WATER DISTRIBUTION NETWORKS DESIGN OPTIMIZATION

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In this paper, a new nonlinear optimization model is developed which enables us to design a water distribution systems supplied by pumping or gravitation from one or more node sources, which satisfies all constraints including pipe diameters, flow velocities, and nodal pressure heads with a minimum total cost. This model could be applied to any looped or tree-shaped network.

The hydraulic analysis of the water distribution system is perfomed by the iterative Newton-Raphson method. This analysis is coupled with a nonlinear optimization technique in order to minimize the design total cost. Expenses of water distribution system consists of:

a) expenses of pipes and their installations;

b) expenses of pressure generating facilities.

The curve fitting technique using real data can be used to find the function that indicates the expenses of (*a*) item. The expenses of (*b*) item can be considered as a function of reservoirs'elevations or pumps'heads. Hence, the general total cost objective function with constraints might be shown as follows:

$$F_c(D_{ij}, Z_{IP,k}) = f_1(D_{ij}) + f_2(Z_{IP,k}) \quad (1)$$

$$\begin{aligned} &D_{\min} \le D_{ij} \le D_{\max} && (ij = 1,\ldots,T) \\ &V_{\min} \le V_{ij} \le V_{\max} && (ij = 1,\ldots,T) \\ &H_{\min} \le H_j \le H_{\max} && (j = 1,\ldots,N) \\ &Z_{\mathrm{IP}\min} \le Z_{IP,k} \le Z_{\mathrm{IP}\max} && (k = 1,\ldots,N_{RP}) \end{aligned} \quad (2)$$

in which: D_{ij} is the diameter of pipe ij; $Z_{IP,k}$ – reservoir elevation or pump total dynamic head; V_{ij} – flow velocity in pipe *ij*; H_j – pressure head at node *j*; $V_{\min}$, $V_{\max}$ – minimum and maximum allowable flow velocities in pipes; $H_{\min}$, $H_{\max}$ – minimum and maximum allowable nodal pressure heads; Z_{IPmin}, Z_{IPmax} – minimum and maximum allowable reservoir elevations or allowable pump dynamic heads.

The objective function with the constraints given in equation (1) can be changed to unconstrained optimization problem by using the following transformation:

– for pipe diameters:

$$D_{ij} = D_{\min} + (D_{\max} - D_{\min}) \sin^2 d_{ij} \quad (3)$$

– for reservoir elevations:

$$Z_{IP,k} = Z_{\mathrm{IP}\min} + (Z_{\mathrm{IP}\max} - Z_{\mathrm{IP}\min}) \sin^2 z_k \quad (4)$$

where d_{ij} and z_k are new transformed variables.

The above transformation means that an unconstrained optimum needs be sought. The velocities and pressure heads constraints should be nondimensionalized as follows:

$$V_{ij} \le V_{max} \Rightarrow 1 - \frac{V_{ij}}{V_{max}} \ge 0 \qquad (ij = 1, \ldots, T)$$

$$V_{ij} \ge V_{min} \Rightarrow \frac{V_{ij}}{V_{min}} - 1 \ge 0 \qquad (ij = 1, \ldots, T) \tag{5}$$

$$H_j \le H_{max} \Rightarrow 1 - \frac{H_j}{H_{max}} \ge 0 \quad (j = 1, \ldots, N)$$

$$H_j \ge H_{min} \Rightarrow \frac{H_j}{H_{min}} - 1 \ge 0 \quad (j = 1, \ldots, N)$$

The concept of the penalty function was used, and, hence, a generalized objective function can be introduced as:

$$\Gamma(D_{ij}, V_{ij}, H_j, Z_{IP,k}, \omega) = f_1(D_{ij}) + f_2(Z_{IP,k}) + \omega \left[\sum_{ij=1}^{T} \left(1 - \frac{V_{ij}}{V_{max}}\right)^2 + \sum_{ij=1}^{T} \left(\frac{V_{ij}}{V_{min}} - 1\right)^2 + \right.$$

$$\left. + \sum_{j=1}^{N} \left(1 - \frac{H_j}{H_{max}}\right)^2 + \sum_{j=1}^{N} \left(\frac{H_j}{H_{min}} - 1\right)^2 \right] \to \min \tag{6}$$

In the function (6), when velocities and pressure heads are in the allowable ranges ω, should be considered equal to zero which means it does not affect the real cost. On the other hand, if velocities and pressure heads are not in the allowable ranges, an extremely large value should be selected for ω. The objective function given by relation (6) can be minimized by the conjugate direction method.

The coupled hydraulic and optimization analysis of networks can be summarized as follows:

a) Assume pipe diameters and reservoir elevations/pump dynamic heads (preliminary design).

b) Do the hydraulic analysis by initially solwing the nonlinear system of equations at nodes via the Newton-Raphson method to get the piezometric heads at nodes. Then, discharges and head-losses of all pipes, and residual pressure heads at the nodes can be determined easily.

c) Compute the objective function of equation (6).

d) Use Powell's conjugate direction method to minimize the total cost objective function. If the objective function is not minimum, pipe diameters and reservoir elevations should be changed. Then, repeat the cycle from stage (*b*).

This technique has the advantage that it uses a specialized optimization algorithm which minimizes directly an objective multivariable function without constraints, implemented in a computer program for IBM-PC compatible systems. This program is capable of handling nonstandard network components such as booster pumps, reservoirs, check valves and pressure-reducing valves. The advantages to use the proposed program are explained from the numerical application for a complex constructive variant of water distribution network. The optimization approach used in this paper does not require calculation of derivatives. The mathematical model expressed by the objective function (6) constitutes a new way of design for complex looped networks based on unconditioned optimization techniques. This makes the method more efficient and consequently helps the designer to get the best design of even the most complicated water distribution systems with fewer efforts.