))$ .By Lemma 4, $((\chi_A, \chi_{A^c}) \cup (\chi_B, \chi_{B^c})) = ((\chi_{A \cap B}, \chi_{(A \cup B)^c})) = (\chi_{<A \cap B>}, \chi_{<A \cup B>^c})$

Thus $(\chi_A, \chi_{A^c}) \vee (\chi_B, \chi_{B^c}) = ((\chi_A, \chi_{A^c}) \cup (\chi_B, \chi_{B^c})) \in IFG(SG))$. More-over, $IFG(S(G)) \subset IFG_{f(G)} \cap IFG_{(1,0)}(G)$. Hence IFG(S(G))is a sublattice of $IFG_{f(G)} \cap IFG_{(1,0)}(G)$.

Proposition 4 allows us to consider the lattice of subgroups *S(G)* of G a group G as a sublattice of the lattice of all intuitionistic fuzzy subgroup IFG*(G)* of G.

Now, in view of Theorems 2 and 3, for each fixed $(s,t) \in I \times I$ with $s+t \leq 1$, $\text{IFN}_{f(1,0)}(G)$ form a modular sublattice $\text{IFN}_{f(1,0)}(G)$ is also modular. It is clear that $\text{IFN}_{f(1,0)}(G)$ ∩ IFN*(S(G))* = IFN*(N(G))*, where *N(G)* denotes the set of all normal subgroups of G and IFN*(N(G))* $= (\chi_N, \chi_{N^c}) : N \in M(G)$. Moreover, IFN*(N(G))* is also modular.

The lattice structure of these sublattices can be visualized by the diagram in Figure 2,

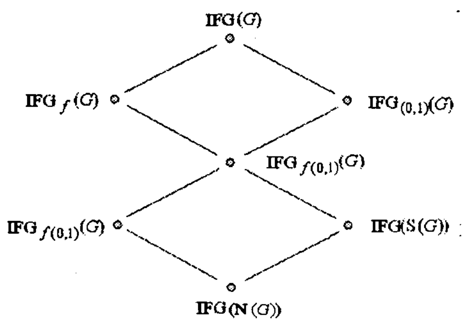

Fig. 2. The lattice diagram of special sublattice of IFG(G)

By using Lemmas 3 and 4, we obtain a well-known classical result.

Corollary 1. Let G be a group. Then *N(G)* forms a modular sublattice of *S(G)*.

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