A LANGRANGE STOCHASTIC MODEL OF SUSPENDED SEDIMENT IN TUBULENT FLUIDS

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The transport of sediment and contaminant has long been one of the most complicated and challenging research topics. With the development of industry, more and more wastes are discharged to the water system. These wastes will dissolve in water or adhere to sediment, and transport downstream in natural rivers. Accurate simulation of suspended sediment transport is essential for water quality management and environmental impact assessment. The suspended sediment transport has been demonstrated to be a stochastic process by Yen 2002. A stochastic hydraulic process can be regarded as a spatial or temporal process involving probability (Yen, 2002).

The movement of a fine particle in fluid is supposed to follow exactly the motion of the surrounding fluid except for a mean downward drift at the terminal fall velocity. If the river flow is turbulent, then the displacement of the particle can be simulated by a SDE. The SDE is composed of a drift term and a random term caused by fluid eddy motions. The SDE describing suspended sediment concentration is also derived with the help of the Ito chain rule. It is assumed that the river is uniform in the longitudinal direction and the bed slope is very small. The longitudinal dispersion is negligible. Here, we only consider the mixing by turbulence in the vertical direction and neglect the molecular microdiffusion.

The Euler-Maruyama numerical scheme is implemented to solve the one-dimensional SDE of a particle trajectory directly. The Euler scheme will also be applied to solve the SDE of the suspended concentration distribution in the equilibrium state. This scheme can attain the order of weak convergence1.0 and the order of strong convergence 0.5 (Kloeden et al 1994). For the stationary case, the streamwise velocity profile is approximated as logarithmic throughout the water depth. In the numerical experiments, the stochastic terms of the two SDEs are simulated by the random number generator.

For a constant dispersion coefficient, the deterministic advection-diffusion equation has an analytical solution in the stationary state. Given the same boundary conditions, the SDE solution is compared with the analytical solution. From figure 1, comparison between the deterministic analytical solution and the SDE solution with a constant value of dispersion coefficient shows fairly good agreement. The ensemble mean is obtained by averaging the concentration over 10,000 samples.

The stochastic model has been derived in the paper using the stochastic diffusion theory. The particle trajectory can be obtained by using the Euler-Maruyama numerical scheme to solve the stochastic model of particle motion. The suspended sediment concentration distribution in the equilibrium state can also be acquired by solving the SDE derived in the paper. The ensemble mean concentration of this stochastic model is compared with the solution of the deterministic advection-diffusion equation, showing good agreement. This study is expected to provide a more accurate alternative scheme to model the suspended sediment.

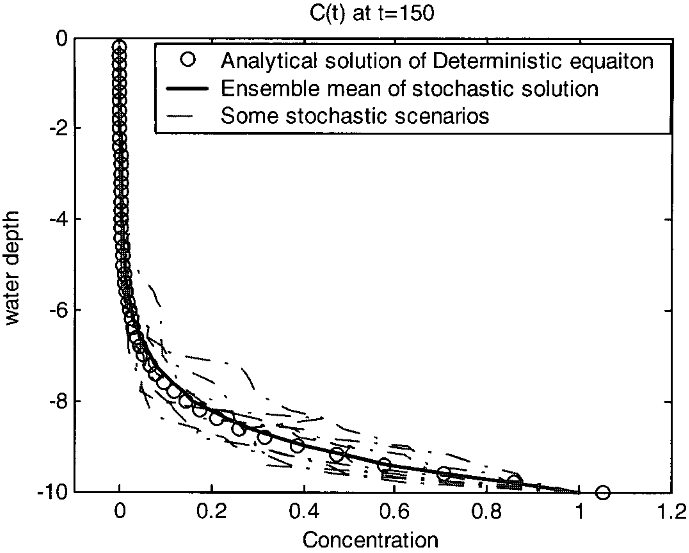

Fig. 1 Comparison of the vertical distribution of concentration between the analytical solution based on deterministic equation and the solutions of SDE given a constant dispersion coefficient.

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