Iterative Learning Control of Discrete-Time Nonminimum-Phase Systems

2005-11

Gu-Min Jeong

School of Electrical Eng.
Kookmin University, Korea
gm1004@kookmin.ac.kr

CONTENTS

1. Introduction

- Nonminimum-phase system
  - Zero-dynamics is unstable
  - Continuous time domain : some of finite zeros being located in the LHP
  - Discrete time domain : some of finite zeros being located outside of the unit circle.
- Control of nonminimum-phase system
  - Approximation of unstable zero to stable zero
  - Stable inversion
  - Pseudo-inverse based inversion
- Iterative Learning Control
  - Find input iteratively which tracks the desired output
  - System model may be unknown
  - Mainly for the minimum-phase system

1. Introduction

- Objective of the research
- Control of nonminimum-phase system
  - Inversion based on output-to-input mapping
  - Time reversal for maximum phase systems
  - Advanced time approach for nonminimum-phase systems
- Learning control of nonminimum phase systems
  - Control of uncertain nonminimum phase systems
  - Stable inverse mapping
  - Simple learning structure
  - Generalized learning law

2. Related Works

- Nonminimum-phase system

  A dynamic system is nonminimum phase if there are some unstable manifolds in the zero dynamics of the system. For linear discrete time systems, this corresponds to some of finite zeros being located outside of the unit circle.

- Stable and unstable manifolds

  $W^s(0) = \{x \in u \| \phi_t(x) \to 0,\ \ as\ \ t \to \infty;\ \phi_t(x) \in U,\ \ \forall t \geq 0\}$
  $W^u(0) = \{x \in u \| \phi_t(x) \to 0,\ \ as\ \ t \to -\infty;\ \phi_t(x) \in U,\ \ \forall t \leq 0\}$
  $U \subset R^n$, $\phi_t(x)$ is the flow of the dynamic system.

2. Related Works

- Feedforward control of linear discrete-time systems
  - Zero Phase Error Tracking Control
  - Pole Zero Cancellation with Series Approximation
  - Pole Zero Cancellation with Modified Series Approximation

  → Approximation of unstable zero to stable zero
  → Difficulty of analysis of output error due to approximation
  → Sensitive to modeling uncertainties
  → Exact model of the system must be given

2. Related Works

- Stable inversion method

Normal form using $z = \Phi(x)$.

$$\begin{aligned} z_1(i+1) &= z_2(i) \\ \vdots \quad &\vdots \quad \vdots \\ z_{\sigma-1}(i+1) &= z_\sigma(i) \\ z_\sigma(i+1) &= R\xi(i) + S\eta(i) + Ku(i) \\ \eta(i+1) &= P\xi(i) + Q\eta(i) \end{aligned}$$

Here $\eta(i) = [z_{\sigma+1}(i), \cdots, z_n(i)]^T$, $\xi(i) = [z_1(i), \cdots, z_\sigma(i)]^T$

– Zero dynamics of the system $\eta(i+1) = Q\eta(i)$

2. Related Works

- Stable inversion method

– Jordan form transformation

$$\bar{\eta}(i+1) = \bar{P}\bar{\xi}(i) + \bar{Q}\bar{\eta}(i)$$

where $Q = \begin{bmatrix} Q_s & 0 \\ 0 & Q_u \end{bmatrix}$

Q_s : all stable eigen values

Q_u : all unstable eigen values

– Solve forwards $\bar{\eta}_s(i+1) = \bar{Q}_s\bar{\eta}_s(i) + C_s(i)$.
Solve backwards $\bar{\eta}_u(i+1) = \bar{Q}_u\bar{\eta}_s(i) + C_u(i)$.

– Boundary condition : $\eta(-\infty) = \eta(\infty) = 0$

2. Related Works

- Pseudo-inverse based inversion

  – Direct inverse :

$$\mathbf{u}_{[0,N-1]} = (\mathbf{J}_a)^{-1}(\mathbf{y}_{[\sigma,N+\sigma-1]} - \mathbf{H}_a x(0))$$

  – Pseudo-inversion

$$\mathbf{u}_{[0,N-1]} = (\mathbf{J}_a^T\mathbf{J}_a)^{-1}\mathbf{J}_a(\mathbf{y}_{[\sigma,N+\sigma-1]} - \mathbf{H}_a x(0))$$

  – Pseudo-inversion for a given α

$$\mathbf{u}_{[0,N-1]} = (\alpha I + \mathbf{J}_a^T\mathbf{J}_a)^{-1}\mathbf{J}_a(\mathbf{y}_{[\sigma,N+\sigma-1]} - \mathbf{H}_a x(0))$$

2. Related Works

- ILC using stable inversion method

- System equation

$$\begin{aligned} \dot{x}(t) &= f(x(t)) + g(x(t)), x(0) = 0 \\ y(t) &= h(x(t)) \end{aligned}$$

- Basic Approach
  - Assume that the linearized model is known around $(x, u) = (0, 0)$.
  - Obtain input u for the linearized system.
  - Find the solution of the nonlinear system using iteration.

2. Related Works

- Stable inversion method
  - Exact knowledge of the system dynamics
  - Truncation error due to the time horizon
  - Difficulty of the analysis of the output error
  - All states must be known
- Pseudo-inverse based inversion
  - Approximate solution
  - Difficulty of the analysis of the output error
- Proposed Method
  - Output to input mapping
  - No truncation error in time interval

2. Related Works

- Limitation of conventional schemes
  - Require precise model or precise linearized model
  - Truncation error or approximate solutions
  - Calculation burden and sensitivity due to the input-to-state mapping
  - Difficulty in the analysis of output error
- Proposed Method

  Control of nonminimum phase systems

  Inversion based on output-to-input mapping

  Advanced time approach

  Learning control of nonminimum phase systems

  Control of uncertain nonminimum phase systems

  Simple learning structure

  Generalized learning law

3. Time Reversal for Nonminimum-Phase Systems

- System :

$$x(i+1) = Ax(i) + Bu(i)$$
$$y(i) = Cx(i)$$

where $u \in \Re^1$, $x = [x_1, \cdots, x_n]^T \in \Re^n$, $y \in \Re^1$

- $\mathbf{u}_{[i,j]} := [u(i), \cdots, u(j)]^T$, $\mathbf{y}_{(i,j)} := [y(i), \cdots, y(j)]^T$.
- Assumptions

  (A3.1) The system is controllable and observable.

  (A3.2) The matrix A is invertible.

  (A3.3) $\beta_n \neq 0$ in $G(z) = \frac{\beta_1 z^{n-1} + \cdots + \beta_n}{z^n + \alpha_1 z^{n-1} + \cdots + \alpha_n}$.

- $\mathbf{u}_{[0,N-1]} \longleftrightarrow \mathbf{y}_{[n,N+n-1]}$
- Input-output relation $\mathbf{y}_{[n,N+n-1]} = \mathbf{H}_e x(0) + \mathbf{J}_e \mathbf{u}_{[0,N-1]}$

3. Time Reversal for Nonminimum-Phase Systems

- **Lemma 3.1** : Nonsingularity of $\mathbf{J}_e$, uniqueness and existence of $\mathbf{u}^d_{[0,N-1]}$.
- **Lemma 3.2** : Time reversal and the stability of the inverse mapping.
  - Time reversal : $N(z) \rightarrow N(z^{-1})$, $N(z^{-1})$ becomes minimum phase.
    - Set $\mathbf{y}^d_{[N+1,N+n-1]}$ to appropriate constants and $\mathbf{u}_{[N,N+n-2]} = 0$
    - $u(N-1) = \frac{y^d(N-1+n) + \cdots + \alpha_n y^d(N-1)}{\beta_n}$.
    - $u(N-2) = \frac{y^d(N-2+n) + \cdots + \alpha_n y^d(N-2) - \beta_{n-1} u(N-1)}{\beta_n}$
    - $u(i)$ is determined backwards.
    - Time reversal : $N(z) \rightarrow N(z^{-1})$.
      $\beta_1 z^{n-1} + \cdots + \beta_n \rightarrow \beta_n z^{n-1} + \cdots + \beta_1$

3. Time Reversal for Nonminimum-Phase Systems

- Input-output relations in conventional ILC

  – $\mathbf{u}_{[0,N-1]} \longleftrightarrow \mathbf{y}_{[\sigma,N+\sigma-1]}$, $\mathbf{y}_{[\sigma,N+\sigma-1]} = \mathbf{H}_a x(0) + \mathbf{J}_a \mathbf{u}_{[0,N-1]}$

  1. input update law :
     $u^{k+1}(i) = u^k(i) + l(y^d(i+\sigma) - y^k(i+\sigma))$
     $\mathbf{u}^{k+1}_{[0,N-1]} = \mathbf{u}^k_{[0,N-1]} + S(\mathbf{y}^d_{[\sigma,N+\sigma-1]} - \mathbf{y}^k_{[\sigma,N+\sigma-1]})$
  2. Convergence condition $\|I - SJ_a\| \le \rho < 1$

- Proposed method for linear maximum-phase systems

  – $\mathbf{u}_{[0,N-1]} \longleftrightarrow \mathbf{y}_{[n,N+n-1]}$, $\mathbf{y}_{[n,N+n-1]} = \mathbf{H}_b x(0) + \mathbf{J}_b \mathbf{u}_{[0,N-1]}$

  1. $\mathbf{y}_{[n,N+n-1]} \to \mathbf{u}_{[0,N-1]}$ is stable
  2. input update law :
     $\mathbf{u}^{k+1}_{[0,N-1]} = \mathbf{u}^k_{[0,N-1]} + S(\mathbf{y}^d_{[n,N+n-1]} - \mathbf{y}^k_{[n,N+n-1]})$
  3. $\mathbf{u}^k_{[0,N-1]} \to \mathbf{u}^d_{[0,N-1]}$
  4. Convergence condition $\|I - SJ_b\| \le \rho < 1$

4. ILC of Maximum-Phase Systems

- System :

$$\begin{aligned} x(i+1) &= f(x(i)) + g(x(i))u(i) \\ y(i) &= h(x(i)). \end{aligned}$$

- ILC with advanced output data

  – $\mathbf{u}_{[0,N-1]} \longleftrightarrow \mathbf{y}_{[n,N+n-1]}$

  – $\mathbf{y}_{[n,N+n-1]} = \mathbf{F}(x(0), \mathbf{u}_{[0,N-1]})$

  1. $\mathbf{y}_{[n,N+n-1]} \to \mathbf{u}_{[0,N-1]}$ is stable
  2. input update law :
     $\mathbf{u}^{k+1}_{[0,N-1]} = \mathbf{u}^k_{[0,N-1]} + S(\mathbf{y}^d_{[n,N+n-1]} - \mathbf{y}^k_{[n,N+n-1]})$
  3. $\mathbf{u}^k_{[0,N-1]} \to \mathbf{u}^d_{[0,N-1]}$

4. ILC of Maximum-Phase Systems

- System :

$$\begin{aligned} x(i+1) &= f(x(i)) + g(x(i))u(i) \\ y(i) &= h(x(i)). \end{aligned}$$

- ILC with advanced output data
  - $\mathbf{u}_{[0,N-1]} \longleftrightarrow \mathbf{y}_{[n,N+n-1]}$
  - $\mathbf{y}_{[n,N+n-1]} = \mathbf{F}(x(0), \mathbf{u}_{[0,N-1]})$

  1. $\mathbf{y}_{[n,N+n-1]} \to \mathbf{u}_{[0,N-1]}$ is stable
  2. input update law :
     $\mathbf{u}^{k+1}_{[0,N-1]} = \mathbf{u}^{k}_{[0,N-1]} + S(\mathbf{y}^{d}_{[n,N+n-1]} - \mathbf{y}^{k}_{[n,N+n-1]})$
  3. $\mathbf{u}^{k}_{[0,N-1]} \to \mathbf{u}^{d}_{[0,N-1]}$

5. Advanced Time Approach for Nonminimum-Phase Systems

- System :

$$\begin{aligned} x(i+1) &= Ax(i) + Bu(i) \\ y(i) &= Cx(i) \end{aligned}$$

where $u \in \Re^1$, $x = [x_1, \cdots, x_n]^T \in \Re^n$, $y \in \Re^1$

- $\mathbf{u}_{[i,j]} := [u(i), \cdots, u(j)]^T, \mathbf{y}_{(i,j)} := [y(i), \cdots, y(j)]^T.$
- Minimum-phase system : $\mathbf{u}_{[0,N-1]} \longleftrightarrow \mathbf{y}_{[\sigma,N+\sigma-1]}$
- Maximum-phase system : $\mathbf{u}_{[0,N-1]} \longleftrightarrow \mathbf{y}_{[n,N+n-1]}$

  - $\mathbf{u}_{[0,N-1]} \longleftrightarrow \mathbf{y}_{[n,N+n-1]}$
  - Input-output relation $\mathbf{y}_{[\sigma+d,N+\sigma+d-1]} = \mathbf{H}_c x(0) + \mathbf{J}_c \mathbf{u}_{[0,N-1]}.$

5. Advanced Time Approach for Nonminimum-Phase Systems

- $\mathbf{u}_{[0,N-1]} \longleftrightarrow \mathbf{y}_{[\sigma+d,N+\sigma+d-1]}$
- Input-output relation $\mathbf{y}_{[\sigma+d,N+\sigma+d-1]} = \mathbf{H}_c x(0) + \mathbf{J}_c \mathbf{u}_{[0,N-1]}$,

$$\mathbf{H}_c = \left[(H_{d+1})^T, \cdots, (H_{N+d})^T\right]^T,$$

$$\mathbf{J}_c = \begin{bmatrix} J_{d+1} & J_d & \cdots & 0 \\ J_{d+2} & J_{d+1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ J_{N+d} & J_{N+d-1} & \cdots & J_{d+1} \end{bmatrix}.$$

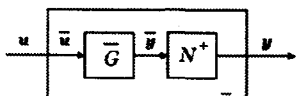
$$H_l = CA^{\sigma+l-1},\ J_l = CA^{\sigma+l-2}B.$$

(A1) The system is stable, controllable and observable.
(A2) The matrix A is invertible.
(A3) $\beta_n \neq 0$ in (2).
(A4) The matrix $\mathbf{J}_c$ is nonsingular.

- **Theorem**

The inverse mapping from $\mathbf{y}^d_{[\sigma+d,N+\sigma+d-1]}$ to $\mathbf{u}^d_{[0,N-1]}$ is stable for $d = d_0$

G(z)



- N^+ : minimum phase zeros of the system, $\bar{G}(z)$: Maximum phase system

$\bar{G}(z) = \frac{y(z)}{u(z)N^+(z)} = \frac{\bar{y}(z)}{u(z)} = \frac{N^-(z)}{D(z)}$

6. ILC of Nonminimum-Phase Systems

- Input update law

$$\mathbf{u}^{k+1}_{[0,N-1]} = \mathbf{u}^{k}_{[0,N-1]} + \mathbf{S}\mathbf{e}^{k}_{[\sigma+d,N+\sigma+d-1]}, \quad 0 \le d \le n-\sigma$$

where $\mathbf{e}^{k}_{[\sigma+d,N+\sigma+d-1]} = \mathbf{y}^{d}_{[\sigma+d,N+\sigma+d-1]} - \mathbf{y}^{k}_{[\sigma+d,N+\sigma+d-1]}$.

- if $d = d_0$, where d_0 is the number of nonminimum phase zeros of the system, the inverse mapping is stable.
- if $d = 0$, it is equivalent to the conventional ILC based on the relative degree

- **Theorem**

The uncertain system (1) satisfies **(A1)–(A4)**. If the condition

$$\|I - \mathbf{S}\mathbf{J}_c\| \le \rho < 1$$

holds, the input $\mathbf{u}^{k}_{[0,N-1]}$ converge to $\mathbf{u}^{d}_{[0,N-1]}$ as $k \to \infty$.

6. ILC of Nonminimum-Phase Systems

- System :

$$\begin{aligned} x(i+1) &= f(x(i)) + g(x(i))u(i) \\ y(i) &= h(x(i)). \end{aligned}$$

- $\mathbf{y}^{d}_{[\sigma+d,N+\sigma+d-1]} = \mathbf{F}(x(0), \mathbf{u}_{[0,N-1]})$

(A1') The system (7) is stable. Also, the relative degree of the system (7) is σ and is well defined $\forall(\mathbf{x}, u) \in \mathrm{R}^{n+1}$ with respect to $u(i)$.

(A2') For the system (7), $\|\mathbf{y}^{d}_{[\sigma+d,N+\sigma+d-1]}\| \le c_1, \forall N$ and $\|\mathbf{x}(0)\| \le c_2$ for some constants c_1 and c_2.

(A3') The linearized system (9) is stable, has d_0 nonminimum-phase zeros and satisfies the assumptions **(A1)-(A4)**.

(A4') For any realizable output trajectory $\mathbf{y}^{d}_{[\sigma+d,N+\sigma+d-1]}$ that corresponds to a given initial condition $\mathbf{x}^{d}(0)$, $\mathbf{F}$ is a one-to-one and continuous mapping.

6. ILC of Nonminimum-Phasse Systems

- Theorem
  Let us assume that the system (7), the desired trajectory and the initial condition satisfy (A1')–(A4'). Let us set $d = d_0$, where d_0 is the number of nonminimum phase zeros. Then the desired trajectory $\mathbf{u}^d_{[0,N-1]}$ is bounded.

- **Input update law**

$$\mathbf{u}^{k+1}_{[0,N-1]} = \mathbf{u}^{k}_{[0,N-1]} + \mathbf{S}^k \mathbf{e}^d_{[\sigma+d,N+\sigma+d-1]}, \quad 0 \le d \le n - \sigma$$

- **Theorem**
  The system satisfies (A1')-(A4') and the system dynamics may not be known completely. If the condition

$$\|I - \mathbf{S}^k \mathbf{J}^k_d\| \le \rho < 1, \text{ for all } k$$

  is satisfied, the input $\mathbf{u}^k_{[0,N-1]}$ converges to bounded $\mathbf{u}^d_{[0,N-1]}$ as $k \to \infty$.

7. Simulation Results

- **System**

$$\begin{aligned} x_1(i+1) &= x_2(i) + 0.1u(i) \\ x_2(i+1) &= -x_1^3(i) + x_3(i) \\ x_3(i+1) &= 4x_1^3(i) + (1+\sin(x_2(i))^2)u(i) \\ y(i) &= x_1(i) + 2.5x_2(i) + x_3(i) \end{aligned}$$

Relative degree :1 Number of nonminimum phase zeros : 1

- **Setting** $z_1 = y, z_2 = x_1, z_3 = x_2$

$$\begin{bmatrix} z_1(i+1) \\ z_2(i+1) \\ z_3(i+1) \end{bmatrix} = \begin{bmatrix} 2.6 & -2.6 & -5.5 \\ 0 & 0 & 1 \\ 1 & -1 & -2.5 \end{bmatrix} \begin{bmatrix} z_1(i) \\ z_2(i) \\ z_3(i) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\sin^2(z_3(i)) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(i)$$

$$y(i) = z_1(i).$$

7. Simulation Results

- **Stable Inversion Method**

o Zero Dynamics

$$\begin{bmatrix} z_2(i+1) \\ z_3(i+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2.5 \end{bmatrix} \begin{bmatrix} z_2(i) \\ z_3(i) \end{bmatrix} + \begin{bmatrix} 0 \\ y^d(i) \end{bmatrix}$$

o Jordan form transformation

$$\tilde{\eta}(i+1) = T^{-1}\mathbf{z}(i+1) = D\tilde{\eta}(i) + T^{-1} \begin{bmatrix} 0 \\ y^d(i) - \sin^2(z_3(i)) \end{bmatrix}$$

o Picard Iteration

$$\begin{aligned} \tilde{\eta}_0(i) &= 0 \\ &\vdots \\ \tilde{\eta}_{m+1}(i) &= \sum_{k=-\infty}^{\infty} \phi(i-k)\{T^{-1} \begin{bmatrix} 0 \\ y^d(k-1) - \sin^2(-0.4472\tilde{\eta}_1 + 0.8944\tilde{\eta}_2) \end{bmatrix} \} \end{aligned}$$

7. Simulation Results

- **Stable Inversion Method**

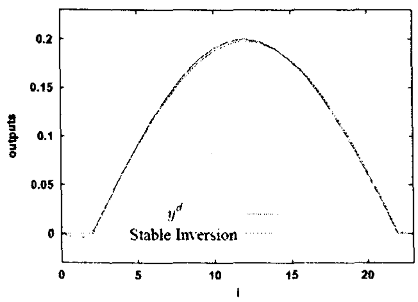


Figure 1: Output using the stable inversion

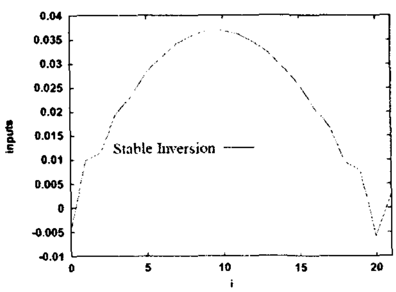


Figure 2: Input using the stable inversion

7. Simulation Results

● **Proposed Method**

o **Model**

$$\begin{aligned} x_1(i+1) &= 1.2x_2(i) + 0.2x_3(i) \\ x_2(i+1) &= -0.1x_1(i) + x_3(i) \\ x_3(i+1) &= 0.4x_3(i) + u(i) \\ y(i) &= x_1(i) + 2.5x_2(i) + x_3(i) \end{aligned}$$

o **Input Update Law**

$$\mathbf{u}^{k+1}_{[0,N-1]} = \mathbf{u}^{k}_{[0,N-1]} + \mathbf{S}^k \mathbf{e}^d_{[\sigma+d_0,N+\sigma+d_0-1]}$$

$$\mathbf{S} = 0.5 \times \hat{\mathbf{J}}_d^{-1}$$

7. Simulation Results

● **Proposed Method**

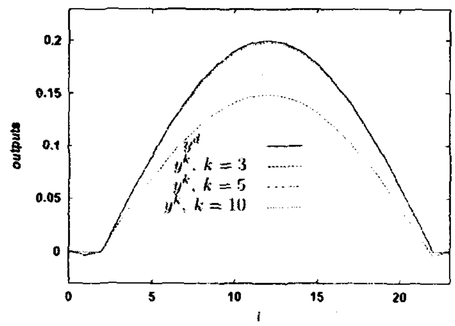


Figure 4: Outputs using the proposed method

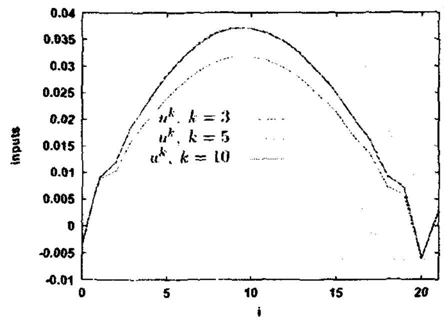


Figure 5: Inputs using the proposed method

7. Simulation Results

- Comparison

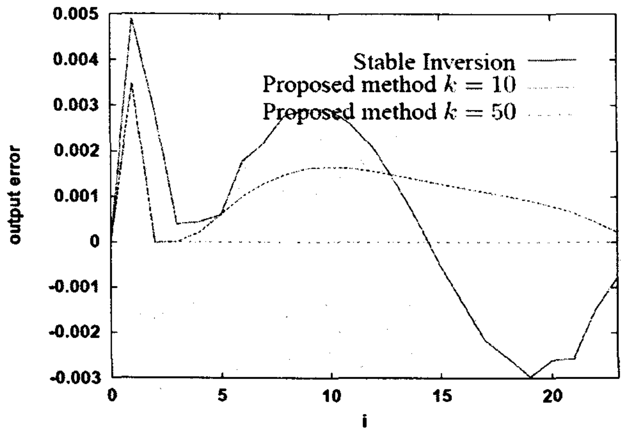


Figure 3: Output error

8. Conclusion and Future Work

▸ Conclusion

Inversion based on output-to-input mapping

Advanced time approach for nonminimum-phase systems

Simple learning structure using input update law

Generalized learning law including both minimum-phase and nonminimum phase systems

No requirement of the exact linearized model of the system

▸ Future work

Extension to learning scheme with feedback controller

Nonsingularity condition of Jc

To make the convergence condition less strict

Neural network / Fuzzy controller design

Thank you !

gm1004@kookmin.ac.kr