퍼지 시스템을 위한 샘플치 데이터 상태 피드백 제어기 설계: 지능형 디지털 재설계 접근

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Design of State Feedback Controller for Fuzzy Systems: Intelligent Digital Redesign

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Abstract - This paper presents a complete solution to intelligent digital redesign problem (IDR) for sampled-data fuzzy systems. The IDR problem is the problem of designing a sampled-data state feedback controller such that the sampled-data fuzzy system is equivalent to the continuous-time fuzzy system in the sense of the state matching. Its solution is simply obtained by linear transformation. Under the proposed sampled-data controller, the states of the discrete-time model of the sampled-data fuzzy system completely matches the state of the discrete-time model of the closed-loop continuous-time fuzzy systems are completely matched at every sampling points.

1. Introduction

Digital redesign (DR) has gained tremendously increasing attention as yet another efficient design tool of sampled-data fuzzy control [1]-[6]. The intelligent digital redesign (IDR) problem is the problem of designing a sampled-data state feedback controller such that the sampled-data closed-loop fuzzy system is equivalent to the continuous-time closed-loop fuzzy system in the sense of the state matching.

There have been fruitful researches in the digital control system focusing on IDR method. Historically, Joo et al. first attempted to develop some intelligent digital redesign methodology for complex nonlinear systems [1]. They synergistically merged both the Takagi-Sugeno (T-S) fuzzy-model-based control and the digital redesign technique for a class of nonlinear systems. Chang et al. extended the intelligent digital redesign to uncertain T-S fuzzy systems [2]. These approach [1,2] to IDR are so called as local approach. The local approach can allows to match the states of the continuous-time and the sampled-data closed-loop fuzzy systems in the analytic way, but it may lead to undesirable and/or inaccurate results. The major reason is that the redesigned digital control gain matrices are obtained by considering only the local state-matching of each sub-closed-loop system [6]. To overcome this weakness, Lee et al. a global state-matching technique based on the convex optimization method, the linear matrix inequalities (LMIs) method, proposed in [6]. Specifically, their method is to globally match the states of the overall closed-loop T--S fuzzy system with the predesigned analog fuzzy-model-based controller and those with the digitally redesigned fuzzy-model-based controller, and further to examine the stabilizability by the redesigned controller in the sense of Lyapunov. However. the IDR problem becomes the overdamped problem according as transferring the local approach to the global one in IDR problem. From that reason, their method is not able to completely match the states between the closed-loop sampled-data and continuous-time systems. It may lead to undesirable and/or inaccurate results.

Motivated by the above observations, we studies the intelligent digital redesign problem (IDR) for sampled-data fuzzy systems. The main features of the proposed method are as follows: First, the multirate control scheme is employed to increase the dimension of input. Second, the proposed approach is to assume that the sampling period is fixed, but the exact discrete-time model can be obtained as integration step size approach zero. Finally, under the proposed sampled-data controller, the states of the discrete-time model of the sampled-data fuzzy system completely matches the state of the discrete-time model of the closed-loop continuous-time fuzzy systems are completely matched at every sampling points.

2. Problem Statement

Consider a nonlinear system described by

$$\dot{x}(t) = f(x(t), u(t)) \tag{1}$$

where $x(t) \in R^n$ is the state vector, and $u_c(t) \in R^m$ is the continuous-time control input.

To facilitate the control design, we will develop a simplified model, which can represent the local linear input·output relations of the nonlinear system. This type of models is referred as T·S fuzzy models. The fuzzy dynamical model corresponding to the nonlinear system (1) is described by the following IF·THEN rules [1,2,3,6]:

$$R_k : IF\ z_1(t)\ is\ about\ \Gamma_{k1}\ and \cdots and\ z_p(t)\ is\ about\ \Gamma_{kp},$$
$$THEN\ \dot{x}(t) = A_k x(t) + B_k u(t) \tag{2}$$

where $R_k, k \in I_q = \{1, 2, \cdots, q\}$, is the kth fuzzy rule, $z_r(t), r \in I_p = \{1, 2, \cdots, p\}$, is the rth premise variable, and $\Gamma_{kr}, (k, r) \in I_q \times I_p$, is the fuzzy set. Then, given a pair $(x(t),u(t))$, using the center-average defuzzification, product inference, and singleton fuzzifier, the overall dynamics of the IF-THEN rules (2) has the form

$$\dot{x}(t)=\sum_{k=1}^{q}\theta_k(z(t))(A_k x(t)+B_k u(t)) \tag{3}$$

where $\theta_k(z(t))=\frac{w_k(z(t))}{\sum_{k=1}^{q}w_k(z(t))}$, $w_k(z(t))=\sum_{r=1}^{p}\Gamma_{kr}(z_r(t))$, and $\Gamma_{kr}(z_r(t))$ is the grade of membership of $z_r(t)$ in Γ_{kr}. The possibly time-varying parameter vector $\theta\in R^q$ belongs to a convex polytope Θ, where

$$\Theta:=\left\{\sum_{k=1}^{q}\theta_k=1,\quad 0\le\theta_k\le 1\right\} \tag{4}$$

It is clear that as θ varies inside Θ, $\sum_{k=1}^{q}\theta_k(z(t))A_k$ and $\sum_{k=1}^{q}\theta_k(z(t))B_k$ range over a matrix polytope

$$\left[\sum_{k=1}^{q}\theta_k(z(t))A_k,\sum_{k=1}^{q}\theta_k(z(t))B_k\right]\in\mathbf{Co}\{(A_k,B_k),k\in I_q\}$$

where **Co** denotes the convex hull. In this note, the stabilization of the polytopic model (3) is equivalent to the simultaneous stabilization of its vertices $(A_k,B_k),k\in I_q$.

In this paper, a well-constructed continuous-time sta te feedback controller, which will be employed in rede signing the digital controller, is given. The controller i s described by the following IF-THEN rules:

$$R_k: IF\ z_1(t)\ is\ about\ \Gamma_{k1}\ and\cdots and\ z_p(t)\ is\ about\ \Gamma_{kp},$$
$$THEN\ u_c(t)=K_k x_c(t), \tag{5}$$

and its defuzzified output is

$$u_c(t)=\sum_{k=1}^{q}\theta_k(z(t))K_k x_c(t) \tag{6}$$

Therefore, main purpose of this paper is to find the di gital equivalent of the following continuous-time close d-loop system:

$$\dot{x}_c(t)=\sum_{k=1}^{q}\sum_{l=1}^{q}\theta_k(z(t))\theta_l(z(t))(A_k+B_kK_l)x_c(t) \tag{7}$$

3. Intelligent Digital Redesign

The task of the sampled-data controller is to stabilize the origin of the closed-loop system. We follow the digital redesign approach to this problem. First, we assume that the stabilizable continuous-time controller is predesigned. Then, we design the sampled-data controller such that the responses of the closed-loop continuous-time and discrete-time systems are closely matched for the same initial conditions. For convenience, we rewrite the continuous-time closed-loop system (7) as

$$\dot{x}_c(t)=f(x_c(t))x_c(t) \tag{8}$$

We consider a zero-order-hold system where u is held constant over the time interval of τ. This type of the system is called as multirate control system [9]. Let T be the sampling period and $\tau=T/M$ for some integers $M>0$. To descretize the plant dynamics, we rewrite (3) as

$$\dot{x}(t)=F(x(t),u(k+j\tau)) \tag{9}$$

Following the discretization procedure of [8], the discrete-time model of (8) over the period $[kT+j\tau,kT+j\tau+\tau]$, $k\times j\in Z_{\ge 0}\times Z_{[0,M-1]}$ is given by

$$x(k+j\tau+\tau)=F_{\tau,h}(x(k+j\tau),u(k+j\tau)) \tag{10}$$

where $h=\tau/N$ for some integers $N>0$.

Remark 1. *A more realistic approach is to assume that the sampling period is fixed (or has positive a lower bound), since the required sampling rate may not be implementable due to hardware limitations. In [8], it is shown that integration step size h determines the accuracy of the discrete-time model.*

By recursive application of (10) defined as

$$F_{\tau,h}^{1}(x(k),u(k)):=F_{\tau,h}(x(k),u(k))$$
$$F_{\tau,h}^{j+1}(x(k),U_{[kT,kT+j\tau]})$$
$$:=F_{\tau,h}(F_{\tau,h}^{j}(x(k),U_{[kT,kT+j\tau-\tau]}),u(k+j\tau))$$
$$F_{T,\tau,h}(x(k),U_{[kT,kT+M\tau-\tau]}):=F_{\tau,h}^{M}(x(k),U_{[kT,kT+M\tau-\tau]}),$$

where

$$U_{[kT,kT+M\tau-\tau]}=\begin{bmatrix}u(k)\\u(k+\tau)\\\vdots\\u(k+M\tau-\tau)\end{bmatrix},$$

we arrive at

$$x(kT+T)=F_{T,\tau,h}(x(k),U_{[kT,kT+M\tau-\tau]}). \tag{11}$$

In the same manner, we can generate the following discrete-time model of (8):

$$x_c(k+1)=f_{T,\tau,h}(x_c(k))x_c(k) \tag{12}$$

by defining

$$f_{\tau,h}^{1}(x_c(k)):=f_{\tau,h}(x_c(k))$$
$$f_{\tau,h}^{j+1}(x_c(k)):=f_{\tau,h}(f_{\tau,h}^{j}(x_c(k)))f_{\tau,h}^{j}(x_c(k))$$
$$f_{T,\tau,h}(x_c(k)):=f_{\tau,h}^{M}(x_c(k)).$$

From (11) and (12), it remains to determine how to construct the sampled-data controller such that

$$F_{T,\tau,h}(x(k),U_{[kT,kT+M\tau-\tau]})=f_{T,\tau,h}(x_c(k))x_c(k) \tag{13}$$

Here, the nonlinear interpolation between $x(k)$ and

$U_{[kT,kT+M\tau+\tau]}$ makes the state matching based on the linear transformation impossible, which leads us to make the following assumption.

Assumption 1. $x(k+j\tau+ih+h)=x_c(k+j\tau+ih+h)$ *for* $k\times j\times h\in Z_{\ge 0}\times Z_{[1,M-1]}\times Z_{[0,N-1]}$.

The decompose model of (11) is given by the following proposition.

Proposition 1. Let Assumption 1 hold; then the dynamics of (11) can be decomposed as

$$x(k+1)=A_{T,\tau,h}(x(k))x(k)+B_{T,\tau,h}(x(k))U_{[kT,kT+M\tau-\tau]} \tag{14}$$

where $n\times n$ matrix $A_{T,\tau,h}(x(k))$ and $n\times mM$ matrix $B_{T,\tau,h}(x(k))$ are given by the following recursive procedure:

$$\begin{aligned}
&A^{1}_{\tau,h}(x(k)):=A_{\tau,h}(x(k))\\
&A^{j+1}_{\tau,h}(x_c(k)):=A_{\tau,h}(f^{j}_{\tau,h}(x(k)))A^{j}_{\tau,h}(x(k))\\
&A_{T,\tau,h}(x(k)):=A^{M}_{\tau,h}(x(k))
\end{aligned}$$

and

$$\begin{aligned}
&B^{1}_{\tau,h}(x(k)):=B_{\tau,h}(x(k))\\
&B^{j+1}_{\tau,h}(x(k))\\
&:=\left[A_{\tau,h}(f^{j}_{\tau,h}(x(k)))B^{j}_{\tau,h}(x(k))\quad B_{\tau,h}(f^{j}_{\tau,h}(x(k)))\right]\\
&B_{T,\tau,h}(x(k)):=B^{M}_{\tau,h}(x(k)),
\end{aligned}$$

respectively.

The following assumption is introduced for ease of control synthesis.

Assumption 2. $\mathrm{rank}\left(B_{T,h,\tau}(x(k))\right)=n$

Remark 2. *From the fact that* $\mathrm{rank}\left(B_{T,h,\tau}(x(k))\right)\le\min\{mM,n\}$, *the* $mM\times n$ *matrix* $B_{T,h,\tau}(x(k))$, $mM<n$ *is necessarily singular. In the proposed approach, this circumstance does not happen because it is possible to set the input multiplicity M such that* $mM\ge n$.

Theorem 1. *Under the sampled-data controller defined as*

$$U_{[kT,kT+M\tau-\tau]}=B^{-}_{T,h,\tau}(x(k))[f_{T,\tau,h}(x(k))-A_{T,\tau,h}(x(k))]x(k), \tag{15}$$

where $B^{-}_{T,h,\tau}(x(k))$ *is the generalized inverse of matrix* $B_{T,h,\tau}(x(k))$ *[10], the state* $x(k)$ *of the discrete-time model of sampled-data system (11) completely matches the state* $x_c(k)$ *of the discrete-time model of closed-loop continuous-time system (12).*

4. Conclusions

This paper proposed the multirate sampled-data control design using the IDR approach for the fuzzy systems. Under some assumption, we develop the sampled-data controller for completely state matching. In addition, the proposed sampled-data controller is available for the long and fixed sampling limit because a family of discrete-time models is employed to deign the controller.

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