헬리콥터 시스템의 지능형 디지털 재설계

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Intelligent Digital Redesign for Helicopter System

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Abstract – We represent an efficient intelligent digital redesign method for a Takagi-Sugeno (T-S) fuzzy system. Intelligent digital redesign means that an existing analog fuzzy-model-based controller converts to equivalent digital counter part in the sense of state-matching. The proposed method performs previous work, moreover, it allows to matching the states of the overall closed-loop T-S fuzzy system with the predesigned analog fuzzy-model-based controller. And the problem of stability represent convex optimization problem and cast into linear matrix inequality (LMI) framework. This method applies to the helicopter systems which are the nonlinear plant and determine the feasibility and effectiveness of the proposed method.

1. Introduction

Most dynamic systems are continuous-time systems until some years ago. So, in the view of practical and theoretical approach, continuous-time controllers have proper territory in continuous-time system. But, as advanced digital implements, represented computer and microprocessor, analog systems with digital systems are often desired. The reason of this phenomenon is that digital devices have more merits than analog devices. In other words, digital devices are easy to implement and have good performance, but the costs are continuously going down.

Digital redesign is the method the that design a suitable analog controller first and then convert the obtained analog controller to the equivalent digital controller maintaining the properties of the original analogously controlled system, by which the benefits of both continuous-time controllers and the advanced digital technology can be obtained. [1] So, it guarantee the possibility of practical use the merits of the digital devices.

In this paper, we use digital redesign method to achieve digital approach. But all of analog systems are not converted equivalent digital systems. There are two conditions, which are required for accomplishing digital redesign successfully, the one is dimension problem and the other is stability problem. The static state feedback controller can be exactly digitally redesigned if the dimension of the state vector is less than or equal to that of the control vector, which is not satisfied in general. Therefore, most digital redesign techniques were developed based on the approximation techniques, in which the discrete system matrix of the original closed-loop analog control system is approximately estimated and used to develop the digitally redesigned controller by state matching.

Another problem, represented stability, is the main issue for the safe and harmless operation of the controlled system, the guaranteed stability of the digitally redesigned system is required for the practical application of the digital redesign techniques. It is possible that the stability of the digitally redesigned system can be verified after the digital redesign procedure. But when the stability of the redesigned system is not satisfied, the only possible remedy is to redesign the original continuous-time controller or adjust the given sampling period. So we need to combine the stability checking algorithm and the digital redesign technique in the unified frame work. [2]

The solution of these combine problem is the need of linear matrix inequality (LMI) setting, which can be effectively solve many controller analysis and synthesis problems. So, casting the digital redesign problem into LMI frameworks seems to be a quite promising way to digitally redesign a continuous-time controller with guaranteed stability. [3]

This paper is organized as follows. At first, we briefly reviews the T-S fuzzy model and PDC controller to analyze the nonlinear plant, helicopter system. Then, introduce the concept of intelligent digital redesign of global approach. This method use to control helicopter system, and to show the effectiveness of the proposed method.

2. Main Subject

2.1 Intelligent digital redesign

The concept of intelligent digital redesign stems from digital redesign method, so two methods are similar to each other. The difference of two methods is that intelligent digital redesign is able to apply complex nonlinear system. In this paper, we analyze the nonlinear by using T-S fuzzy-model-based controller and convert to digital thing.

2.1.1 Takagi-Sugeno (T-S) fuzzy model

Consider a nonlinear dynamic system of the following form:

$$\dot{x}_c(t) = f(x_c(t), u_c(t)) \tag{1}$$

where $x_c(t) \in R^n$ is the state vector, $u_c(t) \in R^m$ is the control input vector. We analyze this nonlinear plant to Takagi-Sugeno (T-S) fuzzy model. The ith rule of T-S fuzzy system is formulated in the following form :

$$\begin{aligned} R^i &: IF\ z_1(t)\ is\ about\ \Gamma_1^i\ and \ldots and\ z_n(t)\ is\ about\ \Gamma_n^i \\ &THEN\ \dot{x}_c(t) = A_i x_c(t) + B_i u_c(t) \end{aligned} \tag{2}$$

$i = 1, 2, \ldots, r$ and r is the number of IF-THEN rules. Γ_n^i are fuzzy sets and $\dot{x}_c(t) = A_i x_c(t) + B_i u_c(t)$ is the output of these IF-THEN fuzzy system's IF-THEN rules. The final output of given fuzzy system is represented as following equations:

$$\dot{x}_c(t) = \sum_{i=1}^{q}\theta_i(z(t))(A_i x_c(t) + B_i u_c(t))$$

$$w_i(z(t)) = \prod_{h=1}^{n}\Gamma_h^i(z_h(t)), \quad \theta_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{q} w_i(z(t))}. \tag{3}$$

Used controller is Parallel Distributed Controller (PDC). We utilize this controller and to design fuzzy controllers to stabilize T-S fuzzy system. The main idea of PDC controller is that the controller also consist of 'IF-THEN' rules and the previous part of the 'IF-THEN' rules are same that of T-S fuzzy system's plant. The PDC controller rule is

R^i : IF $z_1(k)$ is about Γ_1^i and ... and $z_n(t)$ is Γ_n^i

THEN $u_c(t) = K_c^i x_c(t)$. (4)

By using controller, we found the overall continuous-time closed-loop T-S fuzzy system. And the equation is

$$\dot{x}_c(t) = \sum_{i=1}^{q}\sum_{j=1}^{q}\theta_i(z(t))\theta_j(z(t))(A_i + B_i K_c^j)x_c(t). \tag{5}$$

2.1.2 Intelligent digital redesign

Intelligent digital redesign has three processes. At first, design a stabilizable continuous-time controller at nonlinear system. And, discretize the closed-loop continuous-time and the digital fuzzy systems. Then design a digital controller such that the states between the closed-loop continuous-time fuzzy system and the closed-loop digital one will be matched. This process is called 'state matching'. And finally, the stability of the closed-loop discrete-time system is guaranteed.

So we perform the discretization of the continuous-time T-S fuzzy model which represents nonlinear plant. Consider T-S fuzzy models governed by

$$\dot{x}_d(t) = \sum_{i=1}^{q}\theta_i(z(t))(A_i x_d(t) + B_i u_d(t)) \tag{6}$$

where $u_d(t) = u_d(kT)$ is the piecewise-constant control input vector to be determined in the time interval $[kT, kT+T)$, where $T > 0$. For the digital control of the continuous-time T-S fuzzy system, the digital fuzzy-model-based controller is applied. Let the fuzzy rule of the digital control law for the (1) system, we obtain following IF-THEN rules

R^i : IF $z_1(kT)$ is about Γ_1^i and ... and $z_n(kT)$ is Γ_n^i

THEN $u_d(t) = K_d^i x_d(kT)$ (7)

for $t \in [kT, kT+T)$, where K_d^i is the digital control gain matrix to be redesigned for the ith rule, and the overall control law is

$$u_d(t) = \sum_{i=1}^{q}\theta_i(z(kT))k_d^i x_d(kT) \tag{8}$$

for $t \in [kT, kT+T)$.

The next process is that to find digital gains from the analog gains so that the closed-loop state $x_d(t)$ can closely match the closed-loop state $x_c(t)$ at all sampling time instants $t = kT$. Thus it is more convenient to convert the T-S fuzzy system into discrete-time version for derivation of the state-matching condition, but it has some assumption, because T-S fuzzy system is the nonlinear and time-varing system.

So, we need a mathematical foundation for the discretization of the continuous-time T-S fuzzy system [3].

Assumption 1: We assume that the firing strength of the ith rule, $\theta_i(z(t))$ is approximated by its value at time kT, that is

$$\theta_i(z(t)) \approx \theta_i(z(kT)) \tag{9}$$

for $t \in [kT, kT+T)$. Consequently, the nonlinear matrices $\sum_{i=1}^{q}\theta_i(z(t))A_i$ and $\sum_{i=1}^{q}\theta_i(z(t))B_i$ can be approximated as constant matrices $\sum_{i=1}^{q}\theta_i(z(kT))A_i$ and $\sum_{i=1}^{q}\theta_i(z(kT))B_i$, over any interval $[kT, kT+T)$.

If the state maintains this assumption, we can discretize without boundary of LTI system, and obtain the *Theorem 1*:

Theorem 1: The dynamic behavior of the discretized digital T-S fuzzy system can be approximated by *Assumption 1* and the equation is

$$x_d(kT+T) \approx \sum_{i=1}^{q}\theta_i(z(t))(G_i x_d(kT) + H_i u_d(kT)) \tag{10}$$

where $G_i = \exp(A_i T)$ and $H_i = (G_i - I)A_i^{-1}B_i$.

Hence, the discretized version of the closed-loop system is con structed to yield

$$x_d(kT+T) \approx \sum_{i=1}^{q}\sum_{j=1}^{q}\theta_i(z(kT))\theta_j(z(kT))(G_i + H_i K_d^j)x_d(kT) \tag{11}$$

and, The dynamic behavior of the disretized continuous-time T-S fuzzy system can be approximated by *Assumption 1* and the equation is

$$x_c(kT+T) \approx \sum_{i=1}^{q}\sum_{j=1}^{q}\theta_i(z(kT))\theta_j(z(kT))\Phi_{ij}(x_c(kT)) \tag{12}$$

where $\Phi_{ij} = \exp((A_i + B_i K_c^j)T)$.

Our goal is to develop an intelligent digital redesign technique for T-S fuzzy systems so that the global dynamic behavior with the digitally redesigned fuzzy-model-based controller may retain that of the closed-loop T-S fuzzy system with the existing analog fuzzy-model-based controller, and the stability of the digitally controlled T-S fuzzy system is secured [3]. So, for achieving this goal we formulate the following global intelligent digital redesign problem:

Problem 1 (γ-suboptimal Global Intelligent Digital Redesign Problem): Given well-constructed gain matrices K_c^i for the stabilizing analog fuzzy-model-based controller (5), find gain matrices K_d^i, $i = 1, 2, \ldots, q$, for the digital fuzzy control law (8) such that the following constraints are satisfied

1) Minimize γ subject to $\|\Phi_{ij} - G_i - H_i K_d^j\| < \gamma$ $i, j = 1, 2, \ldots, q$ in the sense of the induced 2-norm distance measure.

2) The discretized closed-loop system (11) is globally asymptotically stable in the sense of Lyapunov criterion. [3]

So, we are easy to convert the problem to convert convex optimization problem. This convex optimization problem recasts LMI framework, we guarantee the stability of total system. The main results of this paper are summarized as follows

Theorem 2 (*γ-suboptimal Global Intelligent Digital Redesign*): If there exist symmetric positive definite matrix Q, symmetric positive-semidefinite matrix O, constant matrices F_i and a possibly small positive scalar γ such that the following generalized eigenvalue problem (GEVP) has solutions

Minimize$_{Q,O,F_i}$ γ subject to

$$\begin{bmatrix} -\gamma Q & * \\ \Phi_{ij} Q - G_i Q - H_i F_j & -\gamma I \end{bmatrix} < 0 \tag{13}$$

$$\begin{bmatrix} -Q + (q-1)O & * \\ G_i Q - H_i F_i & -Q \end{bmatrix} < 0, \quad i, j = 1, 2, \ldots, q. \tag{14}$$

$$\begin{bmatrix} -Q - O & * \\ \dfrac{G_i Q + H_i F_j + G_j Q + H_j F_i}{2} & -Q \end{bmatrix} < 0, \quad i = 1, 2, \ldots, q-1, \ j = i+1, \ldots, q. \tag{15}$$

2.2 Helicopter system

In this section, we look around the real plant of the helicopter system. Consider the following nonlinear dynamic system's equation

$$J_p \ddot{p}(t) + B_p \dot{p}(t) = R_p F_p(V_{p(t)} - M_e g(h \sin(p(t)) + R_c \cos(p(t)) + G_p(\tau_y(V_{y(t)}, p(t))$$
$$J_y \ddot{y}(t) + B_y \dot{y}(t) = R_y F_y(V_{y(t)} + G_y(\tau_p(V_{p(t)})) \tag{16}$$

which describe the dynamic behavior of a 2-dimensional helicopter system. Let the state vectors and input vectors for these systems be

$$x(t) = [p(t) \ y(t) \ \dot{p}(t) \ \dot{y}(t) \ \int p(t) \ \int y(t)]^T$$
$$u(t) = [V_p(t) \ V_y(t)]^T \tag{17}$$

Then the state-space of these state and input vectors can be represented as

$$\dot{x}(t) = \begin{bmatrix} x_3(t) \\ x_4(t) \\ -\dfrac{B_p}{J_p} x_3(t) - \dfrac{(M_e g(h \sin x_1(t)) + R_c \cos(x_1(t))}{J_p} \\ -B_y / J_y x_4(t) \\ 10 x_1(t) \\ 10 x_2(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{R_p K_{pp} g}{J_p} & \dfrac{K_{py} g}{Jp} \\ \dfrac{R_p K_{yp} g}{J_p} & \dfrac{K_{yy} g}{J_p} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u(t) \tag{18}$$

The analytic T-S fuzzy system is represented as follows by using the procedure in [12]

R^1 : *IF* $z_1(t)$ *is about* Γ_1, *THEN* $\dot{x}(t) = A_1 x(t) + B_1 u(t)$

R^2 : *IF* $z_1(t)$ *is about* Γ_2, *THEN* $\dot{x}(t) = A_2 x(t) + B_2 u(t)$ (19)

and the membership functions for this subsystem are

$$\Gamma_1(x_1(t)) = \frac{\alpha \sin(x_{11}(t) + \beta) - b(x_{11}(t) + \beta)}{(a - b)(x_{11}(t) + \beta)}$$

$$\Gamma_2(x_1(t)) = \frac{\alpha(x_{11}(t) + \beta) - \alpha \sin(x_{11}(t) + \beta)}{(a - b)(x_{11}(t) + \beta)} \tag{20}$$

2.3 Intelligent digital redesign for helicopter system

We solve the original helicopter problem. By using 'convex optimization technique' and cast into LMI framework, we obtained the continuous-time's gain. The initial value is

$$x_c(0) = x(0) = [1 \ \ 1 \ \ -1 \ \ 1 \ \ 1 \ \ 1]^T \tag{21}$$

and the well-constructed gain matrices for the continuous-time fuzzy-model-based controller is obtained. Before mentioned, digital redesign technique has 'state matching' problem. The solution of this problem is gain matching, applying *Theorem 2*, so we should obtain the gain of discrete-time case. The process is same to continuous case, and the results are also obtained.

These two gains, continuous-time system's gain and discrete-time system's gain, are the result of state-matching technique. LMI framework uses these gains to notice which digital redesign technique is successful or not. At first, we obtain the simulation result of continuous-time case. Fig 1 shows the movement of the continuous-time case's parameter.

We convert this analog system to digital system by using the method of digital redesign. Then, we obtained the results of the discrete-time fuzzy-model-based controller's simulation. And we obtained simulation result. Fig 2 is similar to continuous-time fuzzy-model-based system which are the result of intelligent digital redesign.

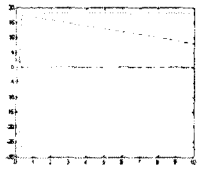

Fig.1 Time response of the controlled continuous-time fuzzy-model-based system

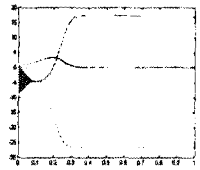

Fig.2. Time response of the discrete-time fuzzy-model-based system

3. Conclusion

This paper discussed the digital redesign method of global approach for a Takagi-Sugeno (T-S) fuzzy system. Intelligent digital redesign means that an existing analog fuzzy-model-based controller converts to equivalent digital counter part in the sense of state-matching. The proposed method performs previous work, moreover, it allows to matching the states of the overall closed-loop T-S fuzzy system with the predesigned analog fuzzy-model-based controller. And the problem of stability represent convex optimization problem and cast into linear matrix inequality (LMI) framework. Finally, we apply this method to helicopter system, and the simulation result are obtained.

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