APPLICATION OF BOLTZMANN THEORY IN HYDRAULICS

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Boltzmann theory can provide a powerful tool to hydraulic modeling and analysis. Boltzmann theory establishes the mathematical link between the gross properties of matter and the motion of the molecules that make up this matter. The probabilistic moments of the Boltzmann equation are known to provide the Navier-Stokes equations (Vincenti and Kruger 1965). In addition, the Boltzmann equation has been shown to be applicable in flow problems where the Navier-Stokes equations fail to apply (Cercignani 1975). Moreover, the Boltzmann approach provides the basis for the theoretical deriving of well known phenomenological relations, such as Newton's law of viscosity. Furthermore, the Boltzmann approach has been successfully exploited by a number of researchers in various fields. Chen et al. (2003) noted ``...it is simpler to model complex physics first at the Boltzammn level, and then use coarse graining procedures only after modeling is done." Such an approach has been successfully used in gas dynamics (e.g., Xu 2001), in multi-phase and multi-component flows as well as porous media flows. Perhaps the two applications that are most relevant to the hydraulic community are: application of Boltzmann theory to turbulence modeling and application of Boltzmann theory to numerical modeling. For example, recent research shows that well known turbulence models can be derived from the Boltzmann theory if one exploits the analogy between molecular fluctuations and turbulent fluctuations.

This paper focuses on illustrating how the Boltzmann theory can be used to formulate numerical algorithms for shallow water equations in vertical plate. The paper begins by showing the classical shallow water equations are obtainable from the moments of the Boltzmann equation. This connection is then exploited to formulate a model in σ-coordinates for shallow flows on the basis of the Boltzmann equation. In particular, since the shallow water equations are moments of the Boltzmann equation, it follows that the discrete form of the shallow water equations can be derived by taking moments of the discrete Boltzmann equation. The proposed numerical model is second order in time and space and is applied to small amplitude waves, large amplitude waves (bores) and to viscous as well as turbulent flows in channels. Good agreements are observed in these verification and validation tests.

Some of the advantages of using the Boltzmann equation as a starting point for the formulation of numerical algorithms for surface water flows include: (i) the fluxes obtained from the Boltzmann model contain both wave and diffusive effects; (ii) there is no need for characteristic decomposition, (iii) the formulation entails solving a single scalar equation rather than a system of nonlinear equation, and (iv) the formulation of Boltzmann-based numerical algorithms in irregular grids is relatively straightforward. The current job can be easily extended to three-dimensional by operator splitting.

It must be noted that Boltzmann-based numerical model are not a panacea. It is found that these schemes are somewhat slower to execute than approximate Riemann-solvers due to the fact that Boltzmann schemes require the evaluation of error functions. In addition, Boltzmann models solve shallow flow water flows as well as compressible flows. The solution of incompressible flows by Boltzmann models are obtained at the limit when Mach number is small while the solution of flows with rigid-lid assumption are obtained at the limit when Froude number is small.

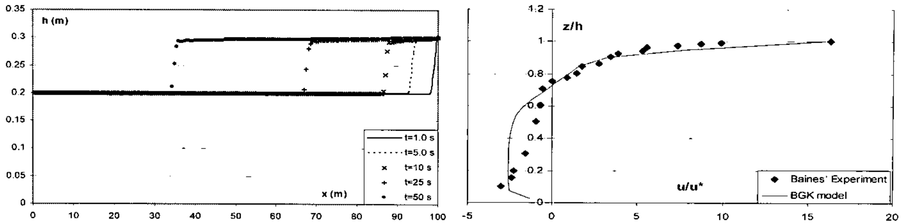

Fig. 1 Validation results: Bore created by sudden downstream blockage (Left); Velocity profile in wind-driven current (Right).

REFERENCES

Chen, H., Kandasamy, S., Orszag, S., Shock, R. Succi, S., and Yakot, V. (2003) "Extended Boltzmann kinetic equation for turbulent flows" *Science*, 301(5633), 633-636.

Ghidaoui, M. S., Deng, J. Q., Gray, W. G. and Xu, K. (2001) "A Boltzmann based model for open channel flows." Int. J. Numer. Methods Fluids. 35, 449-494.

Liang, J. H., Ghidaoui, M. S., Deng, J. Q., and Gray, W. G. (2005) "A Boltzmann-based finite volume algorithm for surface water flows on cells of arbitrary shapes." submitted to *J. Hydraul. Res.*, IAHR.

Vincenti and Kruger (1965) "Introduction to physical gas dynamics" Krieger Publishing company, Malabar, Florida.

Xu, K. (2001) "A gas-kinetic BGK scheme for the Navier-Stokes equations and its connection with artificial dissipations and Godunov methods." J. Comput. Phys. 171, 289-335