보상된 bilinear 변환을 이용한 향상된 LMI 기반 지능형 디지털 재설계

An Improved LMI-Based Intelligent Digital Redesign Using Compensated Bilinear Transform

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ABSTRACT

This paper presents a new linear-matrix-inequality-basedintelligent digital redesign (LMI-based IDR) technique to match he states of the analog and the digital control systems at the intersampling instants as well as the sampling ones. The main features of the proposed technique are: 1) the multirate control is employed, and the control input is changed N times during one sampling period; 2) The proposed IDR technique is based on the compensated bilinear transformation.

Key words : Digital control, Takagi-Sugeno (T-S) fuzzy system, compensated bilinear transform, multirate sampling, linear matrix inequalities (LMIs).

I. Introduction

There have been fruitful researches in the nonlinear system focusing on digital redesign [1-5]. Historically, Joo et al. first attempted to develop some intelligent digital redesign (IDR) methodology for complex nonlinear systems [1]. They synergistically merged both the Takagi-Sugeno (T-S) fuzzy-model-based control and the digital redesign technique for a class of nonlinear systems. Chang et al. extended the intelligent digital redesign to uncertain T-S fuzzy systems [2]. These approaches [1,2] to IDR are so called as the local approach. The local approach enables to match the states of the continuous-time and the digital closed-loop fuzzy systems in the analytic way, but it may lead to undesirable and/or inaccurate results. The major reason is that the redesigned digital control gain matrices are obtained by considering only the local state matching of each sub-closed-loop system [3-5]. To overcome this weakness, Lee et al. a global state-matching technique based on the convex optimization method, the linear matrix inequalities (LMIs) method, proposed in [4,5]. Specifically, their method is to globally match the states of the overall closed-loop T-S fuzzy system with the predesigned analog fuzzy-model-based controller and those with the digitally redesigned fuzzy-model-based controller, and further to examine the stabilizability by the redesigned controller in the sense of Lyapunov. However, the IDR problem cannot be solved by the analytic way according as transferring the local approach to the global one. In addition, a multirate control problem has not been fully tackled in this IDR.

Motivated by the above observations, we study a multirate control for T-S fuzzy systems by using the LMI-based IDR method. The main features of the proposed method are as follows: First, the compensated bilinear transform is proposed to improve the performance of IDR. Second, the multirate fuzzy control is developed, and the control input is changed N times during one sampling period. Second, some sufficient conditions, involved in matching the states and compensating the bilinear transform, and stabilizing the digital fuzzy control system, are provided in the LMI format.

II. Problem Statement

In this section, we consider the problem of matching the responses of an existing analog fuzzy control system with those of the digital fuzzy control system for the same initial conditions.

Consider the system described by the following Takagi−Sugeno fuzzy model:

$$\dot{x}(t) = \sum_{i=1}^{r} \theta_i(z(t))(A_i x(t) + B_i u(t)) \quad (1)$$

where $x(t) \in R^n$ and $u_a(t) \in R^m$, r is the number of model rules, $z(t) = [z_1(t) \cdots z_p(t)]^T$ is the premise variable vector that is a function of states, and $\theta_i(z(t))$ is the normalized weight for each rule, that is $\theta_i(z(t)) \geq 0$ and $\sum_{i=1}^{r} \theta_i(z(t)) = 1$.

For the fuzzy system (1), an existing analog fuzzy controller takes the following form:

$$u_a(t) = \sum_{i=1}^{r} \theta_i(z(t)) K_i x_a(t) \quad (2)$$

where the subscript 'a' means the analog control. By substituting (2) into (1), we obtain

$$\dot{x}_a(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \theta_i(z(t)) \theta_j(z(t))(A_i + B_i K_j) x_a(t) \quad (3)$$

It follows from (3) that

$$x_a(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \theta_i(z(t_0)) \theta_j(z(t_0)) e^{(A_i + B_i K_j)(t - t_0)} x_a(t_0) + \Theta_1(x_a(\mu), x_a(t_0)) \quad (4)$$

where

$$\begin{aligned} &\Theta_1(x_a(\mu), x_a(t_0)) \\ &= \int_{t_0}^{t} \sum_{i=1}^{r} \sum_{j=1}^{r} \theta_i(z(\mu)) \theta_j(z(\mu))(A_i + B_i K_j) x_a(\mu) d\mu \\ &\quad - \int_{t_0}^{t} \sum_{i=1}^{r} \sum_{j=1}^{r} \theta_i(z(t_0)) \theta_j(z(t_0))(A_i + B_i K_j) e^{(A_i + B_i K_j)(t - \mu)} x_a(t_0) d\mu \end{aligned}$$

We consider a multirate digital fuzzy controller where the digital control input $u_d(t)$ is held constant N times between the sampling points. Let T and τ be the sampling time and the control update time, respectively. The relation between T and τ can be defined as $\tau = T/N$. Then, the digital fuzzy controller is implemented by

$$u_d(t) = \sum_{i=1}^{r} \theta_i(z(kT + \kappa\tau)) F_i x_d(kT + \kappa\tau) \quad (5)$$

for the time interval $[kT + \kappa\tau, kT + \kappa\tau + \tau)$, $k \times \kappa \in Z_0 \times Z_{[0,N-1]}$, where the subscript '$d$' denotes the digital control.

Remark 1. *Within a sampling time T, the single−rate controller is static, while the multirate controller is periodically time−varying, i.e., the control action is updated at a small period τ. Clearly, for the single−rate case, this control update period τ is equal to the sampling period T. Specifically, setting $N = 1$ in (5) leads to the following single−rate fuzzy controller:*

$$u_d(t) = \sum_{i=1}^{r} \theta_i(z(kT)) F_i x_d(kT) \quad (6)$$

By interfacing an ideal sampler and a zero−order holder between (1) and (5), the closed−loop system is represented as

$$\dot{x}_d(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \theta_i(z(t)) \theta_j(z(kT + \kappa\tau)) \times (A_i x_d(t) + B_i F_j x_d(kT + \kappa\tau)) \quad (7)$$

for the time interval $[kT + \kappa\tau, kT + \kappa\tau + \tau)$, $k \times \kappa \in Z_0 \times Z_{[0,N-1]}$.

It follows from (7) that

$$\begin{aligned} x_d(kT + \kappa\tau + \tau) = &\sum_{i=1}^{r} \sum_{j=1}^{r} \theta_i(z(kT + \kappa\tau)) \theta_j(z(kT + \kappa\tau)) \\ &\times (\Phi_i + \Gamma_i F_j) x_d(kT + \kappa\tau) \\ &+ \Theta_2(x_d(\mu), x_d(kT + \kappa\tau)) \end{aligned} \quad (8)$$

where $\Phi_i = e^{A_i \tau}$, $\Gamma_i = (\Phi_i - I) A_i^{-1} B_i$, and

$$\begin{aligned}&\Theta_2(x_d(\mu), x_d(kT+\kappa\tau))\\ &= \int_{kT+\kappa\tau}^{kT+\kappa\tau+\tau} \sum_{i=1}^{r}\sum_{j=1}^{r} \theta_i(z(\mu))\theta_j(z(kT+\kappa\tau))\\ &\quad \times (A_i x_d(\mu) + B_i F_j x_d(kT+\kappa\tau))\, d\mu\\ &\quad - \int_{kT+\kappa\tau}^{kT+\kappa\tau+\tau} \sum_{i=1}^{r}\sum_{j=1}^{r} \theta_i(z(kT+\kappa\tau))\theta_j(z(kT+\kappa\tau))\\ &\quad \times \left[A_i e^{A_i(kT+\kappa\tau+\tau-\mu)} + e^{A_i(kT+\kappa\tau+\tau-\mu)} B_i F_j\right] x_d(kT+\kappa\tau)\, d\mu\end{aligned}$$

Now letting $t_0 = kT + \kappa\tau$ and $t = kT + \kappa\tau + \tau$ in (4), we have

$$\begin{aligned}x_a(kT+\kappa\tau+\tau) &= \sum_{i=1}^{r}\sum_{j=1}^{r} \theta_i(z(kT+\kappa\tau))\theta_j(z(kT+\kappa\tau))\\ &\quad \times \Xi_{ij} x_a(kT+\kappa\tau)\\ &\quad + \Theta_1(x_a(\mu), x_a(kT+\kappa\tau))\end{aligned} \quad (9)$$

where $\Xi_{ij}(t-t_0) = e^{(A_i+B_iK_j)\tau}$.

From (8) and (9), the IDR problem is to find the digital gains F_i under the assumption that $x_c(kT+\kappa\tau) = x_d(kT+\kappa\tau)$ such that

$$\Xi_{ij} = \Phi_i + \Gamma_i F_j \quad (10)$$

and

$$\Theta_1(x_a(\mu), x_a(kT+\kappa\tau)) = \Theta_2(x_d(\mu), x_d(kT+\kappa\tau)) \quad (11)$$

are satisfied. However, it may be impossible to solve to (11) because the condition (11) is highly complex nonlinear matrix equality. The following assumption is introduced for ease of control synthesis.

Assumption 1. Assume that $\Theta_1(x_a(\mu), x_a(kT+\kappa\tau)) = 0$ and $\Theta_2(x_d(\mu), x_d(kT+\kappa\tau)) = 0$ for sufficiently small sampling period. Then the equations (8) and (9) can be simplified as

$$\begin{aligned}x_d(kT+\kappa\tau+\tau) &= \sum_{i=1}^{r}\sum_{j=1}^{r} \theta_i(z(kT+\kappa\tau))\theta_j(z(kT+\kappa\tau))\\ &\quad \times (\Phi_i + \Gamma_i F_j)\, x_d(kT+\kappa\tau)\end{aligned} \quad (12)$$

and

$$\begin{aligned}x_a(kT+\kappa\tau+\tau) &= \sum_{i=1}^{r}\sum_{j=1}^{r} \theta_i(z(kT+\kappa\tau))\theta_j(z(kT+\kappa\tau))\\ &\quad \times \Xi_{ij} x_a(kT+\kappa\tau)\end{aligned} \quad (13)$$

respectively.

Remark 2. *It is found in [1,2,3,4,5] that the discrete-time models of (3) and (7) have been described by (13) and (12), respectively.*

Then, in principle, the digital gain F_i can be determined from (10).

Remark 3. *It is noted that the previous results [1,2,3,4,5] have been interested in the single-rate control problem, which refers only to special case,* $N = 1$ *in the multirate control problem. Meanwhile, in several results in the linear control system, it is shown that the multirate control scheme is more realistic approach, which allows us to consider the intersampling points between sampling points.*

III. Main Results

To relax the ESM condition (10), we first obtain a new discretized version of the analog control system (13) by applying the bilinear transform, and then matches the resulting analog system and the digital system in the discrete-time domain.

Proposition 1. IDR based on the block-pulse method The responses of the digital fuzzy system (8) and the analog one (9) will closely match at $t = kT + \kappa\tau + \tau$ for an arbitrary initial state $x_a(kT+\kappa\tau) = x_d(kT+\kappa\tau)$ if there exist the redesigned digital feedback gains F_i such that

$$\frac{1}{2}(I - \frac{1}{2}K_j\Gamma_i)^{-1} K_j(\Phi_i + I) = F_j, \quad (i,j) \in I_R \times I_R \quad (14)$$

Then, the overall digital control

system is redesigned as

$$\dot{x}_d(t)=\sum_{i=1}^{q}\sum_{j=1}^{q}\theta_i(z(t))\theta_j(z(kT))\times\left[\Phi_i x_d\ t\ +\Gamma_i\frac{1}{2}\ I-\frac{1}{2}K_j\Gamma_i\ ^{-1}K_j)\Phi_i+I\ (x_d\ kT\ \right]. \quad (15)$$

Remark 4. *Note that Proposition 1 is more relaxed condition than (10) in the general case,* $m<rn$*. Equation (10) consists of* $(rn)^2$ *scalar equations with* rmn *unknown elements in* $F_i, i\in I_R$*, while (14) is composed of* r^2mn *scalar equations with* rmn *unknown elements in* $F_i, i\in I_R$*. However, we still do not obtain a solution to (14) in an analytic way, except in the case of common* H*, i.e.,* $\Gamma_1=\Gamma_2=\cdots=\Gamma_r$*. Also, the error involved in the bilinear transform causes a performance decline of the state-matching, especially in the slow sampling frequency.*

To avoid the difficulties on the Remark 4, we proposed a new compensated block-pulse function method, and then we reformulate the IDR problem as the minimization problem (MP). The following theorem is the main results of this paper

Theorem 1. *System (1) is stabilizable by the digital feedback gains* F_i *and the norm conditions of realizing the conditions (14) of the corresponding closed-loop system is smaller than a given* γ^2 *if there exist a matrix* $Q=Q^T\succ 0$*, and matrices* $X_{ij}=X_{ij}^T=X_{ji}=X_{ji}^T$*,* E_{ij}*,* S_j *such that the following two MPs:*

MP1: Minimize$_{E_{ij},F_{Ai}}$ γ_1 *subject to*

$$\begin{bmatrix}-\gamma_1 I & (\bullet)^T\\ \Xi_{ij}-\Phi_i-\Gamma_i 0.5(1-0.5K_j\Gamma_i)^{-1}K_j(\Phi_i+E_{ij}) & -\gamma_1 I\end{bmatrix}\prec 0,$$

$$\begin{bmatrix}-\gamma_1 I & (\bullet)^T\\ \Gamma_i 0.5(1-0.5K_j\Gamma_i)^{-1}K_j(\Phi_i+E_{ij})-\Gamma_i F_{Aj} & -\gamma_1 I\end{bmatrix}\prec 0,$$

MP2: Minimize$_{Q,S_i,X_{ij}}$ γ_2 *subject to*

$$\begin{bmatrix}-\gamma_2 Q & (\bullet)^T\\ S_i & -\gamma_2 I\end{bmatrix}\prec 0$$

$$\begin{bmatrix}-I & (\bullet)^T\\ x(0) & -Q\end{bmatrix}\prec 0,$$

$$\begin{bmatrix}-Q+X_{ij} & (\bullet)^T\\ \dfrac{\hat{G}_{ij}Q+\Gamma_i S_j+\hat{G}_{ji}Q+\Gamma_j S_i}{2} & -Q\end{bmatrix}\prec 0$$

$$\left[X_{ij}\right]_{r\times r}\prec 0,\quad (i,j)\in I_J\times I_R$$

are feasible, where $Q=P^{-1}$*,* $S_i=F_{Bi}Q$*, and* $F_i=F_{Ai}+F_{Bi}$*.*

IV. Conclusions

This paper proposed a new LMI-based IDR method control using the compensated bilinear transform. The sufficient conditions for the stabilization problem and IDR problem are derived in the LMI format.

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