# Investigation of physical sensor models for orbit modeling

## Taejung Kim

Dept. of Geoinformatic Engineering, Inha University, tezid@inha.ac.kr

### ABSTRACT:

Currently, a number of control points are required in order to achieve accurate geolocation of satellite images. Control points can be generated from existing maps or surveying, or, preferably, from GPS measurements. The requirement of control points increase the cost of satellite mapping, let alone it makes the mapping over inaccessible areas troublesome. This paper investigates the possibilities of modeling an entire imaging strip with control points obtained from a small portion of the strip. We tested physical sensor models that were based on satellite orbit and attitude angles. It was anticipated that orbit modeling needed a sensor model with good accuracy of exterior orientation estimation, rather then the accuracy of bundle adjustment. We implemented sensor models with various parameter sets and checked their accuracy when applied to the scenes on the same orbital strip together with the bundle adjustment accuracy and the accuracy of estimated exterior orientation parameters. Results showed that although the models with good bundle adjustments accuracy did not always good orbit modeling and that the models with simple unknowns could be used for orbit modeling.

KEY WORDS: Sensor model, Bundle adjustment, SPOT, Orbit-attitude model

### 1. Introduction

Current satellite mapping practice requires accurately measured control points obtained from individual scenes. With these control points one establishes a sensor model to relate image coordinates and their ground coordinates through the process of bundle adjustments. This requirement of control points increases the cost of satellite mapping and makes mapping over inaccessible areas very difficult.

In this paper, we will investigate the feasibility of modeling entire image strips that has been acquired from the same orbital path using control points obtained from a single scene. If feasible, techniques for such orbit modeling can offer many benefits over the current practice including the partial solution of mapping

### inaccessible areas.

In order to model orbits instead of individual scenes, we need a sensor model that represents physical properties of satellite orbit and imaging geometry. For this reason we deliberately choose only one type of models for consideration: the models that are based on satellite orbit and attitude angles [1]. The well-known collinearity-based physical models [2] that are based on platform position and rotation angles are not considered here following from the recent observations that the former model produced better accuracy of estimating exterior orientation of a satellite [3]. However we would like to clarify that we do not preclude other models than the one tested here on their possibilities of orbit modeling. A thorough investigation among different models may be required later.

#### 2. Sensor model tested

The sensor model we tested has been referred to as "orbit-attitude model" [1,3]. It requires the knowledge of satellite orbit reference coordinate system, attitude control scheme and the physical configuration of sensor arrays. Often it is expressed by a series of complicated formulae but can be simplified as the following matrix.

$$\begin{pmatrix} x \\ y \\ -f \end{pmatrix} = \lambda \mathbf{R}_{\eta y}^{\mathsf{T}} \mathbf{R}_{\mathbf{P}, \mathbf{V}}^{\mathsf{T}} \begin{pmatrix} X - X_{s} \\ Y - Y_{s} \\ Z - Z_{s} \end{pmatrix}$$

In the above equation, (x, y) is image coordinates, (X, Y, Z) ground coordinates  $(X_S, Y_S, Z_S)$  satellite position vector, f the focal length,  $\lambda$  a scale factor,  $\mathbf{R}_{rpy}$  the rotation matrix determined by the attitude angles and  $\mathbf{R}_{P,V}$  the rotation matrix that represents the orbit reference coordinate system.  $\mathbf{R}_{P,V}$  can be represented by satellite position  $\mathbf{P}$  and velocity  $\mathbf{V}$ . The exact definition of  $\mathbf{R}_{rpy}$  and  $\mathbf{R}_{P,V}$  are generic to each satellite system.

We regard this model has nine exterior orientation parameters (three in the position vector, three in the velocity vector and three in attitude angles). For the sake of computational convenience we will model the (non-linear) relationship between the parameters of the position and velocity as the following 2nd-order polynomials of time t (or image coordinate x).

$$X_{S} = X_{0} + a_{1}t + b_{1}t^{2}$$

$$Y_{S} = Y_{0} + a_{2}t + b_{2}t^{2}$$

$$Z_{S} = Z_{0} + a_{3}t + b_{3}t^{2}$$

$$V_{X} = V_{X0} + a_{4}t + b_{4}t^{2}$$

$$V_{Y} = V_{Y0} + a_{5}t + b_{5}t^{2}$$

$$V_{Z} = V_{Z0} + a_{6}t + b_{6}t^{2}$$

where  $(V_X, V_Y, V_Z)$  is the velocity vector. And we will model attitude angles (roll, pitch and yaw angles) as piecewise-linear functions of time, whose shapes can be derived from the satellite attitude rate information

provided [4].

We implemented seven orbit-attitude models with difference sets of unknowns as specified in table 1. OA-1 models the  $2^{nd}$  order coefficients of satellite position equations and the attitude biases ( $R_0$  for roll,  $P_0$  for pitch,  $\Psi_0$  for yaw) as unknowns. OA-2 differs from OA-1 in that it models roll bias, drift  $\dot{R}$  and acceleration  $\ddot{R}$  as attitude unknowns. OA-3 models the  $2^{nd}$  order positional coefficients as unknowns and OA-4 the biases, drifts and accelerations of three attitude angles. OA-5 to 7 have much simpler unknowns. OA-5 models position biases and attitude biases, OA-6 only position biases and OA-7 only attitude biases as unknowns. With these models with different unknowns the possibility of orbit modeling was tested.

Table 1. List of unknowns for each sensor model

ID	Unknowns
OA-1	$X_0, a_1, b_1, Y_0, a_2, b_2, Z_0, a_3, b_3, R_0, P_0, \Psi_0$
OA-2	$X_0, a_1, b_1, Y_0, a_2, b_2, Z_0, a_3, b_3, R_0, \dot{R}, \ddot{R}$
OA-3	$X_0, a_1, b_1, Y_0, a_2, b_2, Z_0, a_3, b_3$
OA-4	$R_0, \dot{R}, \ddot{R}, P_0, \dot{P}, \ddot{P}, \Psi_0, \dot{\Psi}, \ddot{\Psi}$
OA-5	$X_0, Y_0, Z_0, R_0, P_0, \Psi_0$
OA-6	$X_0, Y_0, Z_0$
OA-7	$R_0, P_0, \Psi_0$

### 3. Dataset and Experiment Results

We used two stereo strips that were taken by SPOT3 over Daejeon and Junju areas. The two areas are located along the same orbital path. Each strip consists of two scenes, making the length of a strip approximately 120 kms. Table 2 summaries the property of each strip. From Daejeon scene, 27 ground control points (GCPs) were derived by GPS surveying and from Junju scene, 25 GCPs.

Table 2. Properties of image strips used

ID	Strip-1	Strip-2
Satellite	SPOT3	SPOT3
Date of Acquisition	4 April 1995	28 Jan 1995
Tilt Angle	+19.8°	-23.4°
No of GCPs (Daejeon)	27	27
No of GCPs (Junju)	25	25

We performed experiments as follows. First, we divided GCPs for Daejeon scene into 14 points for modeling (model points) and 13 points for independent accuracy check (check points). Next we set up sensor models for the Daejeon scene using the 14 model points. We calculated the errors of modeling points and check points for each sensor model. Then we applied the models derived from Daejeon scenes to Junju scenes. We used the 25 GCPs for Junju scenes to check the accuracy of sensor models. In this way we could check whether sensor models can model the entire orbit of 120km length by using control points acquired from Daejeon scenes only.

Table 3 and 4 summarizes the results of experiments for strip-1 and strip-2, respectively. In two tables, "Model Error" means the error of the 14 model points extracted from Daejeon area and "Check Error" the error of the 13 heck points. "Orbit Error" refers to the error of control points of Junju scenes when the sensor models derived from Daejeon control points were applied to Junju scenes.

The two tables contain very interesting results. The models that included higher degree of coefficients as unknowns (OA1 and OA2) produced better model and check errors for Daejeon scenes. This result agreed with the common practice of using 2nd order coefficients as unknowns when modeling a single scene. However the models with simpler unknowns (OA5 to 7) produced much better accuracy on Junju scenes. In particular, the

model OA7 that only estimated attitude biases produced the most accurate orbit errors, even though OA7 produced less accurate results for modeling Daejeon scenes.

Table 3. Results of orbit modeling of strip-1

Model ID	Model Error	Check Error	Orbit Error
	(rms, pixels)	(rms, pixels)	(rms, pixels)
OA1	0.785	1.189	8.186
OA2	0.775	1.354	9.855
OA3	1.604	2.388	12.329
OA4	1.246	1.175	11.440
OA5	0.926	1.239	1.782
OA6	1.771	2.252	2.429
OA7	1.377	1.486	1.906

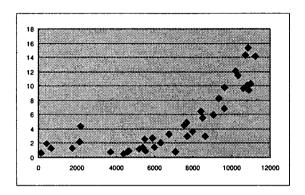
Table 4. Results of orbit modeling of strip-2

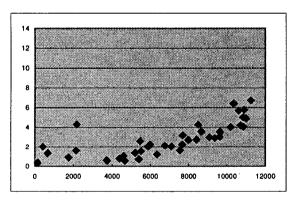
Model ID	Model Error	Check Error	Orbit Error
	(rms, pixels)	(rms, pixels)	(rms, pixels)
OA1	1.117	1.790	8.285
OA2	1.161	1.738	12.381
OA3	1.965	2.462	14.580
OA4	1.329	1.687	8.885
OA5	1.449	1.728	3.874
OA6	2.234	2.415	4.648
OA7	1.715	1.762	1.551

To support the results shown in the table, the magnitude of check errors and orbit errors of OA1, OA5 and OA7 for strip-2 are plotted against the orbital distance (or row coordinates). In figure 1, the horizontal axis indicates image row coordinates. The row coordinate 0 to 6000 indicates the check error for Daejeon scene and 6000 to 12000 the orbit error for Junju scene.

The plot for OA1 shows the clear trend of errors. Errors within Daejeon scene are very low. However, when the distance from the Daejeon scene (or image row

coordinates) increases, the magnitude of error increases rapidly. The plot for OA4 showed the magnitude of orbit errors are reduced greatly compared to that of OA1. However as before, the error of orbit modeling increases with the row coordinates. The plot for OA7 shows that the magnitude of errors does not have any correlation with the image row coordinates, indicating that we have successfully modeled the whole orbital strip.





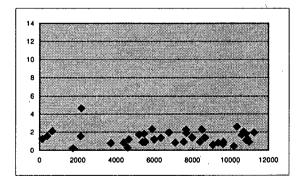


Figure 1. Errors of orbit modeling for OA1 (top), OA4 (middle) and OA7 (bottom) for strip-2

### 4. Conclusions

We presented experiments we devised to test sensor models' ability to model orbital strips and the results. Although through investigation including longer length of image strips must be carried out, the results in this paper seems to suggest that it would be possible to model the whole orbital segments with carefully chosen unknown parameters and that the models with attitude biases as unknowns could be used for orbit modeling.

### Acknowledgements

This research was supported by the Agency for Defense Development, Korea, through the Image Information Research Center at Korea Advanced Institute of Science & Technology.

### References

- [1] Radhadevi, P.V., Ramachandran, R., and Mohan, M., 1998, "Restitution of IRS-1C PAN data using an orbit attitude model and minimum control", ISPRS Journal of Photogrammetry and Remote Sensing, 53(1998):262-271
- [2] Gugan, D. J. and Dowman, I. J., 1988, "Accuracy and completeness of topographic mapping from SPOT imagery", Photogrammetric Record, 12(72):787-796
- [3] Taejung Kim, 2005, "Investigation on the Accuracy of bundle Adjustments and Exterior Orientation Parameter Estimation of Linear Pushbroom Sensor Models", Journal of Korean Surveying Society, 23(2):137-145 (written in Korean)
- [4] SPOT Image, 2002, "SPOT Satellite Geomtry Handbook", S-NT-73-12-SI