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퍼지 기법을 이용한 시간 지연을 가지는 이산시간 비선형 시스템에 대한 강인 제어기 설계

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Design of the Robust Controller for the Discrete-Time Nonlinear System with Time-Delay Via Fuzzy Approach

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Abstract - In this paper, a robust H^∞ stabilization problem to a uncertain discrete-time nonlinear systems with time-delay via fuzzy static output feedback is investigated. The Takagi-Sugeno (T-S) fuzzy model is employed to represent an uncertain nonlinear systems with time-delayed state. Then parallel distributed compensation technique is used for designing of the robust fuzzy controller. Using a single Lyapunov function, the globally asymptotic stability and disturbance attenuation of the closed-loop fuzzy control system are discussed. Sufficient conditions for the existence of robust H^∞ controllers are given in terms of linear matrix inequalities via similarity transform and congruence transform technique.

1. INTRODUCTION

Most plants used in the real world have a strong nonlinearity and uncertainty. Moreover, when this system is controlled, time-delay is generally occurred and disturbance interrupts. Therefore to solve this problem, many efforts have done.

There are many papers that propose the control methodology of the linear system with time-delay. But for the nonlinear system with time-delay, only few papers exist. This arises from the complexity of the nonlinear system. To overcome this difficulty, various schemes have been developed in the last two decades, among which a successful approach is fuzzy control.

Cao et al. first proposed the Takagi-Sugeno (T-S) fuzzy model with time-delay that represents the nonlinear system with time-delay and analyzed the stability of that in [2]. Based on this, Lee et al. [3] proposed a dynamic output feedback robust H^∞ control method for a class of uncertain fuzzy systems with time-varying delay. But in this method, to design the robust controller, bilinear matrix inequality (BMI) must be solved. Therefore the design of the controller is very difficult.

Lo et al. proposed the robust static output feedback control method of the nonlinear system without time-delay via fuzzy control approach in [1]. In this method, controller can be easily designed by solving several linear matrix inequalities (LMI). In [6], we applied the method proposed by Lo for controlling the continuous-time nonlinear system with time-varying delay, and successfully converted to controller design problem to LMIs.

In this paper, we extend the method that proposed in [6] to a discrete-time nonlinear system with time-delay. The procedure is similar to the continuous-time case. We design the H^∞ fuzzy controller that robustly control the discrete-time nonlinear system with time-delay subject to external disturbances. To this end, we first represent the nonlinear system with time-delay to fuzzy model with time-delay as did in [2]. Then parallel distributed compensation technique is applied for the design of the static output feedback fuzzy controller. After selecting one Lyapunov function, we derive the sufficient condition for stability of the fuzzy system. But this condition is composed of BMIs. Therefore we convert it LMI by using similarity transform and congruence transform technique. From this, the H^∞ fuzzy controller can be easily designed by many current convex optimization algorithm tools.

The remainder of the paper is organized as follows: following the introduction, problem formulation is done in Section 2. In Section 3, the sufficient condition for making the discrete-time T-S fuzzy model with time-delay asymptotically stable is derived and the design of H^∞ fuzzy controller is done. Finally, some conclusions are drawn in Section 4.

2. PROBLEM FORMULATION

The T-S fuzzy model is generally known as the universal approximator of nonlinear systems. We consider nonlinear systems represented by the following T-S fuzzy model with time-delay.

Plant Rule i

IF $\theta_1(t)$ is M_{i1} and … and $\theta_n(t)$ is M_{in}
THEN

$$x(t+1)=(A_i+\Delta A(t))x(t)+(A_{di}+\Delta A_d(t))x(t-d(t)) \\ +(B_{1i}+\Delta B_1(t))u(t)+(B_{2i}+\Delta B_2(t))u(t)$$

$$z(t)=(C_i+\Delta C(t))x(t)+(C_{di}+\Delta C_d(t))x(t-d(t)) \\ +(D_{1i}+\Delta D_1(t))u(t)+(D_{2i}+\Delta D_2(t))u(t)$$

$$y(t)=Ex(t), \qquad i=1,\cdots,r$$

$$x(t)=0, \qquad t\le 0. \tag{1}$$

Where $M_{i1},\cdots,M_{in}$ is the fuzzy set, $x\in R^n$ is the state vector, $u(t)\in R^{q1}$ is unknown but the energy-bounded disturbance input, $u(t)\in R^{q2}$ is input vector, $z(t)\in R^s$ is the controlled output, $(A_i,A_d,B_{1i},B_{2i},C_i,C_d,D_{1i},D_{2i})$ are some constant matrices of compatible dimensions, r is the number of IF-THEN rules, d is the time-delay and $\theta(t)=(\theta_1(t),\cdots,\theta_n(t))$ are the premise variables. It is assumed that the premise variables do not depend on the input variables $u(t)$, explicitly.

The time-varying matrices, $(\Delta A,\Delta A_d,\Delta B_1,\Delta B_2,\Delta C,\Delta C_d,\Delta D_1,\Delta D_2)$, is defined as follows:

$$\begin{pmatrix} \Delta A & \Delta A_d & \Delta B_1 & \Delta B_2 \\ \Delta C & \Delta C_d & \Delta D_1 & \Delta D_2 \end{pmatrix}=\begin{pmatrix} M\Delta(t) & (N_1\ N_2\ N_3\ N_4) \\ M_z\Delta_z(t) & (N_{z1}\ N_{z2}\ N_{z3}\ N_{z4}) \end{pmatrix}. \tag{2}$$

where $(M,M_z,N_1,N_2,N_3,N_4,N_{z1},N_{z2},N_{z3},N_{z4})$ is known real constant matrices, and (Δ,Δ_z) are unknown matrix functions with Lebesgue-measurable elements and satisfies $\Delta'(t)\Delta(t)\le I$, $\Delta_z'(t)\Delta_z(t)\le I$ in which I is the identity matrix of appropriate dimension.

Remark 1: The uncertain fuzzy system (1) encompasses the nonlinear system, which it represents.

The defuzzified output of (1) is represented as follows:

$$x(t+1)=\sum_{i=1}^{r}\mu_i(\theta(t))[(A_i+\Delta A(t))x(t) \\ +(A_{di}+\Delta A_d(t))x(t-d(t)) \\ +(B_{1i}+\Delta B_1(t))u(t)+(B_{2i}+\Delta B_2(t))u(t)]$$

$$z(t)=\sum_{i=1}^{r}\mu_i(\theta(t))[(C_i+\Delta C(t))x(t) \\ +(C_{di}+\Delta C_d(t))x(t-d(t)) \\ +(D_{1i}+\Delta D_1(t))u(t)+(D_{2i}+\Delta D_2(t))u(t)]$$

$$y(t)=Ex(t) \tag{3}$$

where $\mu_i(\theta(t))=\dfrac{w_i(\theta(t))}{\sum_{i=1}^{r}w_i(\theta(t))}$, $w_i(\theta(t))=\prod_{k=1}^{n}M_{ik}(\theta(t))$.

The controller is a static output feedback fuzzy controller of the following defuzzifed form:

$$u(t)=\sum_{i=1}^{r}\mu_i(\theta(t))K_iy(t) \tag{4}$$

where K_i are constant control gains to be determined. For simplicity, we represent $\mu_i(\theta(t))$ as μ_i and abbreviate the time index, t, in time-varying matrices.

Substituting Eq. (4) into Eq. (3), the closed-loop system is obtained as follows:

$$x(t+1)=[A_\mu+B_{2\mu}K_\mu E+M\Delta(N_1+N_4K_\mu E)]x(t) \\ +(A_{d\mu}+M\Delta N_2)x(t-d(t)) \\ +(B_{1\mu}+M\Delta N_3)u(t)$$

$$z(t)=[C_\mu+D_{2\mu}K_\mu E+M_z\Delta_z(N_{z1}+N_{z4}K_\mu E)]x(t) \\ +(C_{d\mu}+M_z\Delta_zN_{z2})x(t-d(t)) \\ +(D_{1\mu}+M_z\Delta_zN_{z3})u(t)$$

$$y(t)=Ex(t) \tag{5}$$

where $Y_\mu=\sum_{i=1}^{r}\mu_iY_i$, $Y_i\in\{A_i,A_{di},B_{1i},B_{2i},C_i,C_{di},D_{1i},D_{2i}\}$.

The performance considered here is an H∞ criterion such that the following is satisfied:

$$\sum_0^\infty z'(t)z(t)<\gamma^2\sum_0^\infty w'(t)w(t) \tag{6}$$

Definition 1: H∞ fuzzy controller

1) The controller makes the system (1), (3) robustly stable in the presence of $u(t)$.

2) Given γ, the closed-loop system (5) must satisfy the criterion (6), in which the initial condition is zero.

3. DESIGN OF THE FUZZY CONTROLLER

In this section, a static output feedback fuzzy controller satisfying Definition 1 will be addressed. It should be noted that the result presented in Theorem 1, arising from static output feedback stabilization problems, is only existential and can not be solved by present convex algorithm. Therefore further work is required for easy application.

Theorem 1: Given a constant $\gamma>0$, the system (1) is robustly stabilizable by the controller (4) if there exists the positive symmetric matrices P, S and control gains, K_j satisfied the following matrix inequalities. In other words, (4) is the H∞ fuzzy controller.

$$\begin{cases} M_{ii}<0, & i=1,\cdots,r \\ \dfrac{1}{r-1}M_{ii}+\dfrac{1}{2}(M_{ij}+M_{ji})<0, & 1\le i\ne j\le r \end{cases} \tag{7}$$

$$M_{ij}=\begin{bmatrix} S-P & * & * & * & * & * & * & * & * \\ 0 & -S & * & * & * & * & * & * & * \\ 0 & 0 & -\gamma^2I & * & * & * & * & * & * \\ \Gamma_{ij} & A_d & B_{1i} & -P^{-1} & * & * & * & * & * \\ 0 & 0 & 0 & \varepsilon_1M & -\varepsilon_1I & * & * & * & * \\ \Omega_j & N_2 & N_3 & 0 & 0 & -\varepsilon_1I & * & * & * \\ \Psi_{ij} & C_d & D_{1i} & 0 & 0 & 0 & -I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_2M_z & -\varepsilon_2I & * \\ \Lambda_j & N_{z2} & N_{z3} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2I \end{bmatrix}$$

where $\Gamma_{ij}=A_i+B_{2i}K_jE$, $\Omega_j=N_1+N_4K_jE$, $\Psi_{ij}=C_i+D_{2i}K_jE$, $\Lambda_j=N_{z1}+N_{z4}K_jE$.

Proof: Considet the following Lyapunov function:

$$V(t)=x'(t)Px(t)+\sum_{\sigma=k-d}^{k-1}x'(\sigma)Sx(\sigma).$$

The procedure of the proof is almost identical to that of the continuous-time case. Refer to [6]. **Q.E.D**

The inequality (7) is BMIs which is not solvable by the convex programming technique. Therefore further manipulations are required. Here we use the method that proposed in [1].

First we define new state variables: $x=T\tilde{x}$. Then (5) is converted followings:

$$\tilde{x}(t+1)=[\widetilde{A_\mu}+\widetilde{B_{2\mu}}K_\mu E+\widetilde{M}\Delta(\widetilde{N_1}+N_4K_\mu E)]\tilde{x}(t) \\ +(\widetilde{A_{d\mu}}+\widetilde{M}\Delta\widetilde{N_2})\tilde{x}(t-d(t)) \\ +(\widetilde{B_{1\mu}}+\widetilde{M}\Delta N_3)u(t)$$

$$\begin{aligned} z(t)=[\widetilde{C}_\mu+D_{2\mu}K_\mu\widetilde{E}+M_z\Delta_z(\widetilde{N}_{z1}+N_{z1}K_\mu\widetilde{E})]\widetilde{x}(t) \\ +(\widetilde{C}_{d\mu}+M_z\Delta_z\widetilde{N}_{z2})\widetilde{x}(t-d(t)) \\ +(D_{1\mu}+M_z\Delta_z N_{z3})u(t) \end{aligned} \quad (8)$$

where $\widetilde{A}_\mu=T^{-1}A_\mu T$, $\widetilde{B}_{2\mu}=T^{-1}B_{2\mu}$, $\widetilde{A}_{d\mu}=T^{-1}A_{d\mu}T$, $\widetilde{B}_{1\mu}=T^{-1}B_{1\mu}$, $\widetilde{C}_\mu=C_\mu T$, $\widetilde{C}_{d\mu}=C_{d\mu}T$, $\widetilde{E}=ET$, $\widetilde{M}=T^{-1}M$, $\widetilde{N}_1=N_1T$, $\widetilde{N}_2=N_2T$, $\widetilde{N}_{z1}=N_{z1}T$, $\widetilde{N}_{z2}=N_{z2}T$.

Let $Q=P^{-1}$.

$$Q_{n\times n}=\begin{bmatrix} Q_{1p\times p} & 0 \\ 0 & Q_2 \end{bmatrix}.$$

The transformation matrix, T is selected in order to satisfy the following condition:

$$\widetilde{E}=ET=[I_p\ 0]$$

That is $T=[E'(EE')^{-1}|_{n-p}\ ortc(E)]$.

where $ortc(E)$ denotes orthogonal complement of E. Applying Theorem 1 to (8), the sufficient condition to stabilize (8) is the following:

$$\begin{cases} \widetilde{M}_{ii}<0, & i=1,\cdots,r \\ \frac{1}{r-1}\widetilde{M}_{ii}+\frac{1}{2}(\widetilde{M}_{ij}+\widetilde{M}_{ji})<0, & 1\le i\ne j\le r \end{cases} \quad (9)$$

$$\widetilde{M}_{ij}=\begin{bmatrix} S-P & * & * & * & * & * & * & * & * \\ 0 & -S & * & * & * & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * & * & * & * \\ \Gamma_{ij} & \widetilde{A}_d & \widetilde{B}_{1i} & -P^{-1} & * & * & * & * & * \\ 0 & 0 & 0 & \varepsilon_1 M & -\varepsilon_1 I & * & * & * & * \\ \Omega_j & \widetilde{N}_2 & N_3 & 0 & 0 & -\varepsilon_1 I & * & * & * \\ \Psi_{ij} & \widetilde{C}_d & D_{1i} & 0 & 0 & 0 & -I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_2 M_z & -\varepsilon_2 I & * \\ \Lambda_j & \widetilde{N}_{z2} & N_{z3} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 I \end{bmatrix}$$

where

$\Gamma_{ij}=\widetilde{A}_i+\widetilde{B}_{2i}K_j\widetilde{E}$, $\Omega_j=\widetilde{N}_1+N_1K_j\widetilde{E}$, $\Psi_{ij}=\widetilde{C}_i+D_{2i}K_j\widetilde{E}$, $\Lambda_j=\widetilde{N}_{z1}+N_{z1}K_j\widetilde{E}$

Let $\Theta=diag[Q\ Q\ I\ I\ I\ I\ I\ I\ I]$.

Pre- and post-multiplying (9) by Θ the inequality ex pounded is displayed as

$$\begin{cases} \overline{M}_{ii}<0, & i=1,\cdots,r \\ \frac{1}{r-1}\overline{M}_{ii}+\frac{1}{2}(\overline{M}_{ij}+\overline{M}_{ji})<0, & 1\le i\ne j\le r \end{cases} \quad (10)$$

$$\overline{M}_{ij}=\begin{bmatrix} X-Q & * & * & * & * & * & * & * & * \\ 0 & -X & * & * & * & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * & * & * & * \\ \overline{\Gamma}_{ij} & \widetilde{A}_d Q & \widetilde{B}_{1i} & -Q & * & * & * & * & * \\ 0 & 0 & 0 & \varepsilon_1 M & -\varepsilon_1 I & * & * & * & * \\ \overline{\Omega}_j & \widetilde{N}_2 Q & N_3 & 0 & 0 & -\varepsilon_1 I & * & * & * \\ \overline{\Psi}_{ij} & \widetilde{C}_d Q & D_{1i} & 0 & 0 & 0 & -I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_2 M_z & -\varepsilon_2 I & * \\ \overline{\Lambda}_j & \widetilde{N}_{z2} Q & N_{z3} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 I \end{bmatrix}$$

where

$\overline{\Gamma}_{ij}=\widetilde{A}_i Q+\widetilde{B}_{2i}[F_j\ \ 0]$, $\overline{\Omega}_j=\widetilde{N}_1 Q+N_1[F_j\ \ 0]$,

$\overline{\Psi}_{ij}=\widetilde{C}_i Q+D_{2i}[F_j\ \ 0]$, $\overline{\Lambda}_j=\widetilde{N}_{z1} Q+N_{z1}[F_j\ \ 0]$,

$X=QSQ>0$, $F_j=K_jQ_1$.

Remark 2: the positive definite matrix S is obtained by X and Q

Remark 3: By defining the new variable, $F_j=K_jQ_1$, the inequality (10) is linear matrix inequality that has following 5 variables: $(Q, X, F_j, \varepsilon_1, \varepsilon_2)$.

Therefore, we can easily design the H∞ fuzzy controller by using many current convex optimization tools.

4. CONCLUSION

In this paper, the fuzzy control approach is proposed to robustly control the discrete-time nonlinear system with time-delay. Using Lyapunov theory, the sufficient condition is derived. Through the manipulation, bilinear matrix inequality is converted to linear matrix inequality. Therefore, we can easily design the controller via current convex algorithm.

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