

# 실시간 퍼지 동조 PID 제어 알고리즘

## Real-time Fuzzy Tuned PID Control Algorithm

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### Abstract

In this paper, we proposed a PID tuning algorithm by the fuzzy set theory to improve the performance of the PID controller. The new tuning algorithm for the PID controller has the initial value of parameter  $K_p$ ,  $T_I$ ,  $T_D$  by the Ziegler-Nichols formula that uses the ultimate gain and ultimate period from a relay tuning experiment. We will get the error and the error rate of plant output corresponding to the initial value of parameter and find the new proportion gain( $K_p$ ) and the integral time ( $T_I$ ) from fuzzy tuner by the error and error rate of plant output as a membership function of fuzzy theory. This fuzzy auto tuning algorithm for PID controller considerably reduced the overshoot and rise time as compared to any other PID controller tuning algorithms. And in real parametric uncertainty systems, it constitutes an appreciable improvement of performance. The significant property of this algorithm is shown by simulation

### 1. Introduction

Although more advanced control theories are starting to become available the PID control algorithm will in all likelihood continue to be used for most process control application for some time to come due to its simplicity, familiarity, ready availability, and generally good performance when the application is not demanding. It is commonly recognized that industrial controllers of the PID type are often operating with poor tuning, partly due to the large time constants in many process. The main drawback that prevents PID from being an even more effective control tool is the necessity of tuning the controller (adjusting the gain, integral time constant, and derivative time constant settings) for each control system in order to get the desired control performance.

Significant efforts have been devoted to providing tuning of PID controller. A method for automatic tuning of a simple regulators was introduced in Astrom and Haggland[3,5]. The idea was to determine the critical period and the critical gain from a simple relay feedback experiment and to use Ziegler-Nichols type of control design methods to find the parameters of PID regulators

The purpose of the paper is to investigate an optimal controller design using the fuzzy theory. In this paper, we proposed a new PID tuning algorithm by the fuzzy set theory to improve the performance of the PID controller.

We determined the ultimate gain and the ultimate

period from a simple relay feedback experiment. The new tuning algorithm for the PID controller has the initial value of parameter  $K_p$ ,  $T_i$ ,  $T_d$  by the Ziegler-Nichols formula that used the ultimate gain and ultimate period from a relay tuning experiment. And we compute the error and the error change rate of plant response corresponding to the initial value of parameter. We can find the new proportional gain( $K_p$ ) and the integral time ( $T_i$ ) from fuzzy tuner by the error and the error change rate of plant output.

This fuzzy tuning algorithm for a PID controller considerably reduced the overshoot and rise time as compared to any other PID controller tuning algorithms, such as Ziegler-Nichols tuning method, refinement of the Ziegler-Nichols tuning method and auto tuning methods etc. And it also has been shown that this controller has achievable performance for real parametric uncertainty system.

### 2. Parameter Tuning for PID controller

The PID controller is usually implemented as follow equation (1)

$$u_c = K_c \left( e(t) + \frac{1}{\tau_I} \int_0^t e(t) dt + \tau_D \frac{de(t)}{dt} \right) \quad (1)$$

$$e(t) = y_r(t) - y(t)$$

First, the Fuzzy auto-tuning method obtained the parameter( $K_p$ ,  $T_i$ ,  $T_d$ ) by Zeigler-Nichols tuning formular in Table 1. We will use this one as the

initial value. And the unit step response of plant with PID controller using the initial value is determined. Then we can get the output error and error change rate of the output. The fuzzy auto-tuning method will use the error and the error change rate to make membership function and fuzzy rules.

The Table 1 used to determine the parameters of PID controller in Astrom - Hugglund.

	$K_c$	$\tau_I$	$\tau_D$
P	$0.5K_{cu}$		
PI	$0.45K_{cu}$	$0.85P_u$	
PID	$0.6K_{cu}$	$0.5P_u$	$0.125P_u$

Table 1. Ziegler-Nichols parameter tuning

The tuning method in Astrom and Hagglund is only based on the knowledge of the amplitude and the period of oscillation. The starting point is that a relay feedback experiment gives periodic input output signals as shown Fig. 1. The period of oscillation is approximately the ultimate period under proportional feedback.

We find the critical period( $P_u$ ) from the waveform of the oscillation and calculate the critical gain( $K_{cu}$ ) by equation (2)

$$K_{cu} = \frac{4d}{\pi a} \tag{2}$$

where d is the relay amplitude, and is the amplitude of the waveform of oscillation.

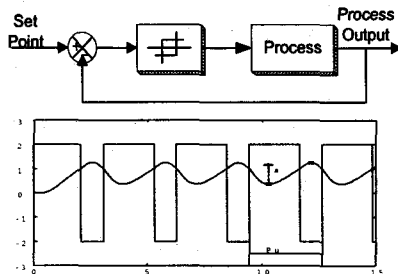


Fig. 1. The relay feedback experiment

### 3. Fuzzy auto tuning

A tuning algorithm proposed in this paper should obtain the output of the plant from the Ziegler-Nichols tuning with the relay feedback experiment in Astrom and Hagglund.

As we compute the error and the change error rate of the response, we can find the proportional gain and the integral time using the error and the error change rate of the output response as membership function of fuzzy auto-tunner.

The error change rate compute from the error of the output by the equation (3)

$$\Delta e(k) = e(k) - e(k-1) \tag{3}$$

Fuzzy auto-tuner determine, the  $K_c(k)$  and  $\tau_I(k)$  by equation (4) from reasoning the change of proportional gain  $\Delta K_c(k)$  and the change of the integral time  $\Delta \tau_I(k)$  from output of the plant

$$\begin{aligned} K_c(k) &= K_c(k-1) + \Delta K_c(k) \\ \tau_I(k) &= \tau_I(k) + \Delta \tau_I(k) \end{aligned} \tag{4}$$

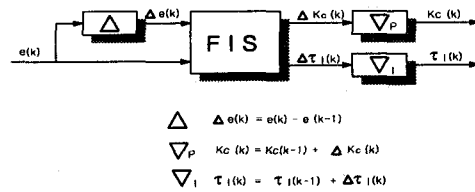


Fig. 2. Structure of Fuzzy Tuner

### 3.1 Fuzzy Rules

The fuzzy rules to determine  $\Delta K_c(k)$  and  $\Delta \tau_I(k)$  are the following equations (5) and (6)

1. The rule for  $\Delta K_c(k)$

$$\begin{aligned} R^i: & \text{ If } e(k) \text{ is } A_i \text{ and } \Delta e(k) \text{ is } B_i \\ & \text{ Then } \Delta K_c(k) \text{ is } C_{i,1} \end{aligned} \tag{5}$$

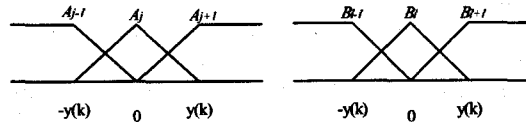
where  $i (i = 1, \dots, 9)$  is the number of rules

2. The rule for  $\Delta \tau_I(k)$

$$\begin{aligned} R^i: & \text{ If } e(k) \text{ is } A_i \text{ and } \Delta e(k) \text{ is } B_i \\ & \text{ Then } \Delta \tau_I(k) \text{ is } D_{i,1} \end{aligned} \tag{6}$$

### 3.2 The membership function of the premise

The membership function of the premise of the error  $e(k)$  and the change of the error  $\Delta e(k)$  are defined in the triangle type in Fig.3 (a) and (b)



(a) error (b) The change of the error  
Fig. 3. Membership Function of Premise

### 3.3 The parameter of the consequence

The parameter of the consequence defined by the fuzzy linguistic variable from the performance index shows that the increased proportional gain  $K_c$  results in large overshoot, a decrease of rise time, and the increased integral time  $\tau_I$  result in a decreased rise time.

The control rules of the consequence of  $K_c$  and  $\tau_I$  defined from the error curve of the step response in Fig.4.

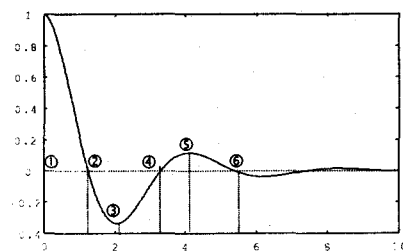


Fig. 4. Error Curve of Output

The control rules of the consequence of  $K_c$  is the following Table 2.

$\Delta e(k) \backslash e(k)$	N	Z	P
N	N	P	N
Z	P	Z	N
P	P	N	P

Table 2. Consequence Variable of  $K_c$

The control rules of the consequence of  $\tau_I$  is the following Table 3.

$\Delta e(k) \backslash e(k)$	N	Z	P
N	P	N	P
Z	N	Z	P
P	N	P	N

Table 3. Consequence Variable of  $\tau_I$

**3.4 Reasoning**

The reasoning in this paper used the simplified reasoning of the consequence[9]

The reasoning of  $\Delta K_c(k)$  is in equation (7)

$$y = \frac{\sum_{i=1}^n w_i C_{j,l}}{\sum w_i} \tag{7}$$

where  $n$  is the number of rules,  $C_{j,l}$  is the value of the center of gravity of  $\Delta K_c(k)$   $w_i$  is the fitness of the  $i$  th rule in the premise, and the fitness take from the following equation (8)

$$w_i = \mu_{A_j}(e(k)) \times \mu_{B_l}(\Delta e(k)) \tag{8}$$

$j = 1, 2, 3, \quad l = 1, 2, 3$

The reasoning of  $\Delta \tau_I(k)$  is similar to (9), like as the reasoning  $\Delta K_c(k)$

$$y = \frac{\sum_{i=1}^n w_i D_{j,l}}{\sum w_i} \tag{9}$$

**4. Computer simulation and result**

Computer simulation was used to determine the control performance of the fuzzy auto tuner and we used electric DC motor as a plant.

The transfer function of the electric DC motor is in equation (10)

$$\frac{\Theta(s)}{V(s)} = \frac{K}{LJs^3 + (RJ + BL)s^2 + (K^2 + RB)s} \tag{10}$$

where  $L$ :armature inductance,  $R$ :armature resistance,  $K$ :motor constant,  $J$ :the moment of inertia and  $B$ :the mechanical friction.

The parameters of the electric DC motor have the following value respectively,  $J=0.042$ ,  $B=0.01625$ ,  $K=0.9$ ,  $L=0.025$ ,  $R=5$  as a nominal value.

The transfer function of the electric DC motor is in equation (11).

$$P(s) = \frac{0.9}{0.00105s^3 + 0.2104s^2 + 0.8913s} \tag{11}$$

To determine the critical gain and the critical period, the system is connected in a feedback loop with a relay as is shown in Fig.1. From the relay tuning experiment, we determined  $a=0.0361$ ,  $P_U=0.26$ ,  $d=4$ , the critical gain and the critical period is thus approximately given by

$$K_{cu} = \frac{4d}{a\pi} = 141.1525 \quad P_u = 0.26$$

where  $a$  is the process output,  $d$  is the relay amplitude.

The initial parameters of the PID controller was computed from Ziegler-Nichols tuning in Table 1 using the  $K_{cu}$  and  $P_u$

We can determine the  $K_c=84.6915$ ,  $\tau_I=0.1300$ , and  $\tau_D=0.0325$ .

**4.1 Computer simulation result**

The noise filtering type PID controller is the following equation (12)

$$u_c = K_c \left( e(t) + \frac{1}{\tau_I} \int e(t) dt - \tau_D \frac{dy_f}{dt} \right) \tag{12}$$

$$e(t) = y_r(t) - y(t)$$

$$y_f(s) = \frac{1}{1 + \tau_d s / N} y(s)$$

The computer simulation results of the noise filtering PID controller using Ziegler-Nichols parameter tuning is in Fig.7. And to consider the change of the set point, input was changed by 2 unit at 4 sec. The overshoot and the rise time of the response is shown in Fig. 5, 46.21% and 0.0697sec respectively. It is improved more than the ideal type of PID controller with a Ziegler-Nichls tuning.

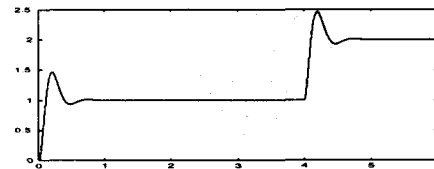


Fig. 5. Process output

The computer simulation results of the Fuzzy auto tuning for the noise filtering PID controller is shown in Fig. 6(a). In this Figure, we can find that the overshoot is 0% and the rise time of the response is the 0.125 sec. It was considerably improved the overshoot, but rise time increased more than Ziegler-Nichols tuning

The parameters( $K_c$ ,  $\tau_I$ ) of PID controller by fuzzy auto tuning is shown in Fig. 6(b), 6(c) respectively. In this figure, we can find that the parameters of PID controller by fuzzy auto tuning adapted to the change of the response.

**5. Computer simulation with real parametric uncertainty**

Consider uncertainty process. the parameter of the electric DC motor has the real parametric uncertainty such as  $J=[0.03 \sim 0.16]$

The computer simulation of the ideal type of PID

ontroller with Ziegler-Nichols tuning is shown in Figure 7. in this Figure, for  $J=[0.03 \sim 0.15]$  we can find that the system displays instability over  $J = 0.12$

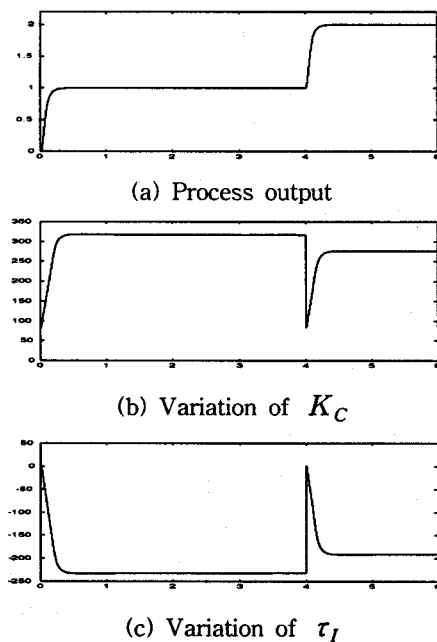


Fig. 6. Process output for the noise filtering PID controller

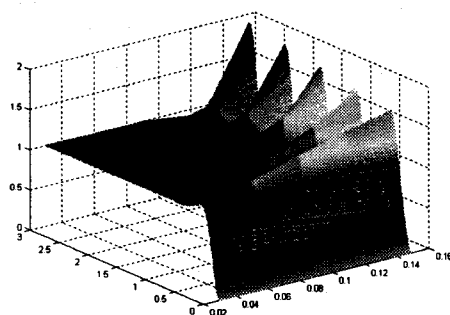


Fig. 7. Z-N tuning for uncertainty process

The computer simulation the ideal type PID controller with Ziegler-Nichols tuning is shown in Fig. 8. In this Figure, for  $J=[0.03 \sim 0.15]$  we can find that the response display stable over  $j = [0.03 \sim .15]$ , it is shown that overshoot, rise time and settling time are maintained constant.

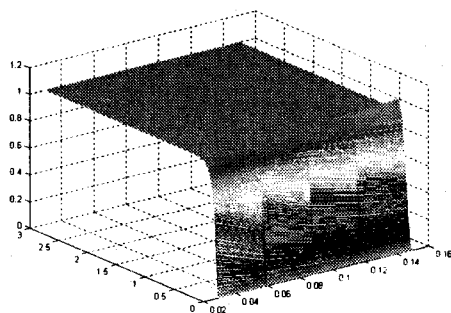


Fig. 8. Fuzzy auto tuning for uncertainty process

## 6. Conclusion

We proposed a new parameter tuning algorithm for PID controller by the fuzzy theory. The tuning algorithm for the PID controller are based on the idea that the fuzzy set theory and the error and the error change rate of output. This method determined to be updated proportional gain,  $K_p'$  and integral time,  $T_i'$  by fuzzy tuner that used the error and the error change rate of plant output, where Ziegler-Nichols tuning method is used to find a initial value of PID parameters

Even though this tuning algorithm for the PID controller has some complex computation procedure to obtain the parameter for a PID controller, it has given a considerable improvement in the overshoot and rise time etc. In real parametric uncertainty systems, the PID controller with fuzzy auto-tuning will give appreciable improvement in the performance.

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