2005년도 대한전기학회 하계학술대회 논문집 2005. 7. 18 - 20

유전체 상수의 유한요소 시뮬레이션

Jeffrey Braunstein*, 곽용수*, 김형석*, 박준석**
*중앙대학교 전자전기공학부, **국민대학교 전자정보통신공학부

A Finite Element Investigation of the Permittivity of Particulates

Jeffrey Braunstein*, Yong-Soo Kwak*, Hyeong-Seok Kim*, Jun-Seok Park**
*School of Electrical & Electronics Engineering, Chungang Univ., **School of Electrical Engineering, Kookmin Univ.

Abstract - In this paper, a finite element (FEM) investigation of composite materials is studied. A pemittivity profile of the material is implemented to correspond to the packing fraction of the physical composite. Curve fitting is applied to the standing wave pattern to determine the effective attenuation coefficient and propagation constant in the composite. The complex permittivity as a function of packing density is then determined. A comparison between the two dimensional and three dimensional measurement simulations is presented. An adaptive scheme is implemented to improve resolution of the finite element particulates.

1. Introduction

In recent years, there has been significant investigation into electromagnetic behavior of composite material. High power delivery to ceramic or metallic particles has been of particular interest[1]. Early studies indicate that extremely fast heating rates can result in materials with fewer impurities and/or unique material properties. Ground penetrating radar (GPR) requires assumption about soil behavior, which is typically a composite material. Low power RF · sensors and passive RFID tags demonstrate environment sensitivity where surround materials can significantly affect antenna performance. Effective design of a heating systems or the placement of an antenna/sensor requires an accurate representation of the effective electric properties. Specifically, the real and imaginary components of the effective permittivity will determine the boundary reflections, internal resonance and the loss component.

In this paper, a simulated mixture is studied using the FEM method. While the concept is not novel, most early efforts investigated geometry domains on the order of the particle size. The packing density is determined by the volume of the particles relative to the volume of the meshed domain. The properties of this smaller region would then be used to assemble a larger analysis. This method has several drawbacks. For many applications, the particulates are not smooth. Both crystal and amorphous molecular structures will form geometric shapes that have edges and points, a characteristic that frequently causes difficulty for numerical simulations. The method also produces some degree of regularity to the larger domain.

2. Effective Permittivity

2.1 Analytic Estimates

A number of researchers have developed models for determining the permittivity of composite materials. Much of the early effort was developed for electrostatic or quasi-static solutions to Laplace's equation. Capacitance calculations for a particle placed inside a parallel plate capacitor were determined. Therfore, the calculation of the effective permittivity is quasistatic and neglects high frequency effects. A simple geometry of the composite was assumed, such as a spherical particle with a spherical inclusion, ie. two concentric spheres. Two independently derived equations using this model are obtained[2][3]

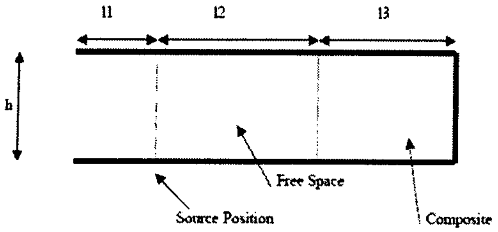
$$\ln(\varepsilon_{eff}) = f\ln(\varepsilon_d) + (1-f)\ln(\varepsilon_o) \qquad (1)$$

$$(\varepsilon_{eff})^{1/3} = f\ln(\varepsilon_d)^{1/3} + (1-f) \qquad (2)$$

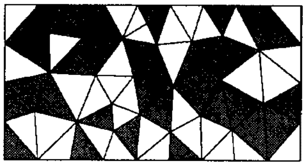
where ε_{eff} represents the effective permittivity, f is the packing fraction and ε_d and ε_o are the permittivity of the dielectric particles and free space, respectively. Despite the different formulation, these models yield similar results. Naturally, subsequent models were more complex, moving the enclosed dielectric off-center, having multiple locations of the interior dielectric, changing the geometric shape. The Maxwell-Garnett[4] model involves a more sophisticated approach to this geometric model. The modern investigation of the Effectivity Medium Theory[5] (EMT) provides a more sophisticated analysis. Recent refinements of the Quasi-Crystaline Approximation[6] (QCA) can be used to model particulates with varied structure and sizes. The more sophisticated applications of the QCA model investigate a combination of these shapes and can therefore represent a larger packing fraction. This model has the most relevance with regard to the FEM investigation.

3. Experimental Analysis

A shorted waveguide, as shown in Figure 1, was implemented to measure the effective permittivity of



<Fig. 1> Experimental Geometry for Permittivity Measurements



<Fig. 2> Example Mesh of Composite with 50% Packing Fraction

the composite material. In the figure, the mixture is placed at the waveguide termination. Optimally, the source frequency is such that only the first mode of propagation will exist in both the free space region and the composite. Therefore, in the composite region, a standing wave pattern will exist with complete reflection from the conducting wall. For large dielectric materials, this preferred situation is likely not possible and measurement errors will be introduced due to mode conversion. The parallel plate field modes are

$$E_y(x,z) = A\sin(\frac{n\pi x}{h})\exp(-\alpha_z z) - A\sin(\frac{n\pi x}{h})\exp(+\alpha_z z) \quad (3)$$

where α_z is the complex propagation constant that needs to be determined. Coaxial, rectangular and circular waveguides are often used for experimental analysis and the lowest order modes are also preferred.

3.1 FEM Method

This investigation utilizes the finite element mesh itself as a collection of particles. Each triangle was randomly assigned a permittivity where the distribution was related to the packing density. Since the particles have sharp corners, field approximations are imprecise. However, the physical particles will also have similar features. The mesh dimensions, and therefore the particles are much smaller than the applied wavelength. For both the FEM particles and the physical particles, the response to wave incidence may be considered an average reflection of multiple scatterers. The error inherent in the FEM approximation of sharp features can be ignored in favor of the average scattering from each particle. In regions of relatively high field magnitude, accuracy can be increased using mesh refinement. An adaptive scheme[7] for subdividing the mesh elements improves the approximation. If necessary, a better estimate of the field at the "corners" of particles can then be obtained while the computational simplicity of the method is still maintained.

3.2 Experimental Measurements

A network analyzer was employed to determine the S11 and S12 parameters of a particulate filled waveguide. The waveguide was 53.34 [mm] in length with a freespace cutoff frequency of 21.1 [GHz]. Ground SiO2 was investigated, with an estimated packing fraction of 50% and real permittivity in the range [2.9,3.2] and imaginary permittivity of approximately j0.0026. The network analyzer swept the frequency range of 25-40 [GHz]. The phase angle of the reflection coefficient at the guide aperture is shown in Figure 4.7a. The effective permittivity is determined from the location of frequencies with the same phase. The phase of the reflection coefficient repeats whenever one half of the wavelength is an integer multiple of the length of the dielectric section of the waveguide. The following formula can then be used to determine the effective permittivity from the experimental measurements,

$$\frac{c}{\varepsilon_{eff}} = 2*L*\left[f_2\sqrt{\left(1-\frac{f_c}{f_2\sqrt{\varepsilon_{eff}}}\right)} - f_1\sqrt{\left(1-\frac{f_c}{f_1\sqrt{\varepsilon_{eff}}}\right)} \right] \quad (4)$$

4. Results

An initial 2D experiment was performed on a simple composite representing a mixture of air and dielectric with a relative permittivity of 2 and an imaginary component of 0.1. Typical field patterns are shown in Figure 3. The cutoff frequency for the second mode was between these two frequencies and mode conversion is easily seen at the higher frequency.

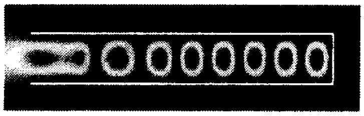
The effective permittivity was determined using a least squares fit of Equation 3. The results are provided in Table 1. The 2D analyis reveals are nearly linear combination for the real component of the effective permittivity. This result is inconsistent with analytic models which predict a result smaller than a linear fit. However, 2D analysis can be considered the geometric equivalent of long strands of dielectric oriented parallel to the eleectric field, in which case the QCA model does predict nearly linear behavior.

Interestingly, the slight increase in the loss tangent relative to the packing density was unexpected and more consistent with the quasi-static measurements where contact in conducting material is equivalent to closing the circuit path. A ray tracing approximation provides some insight into the increased loss. An ideal FEM equilateral triangle can be considered. The mean path length for a ray that is not refracted and a ray that is refracted can be determined at any point along the entry side of the triangle by simple geometric analysis. The mean path length in dielectric particle is 1.2% longer for an equilateral triangle. Since the field in a lossy dielectric has exponential decay, a Taylor approximation if the increased loss tangent provides insight into with the 2D experimental results.

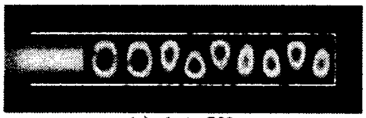
An FEM simulation of the SiO2 experiment was then performed with both 2D and 3D finite elements with results of the measurement data and the FEM simulation shown in Figures 4 and 5, respectively. The 3D experiment was more consistent with analytic models, as would be expected. Furthermore, mesh refinement was applied, with the results of the real component of the permittivity shown in Table 2. Dielectric triangles maintained the properties of their parent triangle.

Packing Fraction	ε'	ε''	% Increase of ε''
50%	1.5029	0.05271	5.4%
65%	1.6586	0.06706	1.1%
75%	1.7529	0.07659	.94%
80%	1.8008	0.08192	2.4%
90%	1.9011	0.09176	1.8%
100%	2.0063	0.1001	0.1%

<Table 1> FEM Effective Permittivities

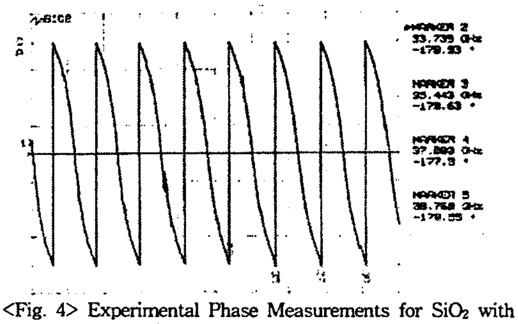


a) 1.2 GHz

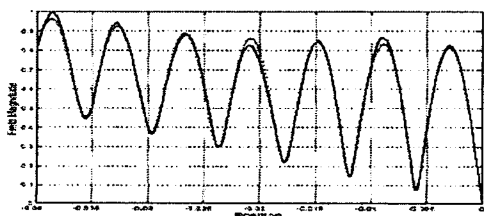


b) 1.4 GHz

<Fig. 3> Electric Field Magnitude in FEM Experiment



<Fig. 4> Experimental Phase Measurements for SiO_2 with approximately 50% Packing Fraction



<Fig. 5> Electric Field Magnitude and Analytic Fit along Axis of the Waveguide

Iteration	1	2	3	4
Permittivity	1.86	1.87	1.88	1.88

< Table 2> Convergence of Effective Permittivity with Mesh Refinement

5. Conclusion

The FEM method was implemented to simulate experimental techniques used in studying the effective permittivity of dielectric particles. The elements in the mesh represented a random distribution of particles, dependent on the packing fraction. Analytic models indicate that material with the lower dielectric constant has a greater effect on the composite permittivity. Experimental data is consistent with these models. However, the experiments have focused on particles that are roughly the same size in all three directions. Recent extensions to the QCA model that consider long thin particles indicate that a linear relationship may exist. The 2D FEM study is consistent with this model. Experiments with a 3D FEM model are more consistent with expectations. The small increase in the 2D FEM loss tangent was an unexpected result and is possible explained by a ray tracing the wave paths.

The FEM model demonstrated several of the difficulties inherent in the experimental measurements. Mode conversion was observed. The evanescent wave will result in miscalculations of the S12 power loss measurements. Also, the Nicholson-Ross-Weir[8] errors were observed when the composite waveguide length corresponded to an integer number of half-wavelengths.

[References]

[1] F. K. Liu, C. J. Ker, Y. C. Chang, F. H. Ko, T. C. Chu, B. T. Dai, "Microwave heating for the preparation of nanometer gold particles", *Jpn. J. of Appl. Phys.*, vol. 42(6B), pp 4152-4158, 2003

[2] A. von Hippel *Dielectric Materials and Applications*, John Wiley and Sons, New York, 1954

[3] H. Looyenga, "Dielectric constants of heterogeneous mixtures", *Physics*, No. 31, pp401-406, 1965

[4] J. C. Maxwell-Garnett, "Colours in metal and glasses in metal films", *Trans. Royal Soc. London*, vol 203, pp385-420, 1904

[5] V. D. A. G. Bruggeman, "Betechnung vershiedener physikalischer konstanten von heterogen substanzen", *Annalen der Physik*, vol. 24(5), pp636-664, 1935

[6] M. Lax, "The multiple scattering of waves II. The effective field in dense systems", *Phys. Rev.*, vol 85, pp621-629, 1952

[7] H. S. Kim, S. P. Hong, K. Choi, H. K. Jung, S. Y. Hahn, "A three dimensional adaptive finite element method for magnetostatic problems", *IEEE Trans on Mag*, vol. 25(5), pp4081-4084, 1991

[8] A. M. Nicolson, G. F. Ross, "Measurement of the intrinisic properties of materials by time domain techniques", *IEEE Trans. on Instrum. Meas.*, vol. IM(17), pp395-402, 1968