퍼지계수를 가지는 미분방정식의 해

The solution of differential equations with fuzzy coefficients

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Abstract

This paper we consider the solution of differential equations with fuzzy coefficients generated by fuzzy number $\tilde{1}$ using two different methods with eigenvalues and eigenvectors.

I. Introduction

The concept of the natural number is easily extended to fuzzy case by max-min convolution.

Let $\tilde{1}$ be a fuzzy number in R^+ with the following membership function.

$$\forall x \in R^+, \ \mu_{\tilde{1}}(x) \in [0,1] \text{ and } \mu_{\tilde{1}}(1) = 1.$$

We now proceed with the successive construction of fuzzy numbers as follows;

$$\tilde{2} = \tilde{1}(+)\tilde{1}, \ \tilde{3} = \tilde{2}(+)\tilde{1}, \ \cdots,$$
$$\tilde{n} = (\widetilde{n-1})(+)\tilde{1}, \cdots.$$

Let us define the α-cut for the fuzzy number $\tilde{1}$ as follows.

$$[\tilde{1}]^\alpha = [\tilde{1}_l^\alpha, \tilde{1}_r^\alpha], \ \alpha \in [0,1].$$

It is constructed from 1 by the use of the intervals of confidence of level α.

$$[\tilde{2}]^\alpha = [\tilde{1}]^\alpha + [\tilde{1}]^\alpha = [2 \cdot \tilde{1}_l^\alpha, 2 \cdot \tilde{1}_r^\alpha] = 2[\tilde{1}_l^\alpha, \tilde{1}_r^\alpha],$$
$$[\tilde{3}]^\alpha = [\tilde{2}]^\alpha + [\tilde{1}]^\alpha = [3 \cdot \tilde{1}_l^\alpha, 3 \cdot \tilde{1}_r^\alpha] = 3[\tilde{1}_l^\alpha, \tilde{1}_r^\alpha],$$
$$\cdots$$
$$[\tilde{n}]^\alpha = [\widetilde{n-1}]^\alpha + [\tilde{1}]^\alpha = [n \cdot \widetilde{1_l^\alpha}, n \cdot \widetilde{1_r^\alpha}]$$
$$= n[\widetilde{1_l^\alpha}, \widetilde{1_r^\alpha}].$$

We now call the fuzzy natural number generated by fuzzy number $\tilde{1}$.

We define the fuzzy real number generated by fuzzy number $\tilde{1}$.

$$\forall k \in R, \ [\tilde{k}]^\alpha = k \cdot [\tilde{1}]^\alpha.$$

We define the multiplication by the fuzzy number for the real matrix.

If $\tilde{a}$ is fuzzy number,

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

where $a_{ij} \in R$ then

$$\tilde{a} \cdot A = \begin{pmatrix} \tilde{a} \cdot a_{11} & \tilde{a} \cdot a_{12} \\ \tilde{a} \cdot a_{21} & \tilde{a} \cdot a_{22} \end{pmatrix}.$$

In this paper we consider the solution of following differential equations with fuzzy coefficients using two different methods

$$(\text{F.D.E.}) \quad \begin{cases} \dot{x_1} = \tilde{a}x_2, \\ \dot{x_2} = \tilde{b}x_1, \\ x_1(0) = \tilde{x}^1, \ x_2(0) = \tilde{x}^2, \end{cases}$$

where $\tilde{a}, \tilde{b}, \tilde{x}^1$ and $\tilde{x}^2$ are the fuzzy natural number generated by fuzzy number $\tilde{1}$.

And we consider the solution of following differential equations with fuzzy coefficients

$$(\text{F.D.E.}) \quad \begin{cases} \dot{x_1} = \tilde{a}x_2, \\ \dot{x_2} = \tilde{b}x_1, \\ x_1(0) = \tilde{x}^1, \ x_2(0) = \tilde{x}^2, \end{cases}$$

where $\tilde{a}, \tilde{b}, \tilde{x}^1$ and $\tilde{x}^2$ are the fuzzy number.

II. The first method

We consider the following differential equations with fuzzy coefficients

$$\text{(F.D.E.)} \quad \begin{cases} \dot{x_1} = \tilde{a}x_2, \\ \dot{x_2} = \tilde{b}x_1, \\ x_1(0) = \tilde{x}^1,\ x_2(0) = \tilde{x}^2, \end{cases}$$

where $\tilde{a}, \tilde{b}, \tilde{x}^1$ and $\tilde{x}^2$ are the fuzzy natural number generated by fuzzy number $\tilde{1}$.

We can be expression as follows:

$$\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \end{pmatrix} = \begin{pmatrix} 0 & \tilde{a} \\ \tilde{b} & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \tilde{1} \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

We find the eigenvalues and eigenvectors of $\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}$.

To expand the determinant in the characteristic equation

$$\det(A - \lambda I) = \begin{vmatrix} -\lambda & a \\ b & -\lambda \end{vmatrix} = 0$$

It follows that

$$\lambda^2 - ab = 0.$$

Hence the eigenvalues are $\lambda_1 = \sqrt{ab}$, $\lambda_2 = -\sqrt{ab}$. To find the eigenvectors, we must now reduce $(A - \lambda I)K = \tilde{0}$ two times corresponding to the two distinct eigenvalues.

For $\lambda_1 = \sqrt{ab}$, we have

$$\begin{pmatrix} -\sqrt{ab} & a \\ b & -\sqrt{ab} \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} \tilde{0} \\ \tilde{0} \end{pmatrix}.$$

And solve the following equation:

$$-\sqrt{ab}\,k_1 + ak_2 = \tilde{0}$$

where zero fuzzy number $\tilde{0}$ satisfies $\sqrt{ab}k_1 - \sqrt{ab}k_1 = \tilde{0}$. Thus we see that $k_2 = \frac{\sqrt{ab}}{a}k_1$.

Choosing $k_1 = \sqrt{ab}\tilde{1}$, we get the eigenvector

$$K_1 = \tilde{1}\begin{pmatrix} \sqrt{ab} \\ b \end{pmatrix}.$$

For $\lambda_2 = -\sqrt{ab}$,

$$\begin{pmatrix} \sqrt{ab} & a \\ b & \sqrt{ab} \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} \tilde{0} \\ \tilde{0} \end{pmatrix}.$$

And solve the following equation:

$$\sqrt{ab}\,k_1 + ak_2 = \tilde{0}$$

where zero fuzzy number $\tilde{0}$ satisfies $\sqrt{ab}k_1 - \sqrt{ab}k_1 = \tilde{0}$. Thus we see that $k_2 = \frac{\sqrt{ab}}{a}k_1$. Choosing $k_1 = \sqrt{ab}\tilde{1}$, then yields the second eigenvector

$$K_2 = \tilde{1}\begin{pmatrix} \sqrt{ab} \\ -b \end{pmatrix}.$$

Hence we know that the fuzzy solution of (F.D.E.) is given by

$$\begin{aligned} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &= c_1 K_1 e^{\lambda_1 t} + c_2 K_2 e^{\lambda_2 t} \\ &= c_1 \tilde{1}\begin{pmatrix} \sqrt{ab} \\ b \end{pmatrix} e^{\sqrt{ab}t} + c_2 \tilde{1}\begin{pmatrix} \sqrt{ab} \\ -b \end{pmatrix} e^{-\sqrt{ab}t} \\ &= \tilde{1}\left(c_1\begin{pmatrix} \sqrt{ab} \\ b \end{pmatrix} e^{\sqrt{ab}t} + c_2\begin{pmatrix} \sqrt{ab} \\ -b \end{pmatrix} e^{-\sqrt{ab}t} \right) \end{aligned}$$

where
$$c_1 = \frac{1}{2}\left(\frac{1}{\sqrt{ab}}x^1 + \frac{1}{b}x^2\right),$$
$$c_2 = \frac{1}{2}\left(\frac{1}{\sqrt{ab}}x^1 - \frac{1}{b}x^2\right).$$

The α-level set of x_1 and x_2 are

$$[x_1]^\alpha = [\sqrt{ab}(c_1 e^{\sqrt{ab}t} + c_2 e^{-\sqrt{ab}t})\tilde{1}_l^\alpha, \sqrt{ab}(c_1 e^{\sqrt{ab}t} + c_2 e^{-\sqrt{ab}t})\tilde{1}_r^\alpha],$$

$$[x_2]^\alpha = [b(c_1 e^{\sqrt{ab}t} - c_2 e^{-\sqrt{ab}t})\tilde{1}_l^\alpha, b(c_1 e^{\sqrt{ab}t} - c_2 e^{-\sqrt{ab}t})\tilde{1}_r^\alpha].$$

III. The second method

We consider the following differential equations with fuzzy coefficients

$$\text{(F.D.E.)} \quad \begin{cases} \dot{x_1} = \tilde{a}x_2, \\ \dot{x_2} = \tilde{b}x_1, \\ x_1(0) = \tilde{x}^1,\ x_2(0) = \tilde{x}^2, \end{cases}$$

where $\tilde{a}, \tilde{b}, \tilde{x}^1$ and $\tilde{x}^2$ are the fuzzy natural number generated by fuzzy number $\tilde{1}$.

For $\alpha \in [0,1]$, we can be expression as follows

$$\begin{cases} \begin{pmatrix} [\dot{x_1}]^\alpha \\ [\dot{x_2}]^\alpha \end{pmatrix} = \begin{pmatrix} 0 & [\tilde{a}]^\alpha \\ [\tilde{b}]^\alpha & 0 \end{pmatrix} \begin{pmatrix} [x_1]^\alpha \\ [x_2]^\alpha \end{pmatrix}, \\ \begin{pmatrix} [x_1(0)]^\alpha \\ [x_2(0)]^\alpha \end{pmatrix} = \begin{pmatrix} [\tilde{x}^1]^\alpha \\ [\tilde{x}^2]^\alpha \end{pmatrix} \end{cases}$$

where $[\tilde{a}]^\alpha = [a \cdot \tilde{1}_l^\alpha, a \cdot \tilde{1}_r^\alpha]$, $[\tilde{b}]^\alpha = [b \cdot \tilde{1}_l^\alpha, b \cdot \tilde{1}_r^\alpha]$, $[\tilde{x}^1]^\alpha = [x^1 \cdot \tilde{1}_l^\alpha, x^1 \cdot \tilde{1}_r^\alpha]$, $[\tilde{x}^2]^\alpha = [x^2 \cdot \tilde{1}_l^\alpha, x^2 \cdot \tilde{1}_r^\alpha]$.

For each $\alpha \in [0,1]$, we consider the equation

$$\begin{cases} \begin{pmatrix} \dot{x}_{1p}^{\alpha} \\ \dot{x}_{2p}^{\alpha} \end{pmatrix} = \begin{pmatrix} 0 & a \cdot \tilde{1}_a^{\alpha} \\ b \cdot \tilde{1}_b^{\alpha} & 0 \end{pmatrix} \begin{pmatrix} x_{1p}^{\alpha} \\ x_{2p}^{\alpha} \end{pmatrix}, \\ \begin{pmatrix} x_{1p}(0)^{\alpha} \\ x_{2p}(0)^{\alpha} \end{pmatrix} = \begin{pmatrix} x^1 \cdot \tilde{1}_1^{\alpha} \\ x^2 \cdot \tilde{1}_2^{\alpha} \end{pmatrix} \end{cases}$$

where $\dot{x}_{ip}{}^{\alpha} \in [\dot{x}_i]^{\alpha}$, $x_{ip}^{\alpha} \in [x_i]^{\alpha}$, $x_{ip}(0)^{\alpha} \in [x_i(0)]^{\alpha}$, $(i=1,2)$, $a \cdot \tilde{1}_a^{\alpha} \in [\tilde{a}]^{\alpha}$, $b \cdot \tilde{1}_b^{\alpha} \in [\tilde{b}]^{\alpha}$, $x^1 \cdot \tilde{1}_1^{\alpha} \in [\tilde{x}^1]^{\alpha}$, $x^2 \cdot \tilde{1}_2^{\alpha} \in [\tilde{x}^2]^{\alpha}$.

We find the eigenvalues and eigenvectors of

$$\begin{pmatrix} 0 & a \cdot \tilde{1}_a^{\alpha} \\ b \cdot \tilde{1}_b^{\alpha} & 0 \end{pmatrix}.$$

From the characteristic equation we obtain the real eigenvalues $\lambda = \pm\sqrt{ab\,\tilde{1}_a^{\alpha}\tilde{1}_b^{\alpha}}$.

For $\lambda_1 = \sqrt{ab\,\tilde{1}_a^{\alpha}\tilde{1}_b^{\alpha}}$ (or $\lambda_2 = \sqrt{ab\,\tilde{1}_a^{\alpha}\tilde{1}_b^{\alpha}}$) we have

$$\begin{pmatrix} -\sqrt{ab\,\tilde{1}_a^{\alpha}\tilde{1}_b^{\alpha}} & a \cdot \tilde{1}_a^{\alpha} \\ b \cdot \tilde{1}_b^{\alpha} & -\sqrt{ab\,\tilde{1}_a^{\alpha}\tilde{1}_b^{\alpha}} \end{pmatrix} \begin{pmatrix} k_{11} \\ k_{12} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

And solve the following equation:

$$-\sqrt{ab\,\tilde{1}_a^{\alpha}\tilde{1}_b^{\alpha}}\, k_{11} + a \cdot \tilde{1}_a^{\alpha}\, k_{12} = 0.$$

Choosing $k_{11} = \sqrt{ab\,\tilde{1}_a^{\alpha}\tilde{1}_b^{\alpha}}$, we get the eigenvector

$$K_1 = \begin{pmatrix} \sqrt{ab\,\tilde{1}_a^{\alpha}\tilde{1}_b^{\alpha}} \\ b \cdot \tilde{1}_{b^{\alpha}} \end{pmatrix}.$$

Similarly, for $\lambda_2 = -\sqrt{ab\,\tilde{1}_a^{\alpha}\tilde{1}_b^{\alpha}}$ (or $\lambda_1 = -\sqrt{ab\,\tilde{1}_a^{\alpha}\tilde{1}_b^{\alpha}}$) yields the second eigenvector

$$K_2 = \begin{pmatrix} \sqrt{ab\,\tilde{1}_a^{\alpha}\tilde{1}_b^{\alpha}} \\ -b \cdot \tilde{1}_{b^{\alpha}} \end{pmatrix}.$$

From the initial value, we find constants c_1, c_2 which satisfy following

$$c_1K_1 + c_2K_2 = (K_1\ \ K_2)\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} x^1 \cdot \tilde{1}_1^{\alpha} \\ x^2 \cdot \tilde{1}_2^{\alpha} \end{pmatrix}.$$

For each $\alpha \in [0,1]$, let

$$\lambda_{il} = \min\{\lambda_i \mid \lambda_i = -\sqrt{ab\tilde{1}_a^{\alpha}\tilde{1}_b^{\alpha}}\},$$

$$\lambda_{ir} = \max\{\lambda_i \mid \lambda_i = \sqrt{ab\tilde{1}_a^{\alpha}\tilde{1}_b^{\alpha}}\},$$

$$k_{ijl} = \min\{k_{ij} \mid k_{ij} = (i,j) \text{ compt. of } K_i,\ (j=1,2)\},$$

$$k_{ijr} = \max\{k_{ij} \mid k_{ij} = (i,j) \text{ compt. of } K_i,\ (j=1,2)\},$$

$$c_{il} = \min\{c_i \mid K \cdot c = x\},$$

$$c_{ir} = \max\{c_i \mid K \cdot c = x\},$$

where $(i=1,2)$ and $K \cdot c = x$ is

$$(K_1\ \ K_2)\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} x^1 \cdot \tilde{1}_1^{\alpha} \\ x^2 \cdot \tilde{1}_2^{\alpha} \end{pmatrix}.$$

For each $\alpha \in [0,1]$, let

$$[\tilde{\lambda}_i]^{\alpha} = [\lambda_{il}, \lambda_{ir}],$$

$$[\tilde{K}_i]^{\alpha} = \left(\begin{matrix} [k_{i1l},\ k_{i1r}] \\ [k_{i2l},\ k_{i2r}] \end{matrix}\right),$$

$$[\tilde{c}_i]^{\alpha} = [c_{il},\ c_{ir}].$$

From the resolution identity, we can construct fuzzy numbers $\tilde{\lambda}_i$, $\tilde{K}_i$, $\tilde{c}_i$.

Hence the fuzzy solution is

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \tilde{c}_1 \otimes \tilde{K}_1 \otimes e^{\tilde{\lambda}_1 t} \oplus \tilde{c}_2 \otimes \tilde{K}_2 \otimes e^{\tilde{\lambda}_2 t}.$$

IV. Example

Consider the fuzzy solution of the following differential equations with fuzzy coefficients generated by fuzzy number $\tilde{1}$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & \tilde{2} \\ \tilde{3} & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} \tilde{1} \\ \tilde{2} \end{pmatrix}$$

where the membership function of $\tilde{1}$ is

$$\int_0^1 x/x + \int_1^2 -x+2/x.$$

The α-level set of $\tilde{1}$ is $[\tilde{1}]^{\alpha} = [\tilde{1}_l^{\alpha}, \tilde{1}_r^{\alpha}] = [\alpha, 2-\alpha]$, $\alpha \in [0,1]$.

From the first method, we obtain the α-level set of the solution x_1 and x_2 are

$$[x_1]^{\alpha} = [(c_1\sqrt{6}\,e^{\sqrt{6}t} + c_2\sqrt{6}\,e^{-\sqrt{6}t})\alpha,\ (c_1\sqrt{6}\,e^{\sqrt{6}t} + c_2\sqrt{6}\,e^{-\sqrt{6}t})(2-\alpha)],$$

$$[x_2]^{\alpha} = [(c_1 3\,e^{\sqrt{6}t} - c_2 3\,e^{-\sqrt{6}t})\alpha,\ (c_1 3\,e^{\sqrt{6}t} - c_2 3\,e^{-\sqrt{6}t})(2-\alpha)],$$

where

$$c_1 = \frac{\sqrt{6}+4}{12},\quad c_2 = \frac{\sqrt{6}-4}{12}.$$

From the second method, we obtain the α-level set of fuzzy number $\tilde{\lambda}_i$, $(i=1,2)$.

$$[\lambda_1]^0 = [-3.00000, 3.46410],$$

$$[\lambda_1]^{0.2} = [-2.80000, 3.26190],$$

$[\lambda_1]^{0.4} = [-2.60000, 3.05941],$

$[\lambda_1]^{0.6} = [2.03960, 2.85657],$

$[\lambda_1]^{0.8} = [2.24499, 2.65329],$

$[\lambda_1]^{1} = [2.44948, 2.44948],$

$[\lambda_2]^{0} = [-3.46410, 3.00000],$

$[\lambda_2]^{0.2} = [-3.26190, 2.80000],$

$[\lambda_2]^{0.4} = [-3.05941, 2.60000],$

$[\lambda_2]^{0.6} = [-2.85657, -2.03960],$

$[\lambda_2]^{0.8} = [-2.65329, -2.24499],$

$[\lambda_2]^{1} = [-2.44948, -2.44948].$

The α-level set of fuzzy number $\tilde{k}_{ij}$, $(i, j = 1, 2)$ is

$[\tilde{k}_{11}]^{0} = [-0.79056, 1.00000],$

$[\tilde{k}_{11}]^{0.2} = [-0.75377, 0.73598],$

$[\tilde{k}_{11}]^{0.4} = [-0.69436, 0.72111],$

$[\tilde{k}_{11}]^{0.6} = [-0.69337, 0.69366],$

$[\tilde{k}_{11}]^{0.8} = [-0.65828, 0.66332],$

$[\tilde{k}_{11}]^{1} = [0.63245, 0.63245],$

$[\tilde{k}_{12}]^{0} = [-0.70710, 1.41411],$

$[\tilde{k}_{12}]^{0.2} = [-0.70710, 1.02062],$

$[\tilde{k}_{12}]^{0.4} = [-0.70710, 0.90128],$

$[\tilde{k}_{12}]^{0.6} = [0.72168, 0.88388],$

$[\tilde{k}_{12}]^{0.8} = [0.74833, 0.83333],$

$[\tilde{k}_{12}]^{1} = [0.77459, 0.77459],$

$[\tilde{k}_{21}]^{0} = [-0.79056, 1.00000],$

$[\tilde{k}_{21}]^{0.2} = [-0.75377, 0.72231],$

$[\tilde{k}_{21}]^{0.4} = [-0.72168, 0.69388],$

$[\tilde{k}_{21}]^{0.6} = [-0.68138, 0.69282],$

$[\tilde{k}_{21}]^{0.8} = [-0.66815, 0.65044],$

$[\tilde{k}_{21}]^{1} = [-0.64549, -0.64549],$

$[\tilde{k}_{22}]^{0} = [-1.41441, 1.04888],$

$[\tilde{k}_{22}]^{0.2} = [-0.70710, 1.00000],$

$[\tilde{k}_{22}]^{0.4} = [-0.70710, 0.94491],$

$[\tilde{k}_{22}]^{0.6} = [0.72111, 0.82915],$

$[\tilde{k}_{22}]^{0.8} = [0.75277, 0.81649],$

$[\tilde{k}_{22}]^{1} = [0.79056, 0.79056].$

The α-level set of fuzzy number $\tilde{c}_i$, $(i, j = 1, 2)$ is

$[\tilde{c}_1]^{0} = [-3.53533, 3.80400],$

$[\tilde{c}_1]^{0.2} = [-1.83847, 3.41658],$

$[\tilde{c}_1]^{0.4} = [-1.55563, 3.04392],$

$[\tilde{c}_1]^{0.6} = [-0.24937, 2.66569],$

$[\tilde{c}_1]^{0.8} = [0.12000, 2.37447],$

$[\tilde{c}_1]^{1} = [2.08156, 2.08156],$

$[\tilde{c}_2]^{0} = [-3.53533, 3.91311],$

$[\tilde{c}_2]^{0.2} = [-3.25269, 3.44302],$

$[\tilde{c}_2]^{0.4} = [-2.96984, 3.00462],$

$[\tilde{c}_2]^{0.6} = [-0.15197, 2.69265],$

$[\tilde{c}_2]^{0.8} = [0.15358, 2.37500],$

$[\tilde{c}_2]^{1} = [0.49031, 0.49031].$

V. The other equation

We consider the following differential equations with fuzzy coefficients

$$\text{(F.D.E.)} \quad \begin{cases} \dot{x}_1 = \tilde{a} x_2, \\ \dot{x}_2 = \tilde{b} x_1, \\ x_1(0) = \tilde{x}^1, \; x_2(0) = \tilde{x}^2, \end{cases}$$

where $\tilde{a}$, $\tilde{b}$, $\tilde{x}^1$ and $\tilde{x}^2$ are the fuzzy number.

For $\alpha \in [0,1]$, we can be expression as follows

$$\begin{cases} \begin{pmatrix} [\dot{x}_1]^\alpha \\ [\dot{x}_2]^\alpha \end{pmatrix} = \begin{pmatrix} 0 & [\tilde{a}]^\alpha \\ [\tilde{b}]^\alpha & 0 \end{pmatrix} \begin{pmatrix} [x_1]^\alpha \\ [x_2]^\alpha \end{pmatrix}, \\ \begin{pmatrix} [x_1(0)]^\alpha \\ [x_2(0)]^\alpha \end{pmatrix} = \begin{pmatrix} [\tilde{x}^1]^\alpha \\ [\tilde{x}^2]^\alpha \end{pmatrix}, \end{cases}$$

where $[\tilde{a}]^\alpha = [a_l^\alpha, a_r^\alpha]$, $[\tilde{b}]^\alpha = [b_l^\alpha, b_r^\alpha]$, $[\tilde{x}^i]^\alpha = [x_l^{i^\alpha}, x_r^{i^\alpha}]$, $(i = 1, 2)$.

For each $\alpha \in [0,1]$, we consider the equation

$$\begin{pmatrix} \dot{x}_{1p}^\alpha \\ \dot{x}_{2p}^\alpha \end{pmatrix} = \begin{pmatrix} 0 & a_p^\alpha \\ b_p^\alpha & 0 \end{pmatrix} \begin{pmatrix} x_{1p}^\alpha \\ x_{2p}^\alpha \end{pmatrix}, \quad \begin{pmatrix} x_{1p}(0)^\alpha \\ x_{2p}(0)^\alpha \end{pmatrix} = \begin{pmatrix} x_p^{1^\alpha} \\ x_p^{2^\alpha} \end{pmatrix},$$

where $\dot{x}_{ip}{}^\alpha \in [\dot{x}_i]^\alpha$, $x_{ip}^\alpha \in [x_i]^\alpha$, $x_{ip}(0)^\alpha \in [x_i(0)]^\alpha$, $(i = 1, 2)$, $a_p^\alpha \in [\tilde{a}]^\alpha$, $b_p^\alpha \in [\tilde{b}]^\alpha$, $x_p^{1^\alpha} \in [\tilde{x}^1]^\alpha$, $x_p^{2^\alpha} \in [\tilde{x}^2]^\alpha$. Using the second method, we find the fuzzy solution.

References

[1] P. Diamond and P. E. Kloeden, Metric space of Fuzzy sets, World scientific, (1994).

[2] P. Diamond and P. E. Kloeden, Optimization under uncertaintly, Proceed.3rd. IPMU Congress, B. Bouchon-Meunier and R. R. Yager, Paris, 247-249, (1990).

[3] P. E. Kloeden, Fuzzy dynamical systems, Fuzzy Sets and Systems, 7, 275-296, (1982).

[4] O. Kaleva, Fuzzy differential equations, Fuzzy Sets and Systems, 24, 301-317, (1987).

[5] A. Kaufmann and M. M. Gupta, Introduction to fuzzy arithmetic, Van Nostrand Reinhold, (1991).

[6] Y. C. Kwun, J. R. Kang, S. Y. Kim, The existence of fuzzy optimal control for the nonlinear fuzzy differential system with nonlocal initial condition, Journal of Fuzzy Logic and Intelligent Systems, 10, No. 1, 6-11, (2000).

[7] M. Mizumoto and K. Tanaka, Some properties of fuzzy numbers, North-Holland Publish Company, (1979).