A HYDRODYNAMIC MODEL OF THE INCOMPRESSIBLE NAVIER–STOKES EQUATIONS FOR FREE SURFACE FLOWS

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In this paper, a three-dimensional numerical method for solving free surface flows with a standard $k-\varepsilon$ turbulence model is presented. In order to couple pressure with velocity directly, the pressure is separated into hydrostatic and hydrodynamic parts and the artificial compressibility method (ACM) is employed to determine the hydrodynamic pressure.

By introducing a time derivative of the hydrodynamic pressure into the continuity equation, the incompressible Navier–Stokes equations are changed from elliptic-parabolic to hyperbolic-parabolic equations. The original continuity equation is then satisfied when numerical solutions reach steady state. For the hyperbolic parts of the equation, it is possible to use highly accurate numerical techniques generally applied to the compressible equations. In this paper, a third-order monotone upstream-centred scheme for conservation laws (MUSCL) method is used. A system of discrete equations is solved implicitly using the lower-upper symmetric Gauss–Seidel (LU-SGS) method.

In order to verify the accuracy and applicability of this method, computed solutions are compared with experimental data for a trench channel (van Rijn, 1982) as shown in Fig. 1 with grid representation of a vertical plane and available measurement locations. Figs. 2 and 3 show comparisons between the computed u velocity and turbulent kinetic energy against experimental data using a time increment of 0.05 sec (Courant Number≈ 35). As shown in these figures, the overall agreement is excellent. Compared with Stansby and Zhou (1998)'s standard $k-\varepsilon$ model, the current model predicts the mean velocity and turbulence much more accurately especially at location 2, where weak separation starts leading to negative velocities near the bottom. This indirectly indicates the accuracy and applicability of the current ACM model against other standard fractional step methods.

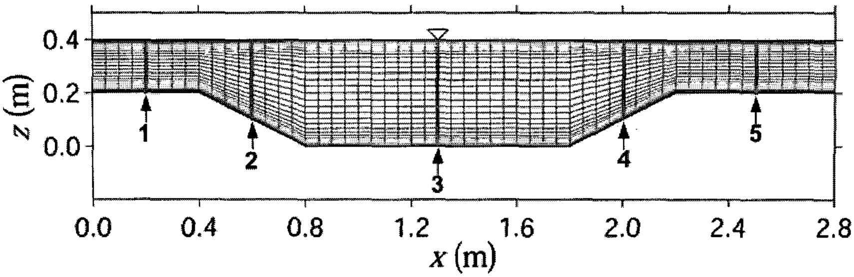

Fig. 1 Geometry of the trench channel model with the grid representation and the five measurement locations.

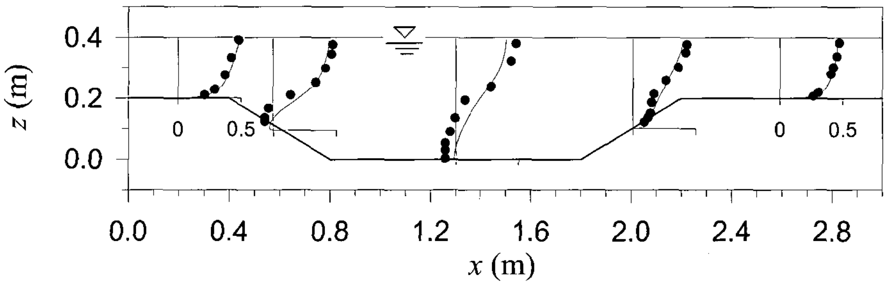

Fig. 2 Comparison between mean velocities (m/s) computed (solid lines) and observed (filled dots).

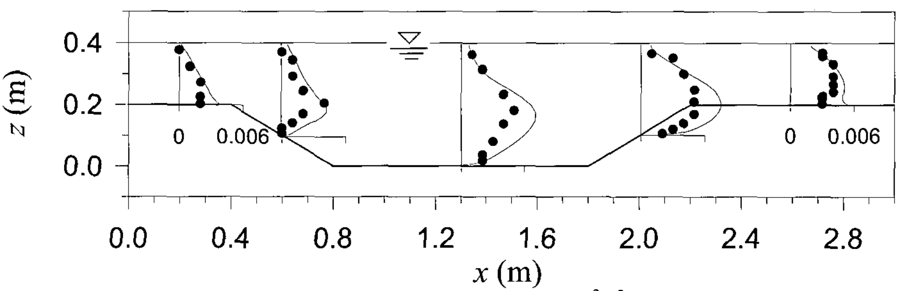

Fig. 3 Comparison between turbulent kinetic energy (m^2/s^2) computed (solid lines) and observed (filled dots).

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