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Implicitly Restarted Arnoldi Method를 이용한 대형전력계통 소신호안정도 적용 기초 연구

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A Basic Study of the application of Implicitly Restarted Arnoldi Method to the Small Signal Stability of Large Power Systems

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Abstract - This paper describes implicitly restarted Arnoldi method (IRAM), which is a technique for combining the implicitly shifted QR mechanism with a k-step Arnoldi factorization to obtain a truncated form of the implicitly shifted QR-iteration. IRAM avoids numerical difficulties and storage problems normally associated with Arnoldi. This paper deals with the basic algorithms of IRAM as an intial research phase for developing the full featured eigenvalue analysis program for large power system up to 30,000 states.

1. Introduction

Conventional QR [1,2] method for small signal stability analysis are not applicable to very large-scale power systems because of limitation of memory capacity, computing time, and computation accuracy. In order to evaluate the small signal stability of power systems, it is usually required to calculate only a specific set of eigenvalues with certain features of interest, for example, local mechanical modes, inter-area modes, etc. Therefore, significant effort has been expended to develop new methods with such basic properties as sparcity based techniques, finding a few specific set of eigenvalues, and mathematical robustness with good convergence characteristics and numerical stability [3,4].

Since the eigenanalysis of modern power systems deals with matrices of very large dimension, sparcity techniques play a key role in the analysis. Two of more popular sparsity-based eigenvalue techniques for general unsymmetrical matrices are S-method [3], which is based on Lanczos method with Cayley transformation, and modified Anoldi method [4]. Lanczos-type method is a very successful method for the symmetrical eigenvalue problem, bus has serious flaws in the case of unsymmetrical eigenvalue problems as the phenomenon of 'breakdown'. The modified Arnoldi method uses complete reorthogonalization and an iterative process with shift-invert transformation [5,6]. However, reorthogonalizaion will requires extensive storage and repeatedly finding the eigensystem of H will become prohibitive at a cost of $O(k^3)$ flops. In order to overcome such difficulties, an alternative has been proposed by Saad [7] to restart the iteration with a vector that has been preconditioned so that it is more nearly in a k-dimensional invariant subspace of interest. This preconditioning takes the form of a polynomial applied to the starting vector that is constructed to damp unwanted components form the eigenvector expansion. This technique is refereed to as explicit (polynomial restarting). One of popular methods is Arnoldi-Chebyshev method.

This paper describes another restarting approach which can be appliable to very large power systems. This approach is called implicitly restarted Arnoldi method (IRAM) [8]. IRAM is a technique for combining the implicitly shifted QR mechanism with a k-step Arnoldi factorization to obtain a truncated form of the implicitly shifted QR-iteration. The numerical difficulties and storage problems normally associated with Arnoldi process is avoided. The algorithm is capable of computing a few (k) eigenvalues with user specified features such as largest real part or largest magnitude. Implicit Restarting provides a means to extract interesting information from very large Krylov subspaces while avoiding the storage and numerical difficulties associated with the standard approach. It does this by continually compressing the interesting information into a fixed size k-dimensional subspace. This is accomplished through the implicitly shifted QR mechanism.

In this paper, the basics algorithms of IRAM are described as an initial research phase for developing sparcity-based eigenvalue program for studying the small signal stability of very large power systems.

2. Algorithms of IRAM

2.1 Implicit Q Theorem

The Hessenberg decomposition is not unique. However, H is unique once the first column of Q is specified [9]. This is essentially the case provided H has no zero subdiagonal entries. Hessenberg matrices with this property are said to be unreduced. A very important theorem that clarifies the uniqueness of the Hessenberg reduction is the implicit Q theorem.

Theorem 1. (Implicit Q Theorem) Suppose $Q=[q_1, \cdots, q_n]$ and $V=[v_1, \cdots, v_n]$are orthogonal matrices with the property that both $Q^TAQ=H$ and $V^TAV=G$ are upper Hessenberg where $A \in \mathbb{R}^{n \times n}$. Let k denote the smallest positive integer for which $h_{k+1,k}=0$, with the convention that $k=n$ if H is unreduced. if $q_1=v_1$, then $q_i = \pm v_i$ and $|h_{i,i-1}| = |g_{i,i-1}|$ for i=2:k. Moreover, if $k<n$, then $g_{k+1,k}=0$.

2.2 The Double Implicit shift QR .

The single shift QR iteration use h_{nn} as the best approximate eigenvalue along the diagonal during each iteration:

for k=1,2,...

$\mu = H(n,n)$

$H - \mu I = UR$ (QR Factorization)

$H = RU + \mu I$

end

However, the eigenvalues a_1 and a_2 of

$$G = \begin{pmatrix} h_{mm} h_{mn} \\ h_{nm}\ h_{nn} \end{pmatrix} \quad m = n-1$$

are complex then h_{nn} would tend to be a poor approximate eigenvalue. A way to get around this difficulty is to perform two single-shift QR steps in succession using a_1 and a_2 as shifts:

$$H - a_1 I = U_1 R_1$$
$$H_1 = R_1 U_1 + a_1 I$$
$$H_1 - a_2 I = U_2 R_2$$
$$H_2 = R_2 U_2 + a_2 I$$

These equations can be manipulated to show that

$$(U_1 U_2)(R_2 R_1) = M$$

where M is defined by

$$M = (H - a_1 I)(H - a_2 I)$$

Note that M is a real matrix since

$$M = H^2 - sH + tI$$

where

$$s = a_1 + a_2 = h_{mn} + h_{nn} = trace(G) \in \mathbb{R}$$

and

$$t = a_1 a_2 = h_{mm} h_{nn} - h_{mn} h_{nm} = det(G) \in \mathbb{R}$$

Since this step requires $O(n^3)$ flops to compute H_2 from H, this is not a practical course of action to compute $H_2 = Z^T HZ$, where Z is computed from real QR factorization, $M = ZR$. However, by applying to the Implicit Q theorem, the double shift step with $O(n^2)$ flops can be implemented. In particular we can effect the transition from H to H_2 in $O(n^2)$ flops if we compute Me_1, the first column of M. The first column of M is $Me_1 = [x, y, z, 0, \cdots, 0]^T$ where

$$x = h_{11}^2 + h_{12} h_{21} - s h_{11} + t$$
$$y = h_{21}(h_{11} + h_{22} - s)$$
$$z = h_{21} h_{32}$$

Then we can determine a Householder matrix P_0 such that $P_0(Me_1)$ is a multiple of e_1 and compute Householder matrices $P_1, \ldots, P_{n-2}$ such that if Z_1 is the product $Z_1 = P_0 P_1 \cdots P_{n-2}$, then $Z_1^T H Z_1$ is upper Hessenberg and the first columns of Z and Z_1 are the same.

2.3 k-step Arnoldi Factorization

If $A \in C^{n \times n}$ then a relation of the form

$$AQ_k = Q_k H_k + r_k e_k^T$$

where $Q_k \in C^{n \times k}$ has orthonormal columns, $Q_k^H r_k = 0$ and $H_k \in C^{k \times k}$is upper Hessenberg with non-negative subdiagonal elements is called a k-step Arnoldi Factorization of A. This equations are obtained from Arnoldi process. In particular, if $Q = [q_1, \ldots, q_n]$ and we compare columns in $AQ = QH$, then

$$Aq_k = \sum_{i=1}^{k+1} h_{ik} q_i \quad 1 \le k \le n-1$$

Isolating the last term in the summation gives

$$h_{k+1,k} q_{k+1} = Aq_k - \sum_{i=1}^{k} h_{ik} q_i := r_k$$

where $h_{ik} = q_i^T Aq_k$ for i=1:k. It follows that if $r_k \neq 0$, then q_{k+1} is specified by

$$q_{k+1} = r_k / h_{k+1,k}$$

where $h_{k+1,k} = \| r_k \|_2$.

The q_k are called the Arnoldi vectors and they define an orthonormal basis for the Krylov subspace $\kappa(A, q_1, k)$:

$$span\{q_1, \cdots, q_k\} = span\{q_1, Aq_1, \cdots, A^{k-1} q_1\}$$

2.4 Implicit Restarted Arnold Method

The IRAM determines the restart vector implicitly using the QR iteration with shifts. The restart occurs after every m steps and we assume that $m > j$ where j is the number of sought-after eigenvalues. The choice of the Arnoldi length parameter m depends on the problem dimension n, the effects of orthogonality loss, and system storage constraints. After m steps we have the Arnoldi factorization

$$AQ_C = Q_C H_C + r_C e_m^T$$

The subscript "c" stands for "current". The QR iteration with p shifts is then applied to H_C. Here $p = m - j$ and we have $H_+ = V^T H_C V$ because $V_i^T H^{(i)} V_i = H^{(i+1)}$. The orthogonal matrix $V = V_1 \cdots V_p$, with V_i the orthogonal matrix associated with the shift μ_i, has two crucial properties:

(1) $[V]_{mi} = 0$ for i=1:j-1. This is because each V_i is upper Hessenberg and so $V \in \mathbb{R}^{m \times m}$ has lower bandwidth $p = m - j$.

(2) $Ve_1 = \alpha(H_C - \mu_p I)(H_C - \mu_{p-1} I) \cdots (H_C - \mu_1 I)e_1$ where α is a scalar.

We obtain the following transformation:

$$AQ_+ = Q_+ H_+ + r_C e_m^T V$$

where $Q_+ = Q_C V$. In view of property (1),

$$AQ_+(:, 1:j) = Q_+(:, 1:j) H_+(1:j, 1:j) + v_{mj} r_C e_m^T$$

is a length-j Arnoldi factorization. Back to the basic Arnold j iteration at step $j+1$ and performing p steps, we can have a new length-m Arnoldi factorization.

Figure 1-3 picture one cycle of the iteration for clear understanding of each step of IRAM.

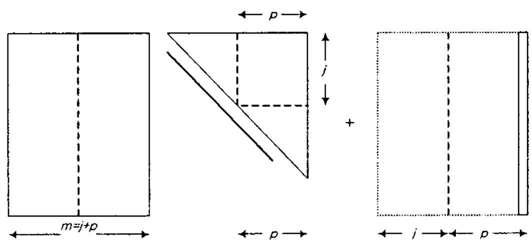

Fig. 1. Step 1: m-step Arnoldi Factorization, $Q_{j+p} H_{j+p} + r_m e_m^T$

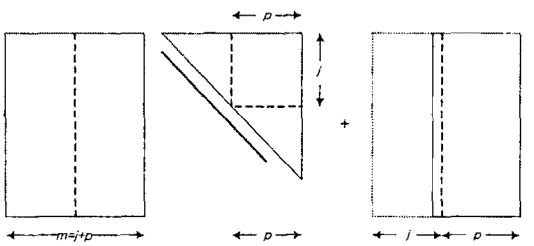

Fig. 2. Step 2: Applying the implicitly shifted QR Step, $Q_{j+p}VV^{T}H_{j+p}V + r_{m}e_{m}^{T}V$

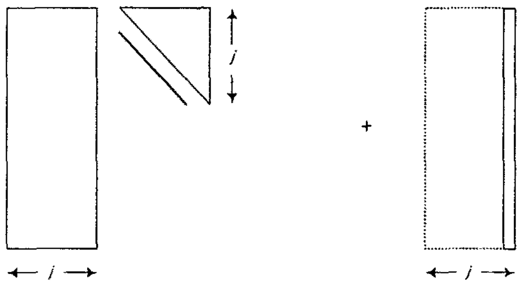

Fig. 3. Step 3: j-step Arnoldi factorization after discarding the last p columns, $Q_{j}H_{j} + v_{mj}r_{m}e_{j}^{T}$

3. Conclusions

This paper described implicitly restarted Arnoldi method (IRAM). IRAM combines the implicitly shifted QR mechanism with a k-step Arnoldi factorization to obtain a truncated form of the implicitly shifted QR-iteration. Implicit Restarting provides a means to extract interesting information from very large Krylov subspaces while avoiding the storage and numerical difficulties associated with the standard approach. It does this by continually compressing the interesting information into a fixed size k-dimensional subspace. In the on-going research phase, a new sparcity-based eigenvalue algorithm applicable to very large power system will be developed using IRAM algorithm.

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