Bayesian Method for Sequential Preventive Maintenance Policy

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Abstract

In this paper, we propose a Bayesian approach to determine the adaptive preventive maintenance(PM) policy for a general sequential imperfect PM model proposed by Lin, Zuo and Yam(2000) that PM not only reduces the effective age of the system but also changes the hazard rate function. Assuming that the failure times follow Weibull distribution, we adopt a Bayesian approach to update unknown parameters and determine the Bayesian optimal sequential PM policies. Finally, numerical examples of the optimal adaptive PM policy are presented for illustrative purposes.

1. Introduction

The preventive maintenance(PM) is an action taken on a repairable system while it is still operating, which needs to be carried out in order to keep the system at the desired level of successful operation. The PM action may include minimal repair, perfect repair or replacement of the system as well as its components and even the inspection of the system may be considered as part of the PM works.

Many authors have proposed several PM models and obtained the optimal PM policies by optimizing several criteria regarding the operating cost. Nakagawa(1986) considers several periodic and sequential PM policies for the system with minimal repair at each failure. For these models, the PM action is conducted at periodic times kx for periodic PM and at sequential times $x_1, x_1+x_2, \cdots, x_1+\cdots+x_N$, where $x_1, x_2, \ldots, x_N$ are not necessarily equal. When a failure occurs between the PM's, a minimal repair is done and the system remains in the same state as it was before failure. Canfield(1986) considers a periodic PM policy for which the PM slows the degradation process of the system, while the hazard rate keeps monotone increase although the hazard rate is reduced to that of a reduced virtual age. Park, Jung and Yum(2000) derive the optimal periodic PM schedules by incorporating various cost structures into Canfield's model. Lin, Zuo and Yam(2000) propose a general sequential imperfect PM model that PM not only reduces the effective age of the system but also changes the hazard rate function. The models are hybrid in the sense that they are combination of the age reduction PM model and the hazard rate adjustment PM model.

In this paper, we consider the Bayesian approach to update the unknown parameters and to derive the necessary mathematical formulas for determining the optimal adaptive sequential PM policy for a general sequential PM model proposed by Lin, Zuo and Yam(2000).

The Bayesian approach could be quite effective when the failure distribution of the system is either unknown or contains uncertain parameters, which is common in most practical applications. Mazzuchi and Soyer(1996) adopt a Bayesian approach to solve the optimal replacement problem for both the block replacement protocols with minimal repair and the age replacement protocols by minimizing the expected long-run average cost when the underlying distribution is Weibull. Their models have been extended by Juang and Anderson(2004), in which the minimal repair cost is assumed to be random in addition to the random parameters of Weibull model discussed in Mazzuchi and Soyer(1996).

The remainder of this paper is organized as follows. In section 2, Lin, Zuo and Yam's(2000) general sequential preventive maintenance model is presented. In Section 3, a Bayesian approach is adopted for the Weibull model by assigning appropriate prior distributions on both shape and scale parameters and the method to derive the optimal adaptive sequential PM schedules is discussed. Section 4 presents numerical examples for Bayesian approach to illustrate the proposed procedures.

2. General sequential imperfect preventive maintenance model

Lin, Zuo and Yam(2000) propose a general sequential imperfect preventive maintenance model incorporating adjustment/ improvement factors in hazard rate and effective age. The models are hybrid in the sense that they are combinations of the age reduction PM model and the hazard rate adjustment PM model. The PM not only reduces the effective age to a certain value but also changes the slope of the hazard rate, while the hazard rate increase with the number of PMs. PM is performed in a sequence of intervals.

Notations.

T	time to failure
$h(t)$	hazard rate without PM
$h_{pm}^{k}(t)$	hazard rate between the k th and the $(k+1)$ st PM
x_k	interval length between the $(k-1)$ st and the k th PM
z_k	$=\sum_{i=1}^{k} x_i$, the k th PM time
y_k	effective age of the system just before the k th PM
a_k	adjustment factor in hazard rate after the k th PM, $1=a_0 \le a_1 \le \cdots \le a_{N-1}$
A_k	$=\prod_{i=0}^{k-1} a_i, \quad k=1,2,\cdots,N$
b_k	improvement factor in effective age after the k th PM, $0=b_0 \le b_1 \le \cdots \le b_{N-1} < 1$
N	number of PM's conducted before replacement
C_{pm}	cost of PM
C_{re}	cost of replacement
C_{mr}	cost of minimal repair

Lin, Zuo and Yam(2000) consider the following assumptions.

1) The planning horizon is finite.

2) The hazard rate function of the system when there is no repair or PM, $h(t)$, is continuous and strictly increasing function.

3) The time for PM, minimal repair and replacement are negligible.

4) PM is performed at $z_1, z_2, \cdots, z_{N-1}$, and the system is replaced at z_N as new system.

5) The hazard rate function becomes $a_k h(bz_k)$ right

after the k th PM when it was $h(z_k)$ just before the PM. After the PM, the hazard rate function is expressed as $a_k h(bz_k + x)$ for $x > 0$. Here we have $1 = a_0 \le a_1 \le \cdots \le a_{N-1}$ and $0 = b_0 \le b_1 \le \cdots \le b_{N-1} < 1$.

Lin, Zuo and Yam(2000) consider the situation where a system preventively maintained at $z_1, z_2, \cdots, z_{N-1}$ and is replaced at z_N. Minimal repair is performed at failure between PMs. The system has the hazard rate $A_k h(t)$ between the $(k-1)$ th and k th PMs. When $a_k = 1,\ k = 1,2,\cdots,N-1$, the hybrid model reduces to the age reduction model. When $b_k = 0,\ k = 1,2,\cdots,N-1$, the hybrid model reduces to the hazard rate adjustment.

The effective age of the system becomes $b_{k-1}y_{k-1}$ right after the $(k-1)$ th PM and then becomes $y_k = x_k + b_{k-1}x_{k-1} + \cdots + b_{k-1}b_{k-2}\cdots b_2 b_1 x_1$ immediately before the k th PM. Obviously, we have $y_k = x_k + b_{k-1}y_{k-1}$ or $x_k = y_k - b_{k-1}y_{k-1}$.

Following Ref. [5], the mean cost rate is

$$C(y_1, y_2, \ldots, y_N, N) = \frac{C_{mr} \sum_{k=1}^{N} A_k \left[H(y_k) - H(b_{k-1}y_{k-1})\right] + (N-1)\, C_{pm} + C_{re}}{\left[\sum_{k=1}^{N-1} (1-b_k) y_k + y_N\right]}. \quad (1)$$

Using the proposed hybrid model, Lin, Zuo and Yam(2000) develop optimal PM policies to minimize the mean cost rate when the parameters of hazard rate are given.

3. Bayesian method for general sequential imperfect preventive maintenance model

In practical applications the failure distribution of a system is usually either unknown or contains uncertain parameters. In this case, it is necessary to select an appropriate estimation method to accurately calculate the parameters of a given distribution and the mean cost rate.

In this section, we discuss a Bayesian approach for a general sequential PM model proposed by Lin, Zuo and Yam(2000). We consider that the failure times follow a Weibull distribution with the following hazard rate function.

$$h(t) = \alpha \beta t^{\beta-1}, \qquad t \ge 0,\ \alpha > 0,\ \beta > 1, \quad (2)$$

where α and β are the scale and shape parameters, respectively. For this model, the hazard rate function can be expressed as

$$h_{pm}^{k}(t) = \begin{cases} \alpha \beta t^{\beta-1} & for\ \ 0 \le t \le z_1 \\ A_{k+1} \alpha \beta \left(t - \sum_{i=1}^{k} (1-b_i) y_i\right)^{\beta-1} & for\ \ z_k < t \le z_{k+1}, \end{cases} \quad (3)$$

for $k = 0,1,\ldots,N-1$, $h_{pm}^{0}(t) = h(t)$ and $x_0 = y_0 = z_0 = 0$. Substituting the expression (3) into the formula (1), we obtain the following formula for the mean cost rate.

$$C(y_1, y_2, \ldots, y_N, N) = \frac{C_{mr} \sum_{k=1}^{N} A_k \left\{\alpha y_k^{\beta} - \alpha (b_{k-1}y_{k-1})^{\beta}\right\} + (N-1)\, C_{pm} + C_{re}}{\left[\sum_{k=1}^{N-1} (1-b_k) y_k + y_N\right]}. \quad (4)$$

To determine the optimal sequential PM schedules for the Weibull model, we apply the adaptive estimation schemes to update uncertainty about α and β based on the empirical data observed during the current life cycle and thus, reevaluate the optimal schedules for the next life cycle.

As in Juang and Anderson(2004), we use a gamma distribution and a discrete beta distribution as prior distributions for scale parameter α and shape parameter β, respectively. Such priors are also suggested by Mazzuchi and Soyer(1996). Since the discrete beta distribution allows for great flexibility in representing prior uncertainty, it has been used as a prior for β in several Bayesian PM models. The prior distributions for α and β are given by

$$f(\alpha) = \frac{b^a}{\Gamma(a)} \alpha^{a-1} e^{-b\alpha}, \qquad \alpha \ge 0,\ b > 0, \tag{5}$$

and

$$\Pr(\beta = \beta_l) = \int_{\beta_l - \delta/2}^{\beta_l + \delta/2} g(\beta)d\beta \equiv P_l, \tag{6}$$

where $\beta_l = \beta_L + \delta(2l-1)/2$ and $\delta = (\beta_U - \beta_L)/m$ for $l = 1,2,\ldots,m$. Here, $\sum_{l=1}^{m} P_l = 1$ and $g(\beta)$ is a beta density to be defined as

$$g(\beta) = \frac{\Gamma(c+d)}{\Gamma(c)\Gamma(d)} \frac{(\beta-\beta_L)^{c-1}(\beta_U-\beta)^{d-1}}{(\beta_U-\beta_L)^{c+d-1}}, \quad 0 \le \beta_L \le \beta \le \beta_U,\ c, d > 0. \tag{7}$$

Initially, α and β are prior independent and thus the joint prior distribution of α and β is the product of two marginals given in (5) and (6). That is,

$$p(\alpha,\beta) = f(\alpha)\Pr(\beta=\beta_l) = \frac{b^a}{\Gamma(a)} \alpha^{a-1} e^{-b\alpha} \times P_l.$$

To obtain the optimal sequential PM schedules based on the prior distributions of α and β, we need to formulate the mean cost rate by taking the expectation on (4) with respect to α and β. Given the priors (5) and (6), the mean cost rate can be expressed as

$$C_B(y_1,y_2,\ldots,y_N,N) = E_{\alpha,\beta}\, C(y_1,y_2,\ldots,y_N,N)$$
$$= E_{\alpha,\beta}\left[\frac{C_{mr}\sum_{k=1}^{N} A_k\left\{\alpha y_k^{\beta} - \alpha(b_{k-1}y_{k-1})^{\beta}\right\} + (N-1)C_{pm} + C_{re}}{\sum_{k=1}^{N-1}(1-b_k)y_k + y_N}\right]$$
$$= \frac{\sum_{l=1}^{m}\left[C_{mr}\frac{a}{b}\sum_{k=1}^{N} A_k\left\{y_k^{\beta_l} - (b_{k-1}y_{k-1})^{\beta_l}\right\} + (N-1)C_{pm} + C_{re}\right]P_l}{\sum_{k=1}^{N-1}(1-b_k)y_k + y_N}. \tag{8}$$

If we differentiate the mean cost rate given in (8) with respect to each y_k, $k = 1,2,\ldots,N$ and set them equal to 0, then we have

$$C_{mr}\frac{a}{b}\sum_{l=1}^{m} A_N \beta_l y_N^{\beta_l - 1} P_l = C_p(y_1,y_2,\ldots,y_N,N) \tag{9}$$

and

$$\sum_{l=1}^{m}\beta_l\left\{A_k y_k^{\beta_l - 1} - A_{k+1}b_k(b_k y_k)^{\beta_l - 1}\right\}P_l = \sum_{l=1}^{m}\beta_l A_N(1-b_k)y_N^{\beta_l - 1}P_l$$
$$(k = 1,2,\ldots,N-1). \tag{10}$$

Theorem 1. For a fixed $y_N (0 < y_N < \infty)$, the solution of equation (10) with respect to $y_k (y_k > 0)$ exists and is unique if $1 - a_k b_k > 0$, $k = 1,2,\ldots,N-1$ and $\beta_l > 1$, $l = 1,2,\ldots,m$.

Proof. The left hand side of equation (10) is 0 when $y_k = 0$. If $1 - a_k b_k > 0$ and $\beta_l > 1$, the derivative of equation (10) with respective to y_k is

$$\sum_{l=1}^{m}\beta_l(1-\beta_l)y_k^{\beta_l - 2}A_k\left(1 - a_k b_k^{\beta_l}\right)P_l$$
$$\ge \sum_{l=1}^{m}\beta_l(1-\beta_l)y_k^{\beta_l - 2}A_k\left(1 - a_k b_k\right)P_l > 0.$$

i.e. the left hand side of equation (10) is a strictly increasing function of y_k. Therefore, the solution of equation (10) with respective to y_k is unique. □

Lin, Zuo and Yam(2000) mentioned that the condition , $1 - a_k b_k > 0$, is a reasonable one in their paper. The condition $1 - a_k b_k > 0$ means that the hazard rate adjustment factor a_k should be smaller than the improvement factor b_k.

Substituting each solution of equation (10), y_k $(k = 1,2\cdots,N-1)$, into equation (9), we obtain

$$\frac{a}{b}\sum_{l=1}^{m}\left[A_N\beta_l y_N^{\beta_l - 1}\left\{\sum_{k=1}^{N-1}(1-b_k)y_k + y_N\right\} - \sum_{k=1}^{N} A_k\left\{y_k^{\beta_l} - (b_{k-1}y_{k-1})^{\beta_l}\right\}\right]P_l = \frac{(N-1)C_{pm} + C_{re}}{C_{mr}}. \tag{11}$$

where each $y_k (k = 1,2,\cdots,N-1)$ is some function of y_N. Then, the left hand side of equation (11) is a function only of y_N.

Theorem 2. If $1 - a_k b_k > 0$, $k = 1,2,\cdots,N-1$ and $\beta_l > 1$, $l = 1,2,\ldots,m$, the solution of equation (11) with respective to y_N $(y_N > 0)$ exists and is unique.

Proof. Since

$$\sum_{l=1}^{m}\beta_l y_k^{\beta_l - 1}A_k\left(1 - a_k b_k^{\beta_l}\right)P_l \ge \sum_{l=1}^{m}\beta_l y_k^{\beta_l - 1}A_k\left(1 - a_k b_k\right)P_l > 0,$$

for $k = 1,2,\cdots,N-1$, the solution of equation (10) with respect to y_k is zero when $y_N = 0$. Thus, if $y_N = 0$, the left hand side of equation (11) equals zero and is smaller than the right hand side of (11). On the other hand, the derivative of equation (11) with respective to y_N is obtained as follows.

$$\frac{a}{b}\sum_{l=1}^{m}\left[A_N\beta_l(\beta_l - 1)y_N^{\beta_l-2}\left\{\sum_{k=1}^{N-1}(1-b_k)y_k + y_N\right\}\right]P_l > 0.$$

i.e. the left hand side of equation (11) is a strictly increasing function of $y_N (y_N \geq 0)$ and equals zero at $y_N = 0$. Therefore, the solution of equation (11) with respective to $y_N (y_N > 0)$ exists and is unique. □

Based on the above results, we obtain y_k, $k = 1,2,\cdots,N$ and x_k from $x_k = y_k - b_{k-1}y_{k-1}$, $k = 1,2,\cdots,N$.

As for the case when neither N nor $x_1, x_2, \ldots, x_N$ is known, we may proceed similarly. Park, Jung and Yum(2000) discuss the similar situations based on Canfield's(1986) periodic PM model. Firstly, we assume that N is fixed and determine the values of $x_1^*, x_2^*, \ldots, x_N^*$ as a function of N alone by applying the method discussed in the earlier part of this section. Then, the value of N^* is determined as

$$N^* = \min_{N \geq 1} C_B(x_1^*(N), x_2^*(N), \ldots, x_N^*(N), N).$$

Once the value of N^* is determined, then $x_1^*, x_2^*, \ldots, x_{N^*}^*$ can be obtained by replacing N by N^* in its expressions.

In the next, we discuss the concept of an adaptive sequential PM strategy which is based on the posterior distributions of α and β. When the failure data is recorded at the end of each life cycle, the priors for α and β are adaptively updated and hence become the prior distributions for the next life cycle. Let N_k denote the number of failures and let $\underline{\mathbf{T_k}} = (T_{k1}, T_{k2}, \cdots, T_{kn_k})$ denote the failure times between the k th PM and the $(k+1)$ st PM for $k = 0,1,\cdots,N-1$. Then the joint probability density of $(\underline{\mathbf{T_k}}, N_k)$ is written as

$$f(t_{k1}, t_{k2}, \cdots t_{kn_k}) = \left\{\prod_{j=1}^{n_k} h_{pm}^k(t_{kj})\right\}\exp\left\{-H_{pm}^k(z_{k+1})\right\},$$

where $H_{pm}^k(t) = \int_{z_k}^{t} h_{pm}^k(s)ds$. To simplify the notations, we let $\underline{\mathbf{t}} = \{\underline{\mathbf{t}}_0, \underline{\mathbf{t}}_1, \cdots, \underline{\mathbf{t}}_{N-1}\}$ be the vector of observed failure times throughout the life cycle of the system. Given $\underline{\mathbf{t}}$, the posterior distributions of α and β are derived. Then, it becomes the prior distributions for the next life cycle of the system. The likelihood function of α and β can be obtained by

$$\begin{aligned} L(\alpha, \beta \mid \underline{\mathbf{t}}) &= \prod_{k=0}^{N-1} f(t_{k1}, t_{k2}, \cdots, t_{kn_k}) \\ &= \left\{\prod_{k=0}^{N-1}\prod_{j=1}^{n_k} A_{k+1}\alpha\beta\left(t_{kj} - \sum_{i=1}^{k}(1-b_i)y_i\right)^{\beta-1}\right\} \\ &\quad \cdot \exp\left\{-\sum_{k=0}^{N-1} A_{k+1}\alpha\left\{\left(z_{k+1} - \sum_{i=1}^{k}(1-b_i)y_i\right)^{\beta} - \left(z_k - \sum_{i=1}^{k}(1-b_i)y_i\right)^{\beta}\right\}\right\}. \end{aligned} \tag{12}$$

Using Bayes' theorem, the joint posterior distribution of α and β can be expressed as follows,

$$f(\alpha, \beta_l \mid \underline{\mathbf{t}}) = \frac{\alpha^{\left(a + \sum_{k=0}^{N-1} n_k - 1\right)} \beta_l^{\left(\sum_{k=0}^{N-1} n_k\right)} g_1(\beta_l) \cdot \exp\{-a \cdot (g_2(\beta_l))\} P_l}{\sum_{h=1}^{m}\left[P_h \beta_h^{\left(\sum_{k=0}^{N-1} n_k\right)} g_1(\beta_h)\Gamma\left(a + \sum_{k=0}^{N-1} n_k\right) \Big/ (g_2(\beta_h))^{\left(a + \sum_{k=0}^{N-1} n_k\right)}\right]},$$

where

$$g_1(\gamma) = \prod_{k=0}^{N-1}\prod_{j=1}^{n_k} A_{k+1}\left(t_{kj} - \sum_{i=1}^{k}(1-b_i)y_i\right)^{\gamma-1},$$

$$g_2(\gamma) = b + \sum_{k=0}^{N-1} A_{k+1}\left\{\left(z_{k+1} - \sum_{i=1}^{k}(1-b_i)y_i\right)^{\gamma} - \left(z_k - \sum_{i=1}^{k}(1-b_i)y_i\right)^{\gamma}\right\}$$

Since $f(\alpha \mid \beta_l, \underline{\mathbf{t}}) = f(\alpha, \beta_l, \underline{\mathbf{t}}) / \Pr(\beta = \beta_l, \underline{\mathbf{t}})$, the conditional posterior distribution is a gamma distribution with parameters of a^* and b^* which has the following density function.

$$f(\alpha \mid \beta_l, \underline{\mathbf{t}}) = \frac{(g_2(\beta_l))^{\left(a + \sum_{k=0}^{N-1} n_k - 1\right)}}{\Gamma\left(a + \sum_{k=0}^{N-1} n_k\right)} \alpha^{\left(a + \sum_{k=0}^{N-1} n_k - 1\right)} \cdot \exp[-a \cdot (g_2(\beta_l))],$$

where $a^* = a + \sum_{k=0}^{N-1} n_k$ and $b^* = g_2(\beta_l)$ are the updated shape and scale parameters. In addition, since $\Pr(\beta = \beta_l, |\underline{t}) = f(\alpha, \beta_l|\underline{t}) / f(\alpha, |\beta_l, \mathbf{t})$, the posterior distribution of β can be written as

$$\Pr(\beta = \beta_l, |\underline{t}) = P_l^* = P_l \cdot \frac{\beta_l^{\left(\sum_{k=0}^{N-1} n_k\right)} g_1(\beta_l) \Big/ \{g_2(\beta_l)\}^{a+\sum_{k=0}^{N-1} n_k}}{\sum_{h=1}^{m} \left[P_h \beta_h^{\left(\sum_{k=0}^{N-1} n_k\right)} g_1(\beta_h) \Big/ \{g_2(\beta_h)\}^{a+\sum_{k=0}^{N-1} n_k} \right]}.$$

Note that the posterior distributions of α and β are no longer independent. The optimal sequential PM schedules based on the posterior distributions of α and β can be calculated in a straightforward manner by replacing a, b and P_l in the equation (8) by a^*, b^* and P_l^*, respectively, and by finding N^* and the optimal PM intervals $x_1^*, x_2^*, \cdots, x_{N^*}^*$.

4. Numerical example

This section presents numerical examples to illustrate a Bayesian approach for a general sequential PM model proposed by Lin, Zuo and Yam(2000) that the failure times follow a Weibull distribution and derive the optimal PM schedules based on both prior distribution and posterior distribution on two parameters α and β. We take $C_{mr} = 1$, $C_{pm} = 1.5$, $a_k = (6k+1)/(5k+1)$, $b_k = k/(2k+1)$, $k = 0,1,2,\cdots$. For the Bayesian sequential PM policy we take $a = 2.0$, $b = 3.0$, $c = d = 2.0$, $\beta_L = 2.0$, $\beta_U = 4.0$, $m = 20$. Thus, δ equals to $(4.0-2.0)/20 = 0.1$.

Table 4.1 illustrates the adaptive nature of our approach by considering three life cycles of the system. At the end of each cycle, the failure data is used to update the parameters a, b and P_l and thus to renew the optimal PM schedules during the next life cycle. Instead of real data, we use the simulated data based on the hazard function with PM given in (2) with $\alpha = 1$ and $\beta = 3$ and the computation results are listed in Table 4 when $C_{re} = 7$. In our approach, we first derive the optimal PM schedules with no failure data and then, based on that schedule, the next failure data are generated.

Using the failure data, the updated values of parameters a, b and P_l are obtained and we renew the sequential optimal PM schedules for the next life cycle. Table 1 shows that the optimal number of PM's seems stabilized as the number of cycles increases.

<Table 1> Optimal adaptive Bayesian PM schedules with $C_{re} = 7$

Cycle	Failure Times	Optimal PM Number	Optimal PM Interval	Mean Cost
0		4	$x_1^* = 1.30549$ $x_2^* = 0.73815$ $x_3^* = 0.59921$ $x_4^* = 0.76896$	5.01761
1	0.93950 1.95339 1.99763 2.99409 3.11745 3.13349 3.35542	5	$x_1^* = 1.19893$ $x_2^* = 0.68365$ $x_3^* = 0.55860$ $x_4^* = 0.48661$ $x_5^* = 0.64529$	5.29110
2	0.46367 1.05105 1.11849 1.46946 1.55493 2.33647 2.66456 3.46906 3.53222	4	$x_1^* = 1.19977$ $x_2^* = 0.67145$ $x_3^* = 0.54093$ $x_4^* = 0.70234$	5.63610
3	1.10643 1.19391 1.19943 1.36987 2.64637 2.91597 3.02493 3.11167	4	$x_1^* = 1.22273$ $x_2^* = 0.67442$ $x_3^* = 0.53729$ $x_4^* = 0.70767$	5.79342

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