A LOW NUMERICAL DIFFUSIVE SCHEME FOR POLLUTANT TRANSPORT IN SHALLOW WATER SYSTEMS

MARCO PETTI [1] and SILVIA BOSA [2]

[1] Professor, Dipartimento di Georisorse e Territorio,
University of Udine, via Cotonificio 114, 33100 Udine, Italy
(Tel: +39-0432-558712, Fax: +39-0432-558700, e-mail: petti@dgt.uniud.it)
[2] PhD student, Dipartimento di Georisorse e Territorio,
University of Udine, via Cotonificio 114, 33100 Udine, Italy
(Tel: +39-0432-558715, Fax: +39-0432-558700, e-mail: bosa@dgt.uniud.it)

In recent years there has been a growing interest for numerical methods, which are able to simulate natural phenomena such as the advection and diffusion of pollution in hydrodynamic environments as rivers, lakes and lagoons. For a wide class of dispersed substances the classical advection-diffusion equation based on passive tracer concept is appropriate to describe the motion of the pollutant. The hydrodynamic field can be described by the shallow water equations.

In literature many works are devoted to finite volume methods to integrate these equation system. Their main aspect is the peculiar way used to evaluate the advective intercell fluxes. Particularly many authors have adopted numerical models, that exploit Godunov's procedure, in order to evaluate the intercell flux as the result of a local Riemann problem between neighboring cells. The results obtained through classical second order schemes in pure hydrodynamic problems are good and assure the models to be shock-capturing.

Nevertheless to control the spurious oscillations introduced by a second order scheme, it is necessary that the numerical scheme is total variation diminishing (TVD). Considering the shallow water equations coupled with the advection-diffusion equation, the TVD property can lead to a rise in numerical damping and diffusion (Leonard, 1991). This problem could reveal very misleading in environmental engineering studies. The main aim of this paper is to present a numerical technique, that can help in reducing numerical diffusion, maintaining a coupled shock-capturing model at the same time. In fact, one of the advantages of the proposed method is the simultaneous advancement in time of the hydrodynamic and the solute transport solution.

To integrate the equation system a standard predictor-corrector method is used, that makes the model second order accurate in time. To evaluate the numerical fluxes the two-dimensional problem is reduced to a sequence of augmented one-dimensional problems, and a weighted average flux (WAF) Riemann solver with MINBEE limiters is applied.

The method described in the present paper is based on a particular technique of higher order to reconstruct the left and right states of the local Riemann problem, i.e. a fifth order one-dimensional WENO reconstruction.

A fifth order WENO scheme takes into account polynomials that give rise to a third order reconstruction for cell i and combines them to get a fifth order reconstruction (Jiang and Shu, 1997, Shu, 1997). Considering cell i, it is possible to create 3 different third order reconstructions. These are combined to obtain a fifth order one. A similar operation must be done to extrapolate the state to the left and to the right of the intercell.

The choice of a one-dimensional reconstruction represents a compromise between accuracy and computational efficiency, useful to study a large real domain for which relatively inexpensive numerical method is needed. Even if the overall accuracy of the proposed scheme remains of second order, the one-dimensional WENO reconstruction leads to a large benefit in reduction of numerical diffusion of the model.

The model is compared with a pure second order WAF scheme, that simply considers as the states involved in the local Riemann problem, the states of the cells to the left and to the right of the intercell.

The main aim of the present model is to reduce numerical diffusion, thus the numerical scheme is tested on pure advection problems.

The model is verified on a bench-mark test proposed by Leonard (1991) that represents a 1D pure advective problem of a distribution of pollutant concentration consisting in a discontinuous step, an isolated sine-squared wave and a semi-ellipse (Fig.1).

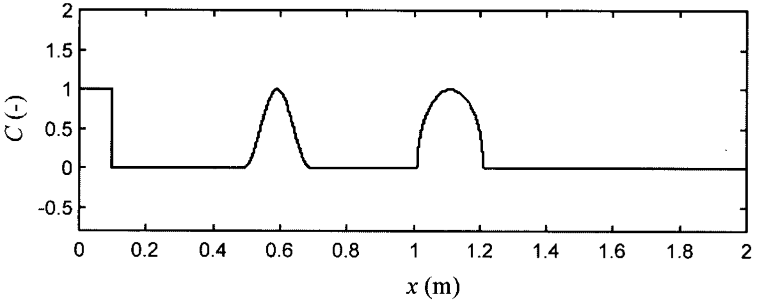

Fig. 1 Initial condition; from left to right the unit step, the isolated sine-squared profile and the semi-ellipse profile.

Another test carried out is the 2D advection of block-shaped pulse (Stefanovic and Stefan, 2001). Both tests have an exact solution documented in literature.

The great advantage of the method proposed is that it gives a good accuracy with computational times sensitively smaller of those obtainable with a pure bidimensional WENO reconstruction.

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