Identification of the Movement of Underlying Asset in Real Option Analysis: Studies on Industrial Parametric Table

Jeong-Dong Lee* ·Ari Kang** ·Jong-Wook Jeong***

Abstract: This paper has an intention of proposing useful parametric tables of each industry group within Korea. These parametric tables can be insightful criteria for those who are dealing with the exact valuation of company, technology or industry through Real Option Analysis (ROA) since the identification of the movement of underlying asset is the very first step to be done. To give the exact estimations of parameters and the most preferred model in each industry group, we cover topics on ROA, stochastic process, and parametric estimation method like Generalized Method of Moments (GMM) and Maximum Likelihood Estimation (MLE). Additionally, specific industry groups, such as, Internet service group and mobile telecommunication service group defined independently in this paper are also examined in terms of its property of movement with the suggesting of the most fitting stochastic model.

Key words: ROA, stochastic process, GMM, MLE, parametric table

1. INTRODUCTION

Valuation of technology, company or industry is one of the most important concerns nowadays. In the world of highly advanced technology and complex social relationships, many decisions of business affair such as new investment, Mergers And Acquisitions (M&A) or industry policy can be implemented correctly only when the relevant company or industry is evaluated exactly. So far, there have been many valuation approaches, such as Industry Standards, Ranking methods, auctions, Discounted Cash Flow method (DCF) and so on[1]. However, in the view of the great importance of exact valuation, those approaches are not so confirmable because they usually take the several wild assumptions granted but still leave lots of uncertainty unresolved. On the other hand, the real option analysis (ROA) which has recently emerged as the powerful tool of the valuation task has shown more realistic evaluation result.

The most significant advantage of ROA is the curtailment of uncertainty. The uncertainty comes from the future decisions which might be altered without notification according to various situations affected by trends, financial crisis, political issues and so on. ROA can deal with the uncertainty more

* Professor, Techno-Economics and Policy Program in Seoul National University
** Techno-Economics and Policy Program in Seoul National University (e-mail:onlytot0@snu.ac.kr)
*** Techno-Economics and Policy Program in Seoul National University (e-mail:mirue@snu.ac.kr)

[1] Razgaitis (1999)

realistically because the method encompasses the possible changes of future decision. There have been many applications of ROA in practice. Ottoo (1998) valued the internal growth opportunities of Biotechnology company with a real option framework, Lee (2001) applied option pricing model to the project of natural resource development, Insely (2002) examined the value of forestry investment using ROA, Schwartz and Moon (2000) evaluated the company using ROA with Monte-Carlo simulation and Baranzini, Chesney and Morisset (2003) interestingly studied the impact of possible climate catastrophes on global warming policy with the aid of ROA.

Unfortunately, however, the advantage of considering uncertainty of ROA requires not a small amount of information. Most importantly, the exact valuation through ROA can be obtained when the movement of underlying asset is fully identified. However, figuring out the exact movement is not an easy task because the estimation methodology has not been fully developed. So far, as a part of estimating the movement of financial underlying asset, alternative estimation methods have been released. The parametric estimation techniques estimate the value of parameters after assuming the form of the function with the parametric restrictions. Generalized Method of Moments (GMM) and Maximum Likelihood Estimation (MLE) based methods are proposed by Chan, Karolyi, Longstaff and Sanders (1992, hereafter CKLS), Nowman (1997), Aït-Sahalia (1996, 1998) and Yu-Philips (2001) etc. CKLS enabled the comparison of alternative models within one framework without considering the distribution of the data. Nowman and Yu-Philips exploited the Gaussian approach and Aït-Sahalia introduced the approximation of Hermite polynomial expansion to implement MLE. Additionally some recent researchers like Gordon (2000) and Laskin (2000) focus on fractal property of data.

The intention of this paper is to suggest estimation methods which show good performance and apply it to the semi-annual sales and daily industry index of Korean industry groups. This study will contribute to have a general look on how the underlying asset of each industry has moved. Also suggested industrial parametric table serves as a benchmark of individual company.

The paper is organized as follows. Section 2 describes several estimation methodologies putting quite huge interests on GMM and MLE approach. Section 3 presents the results of simulation on identifying best estimating method. Section 4 deals with empirical results. Section 5 concludes.

2. METHODOLOGIES

2.1. Real Option Analysis

Because of the intense competition in the market with the importance of technology and industry, the more accurate analysis on real property has been required and ROA becomes one of the most powerful tools. The basic knowledge of ROA is similar to that of financial option pricing and the parameters used in option pricing can be compared to that of ROA. For example, current stock price, exercise price and maturity data in financial option are present value of possible future earnings, further investment expenditure and expected time of research in ROA. Even though ROA deals with the non-trade property and it is the main difference between ROA and financial option, the option pricing concept can still be applicable (Lee, 2003). Especially ROA is much more applicable if the object has possible changes in future decision. For example, if a company has pliant strategies in R&D stage, producing stage or commercializing stage, ROA has a great power by the feasibility of considering such pliancy.

The identification of movement of underlying is the most fundamental factor to get the exact value through ROA. Both analytical and numerical approaches in ROA define the identification of underlying asset as the first step. However, in spite of many applications of ROA in practical issues, those studies have limitation caused by misidentified stochastic process of the value of underlying asset. From the viewpoint of important identification issues related to the valuation of technology, company or industry, the examination of several estimation of stochastic process which correctly expresses the movement of underlying asset deserves special emphasis.

2.2. Stochastic process

Stochastic process is expressed by the following continuous and differential equation.

$$dX_t = \mu(X_t,t)\,dt + \sigma(X_t,t)\,dZ_t \qquad (2.1)$$

$\mu(X_t,t)$ represents the drift and $\sigma(X_t,t)$ for diffusion trends. X_t is a variable in interest, specially the value of underlying asset of ROA in this paper. Z_t is a Brownian motion whose differences follow Normal distribution. It can be rewritten as equation (2.2) in the case of homogeneous in time form.

$$dX_t = \mu(X_t)dt + \sigma(X_t)dZ_t \qquad (2.2)$$

Under this stochastic process modeling, many studies have been done to identify the appropriate model by setting the linear drift term and non-linear diffusion term like equation (2.3)

$$dX_t = (\alpha + \beta X_t)dt + \sigma X_t^{\gamma} dZ_t. \qquad (2.3)$$

This equation shows the property of mean reverting towards the long run mean $-\alpha / \beta$ and the speed of the reversion $-\beta$. The γ gives the information of level effect that how the movement of process depends on the value of variable X itself.

Many researchers like Merton (1973), Vasicek (1977), Cox, Ingersoll and Ross (CIRSR) (1985), Dothan (1978), Black and Scholes (1973), Brennan and Schwartz (BS) (1980), and Cox (1975) proposed various types of models with restrictions on equation (2.3). Table 1 presents theirs models and its parameter restrictions according to the order of elasticity of variance[2] (CKLS,1992.)

Table 1 Various models of stochastic process

	Model	**K**	**2**	**L**	**3**
Merton(1973)	$dX_t = \alpha dt + \sigma dZ_t$		**0**		**0**
Vasicek(1977)	$dX_t = (\alpha + \beta X_t)dt + \sigma dZ_t$				**0**
CIR SR(1985)	$dX_t = (\alpha + \beta X_t)dt + \sigma X_t^{1/2} dZ_t$				**1/2**
Dothan(1978)	$dX_t = \sigma X_t dZ_t$	**0**	**0**		**1**
GBM(1973)	$dX_t = \beta X_t dt + \sigma X_t dZ_t$	**0**			**1**
BS(1980)	$dX_t = (\alpha + \beta X_t)dt + \sigma X_t dZ_t$				**1**
CIR VR(1980)	$dX_t = \sigma X_t^{3/2} dZ_t$	**0**	**0**		**3/2**
CEV(1975)	$dX_t = \beta X_t dt + \sigma X_t^{\gamma} dZ_t$	**0**			

Note: Various models are listed with the information of restrictions imposed by each model. The unrestricted model is as follows.

$$dX_t = (\alpha + \beta X_t)dt + \sigma X_t^{\gamma} dZ_t$$

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2 $\dfrac{\ln \sigma(X_t)}{\ln X_t}$

2.3. Estimation method of Stochastic process

Parameters of the models summarized in Table 1 can be estimated by various estimation methods. Though respective researchers have studied the estimation of particular model, there has been aroused the necessary of choosing the best explanatory model and estimating parameters of that model. In this respect the former studies of CKLS and Nowman (1997) which dealt with comparison between several alternative stochastic models bear great significance.

2.3.1. Generalized Method of Moments (GMM)[3]

CKLS used the technique of GMM of Hansen (1982). To implement the discretely observed data in continuous stochastic process, they set a discrete-time econometric specification.

$$X_{t+1} - X_t = \alpha + \beta X_t + \varepsilon_{t+1} \quad (2.4)$$

$$E[\varepsilon_{t+1}] = 0 \quad , \quad E[\varepsilon_{t+1}^2] = \sigma^2 X_t^{2\gamma} \quad (2.5)$$

These specifications make the expectation of following equation set to be zero[4].

$$f_t(\theta) = \begin{bmatrix} \varepsilon_{t+1} \\ \varepsilon_{t+1} X_t \\ \varepsilon_{t+1}^2 - \sigma^2 X_t^{2\gamma} \\ \left(\varepsilon_{t+1}^2 - \sigma^2 X_t^{2\gamma}\right) X_t \end{bmatrix} \quad (2.6)$$

where θ is the parameter vector with $\alpha, \beta, \sigma^2, \gamma$.

To implement GMM, one should compose sample expectation of $f_t(\theta)$, say $G_T(\theta)$ with T observation[5].

The estimated parameters are the values that minimize the next quadratic equation (2.7)[6].

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[3] CKLS (1992)

[4] These become the null hypothesis of GMM procedure.

[5] As the number of samples increases, sample expectation can satisfy the condition of $E[f_t(\theta)] = 0$.

$$J_T(\theta) = G_T'(\theta) W_T(\theta) G_T(\theta) \qquad (2.7)$$

where $W_T(\theta)$ is positive definite weighting matrix.

After estimating parameters, though the GMM, one can figure the appropriateness of each model using the value of $J_T(\theta)$. That is, every restricted model has different value of $J_T(\theta)$ and the degree of its closeness to that of unrestricted model can be a criterion to identify the explanatory power of each model[7]. Since GMM allows the parameter estimation without considering different distribution of each model and gives the information of goodness of fit to real data, it is very useful. However, it nests some restriction on the requirement of stationary of data and causes aggregation bias.

2.3.2. Gaussian Estimation by Nowman[8]

Nowman tried to apply Gaussian estimation to various stochastic models listed in Table 1 and compare the results with the study of CKLS. The main limitation of applying Gaussian estimation on various models is the non-Gaussian distribution of models with conditional heteroscedasticity[9]. However, the Gaussian estimation method of Bergstrom (1984) can overcome this limitation. Furthermore the difficulty of applying Gaussian estimation caused by non-constant conditional volatility[10] is also released by Nowman.

The stochastic process equation (2.3) can be modified when applying constant volatility in unit period.

$$dX_t = (\alpha + \beta X_t)dt + \sigma (X_{t'-1})^{\gamma} dZ \qquad (2.8)$$

where $t' - 1$ is the largest integer less than t.

As Bergstrom (1984) provides, the process of X_t can be given by

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[6] It assures $G_T(\theta)$ approaches $f_t(\theta)$. The parameters can be obtained simply by differentiating equation (2.8).

[7] $J_T(\theta)$ of unrestricted model is zero because the number of parameters to be estimated and that of moments are the same. The minimized value of $J_T(\theta)$ follows χ^2 with degree of freedom (number of moments –number of parameters to be estimated) under the null hypothesis of the model is true.

[8] Nowman (1997)

[9] The changes of X in CIRSR model are proportional to a non-central χ^2 variate.

[10] While the Gaussian estimation method developed by Bergstrom assumes the constant volatility, the general model of stochastic process equation (2.3) allows the level effect caused by γ.

$$X_t = e^{\beta} X_{t-1} + \frac{\alpha}{\beta}\left(e^{\beta} - 1\right) + \eta_t \qquad (2.9)$$

where $E(\eta_s \eta_t) = 0$ and $E\left(\eta_t^2\right) = \int_{t-1}^{t} e^{2(t-\tau)\beta} \sigma^2 \left(X_{t-1}\right)^{2\gamma} d\tau = \frac{\sigma^2}{2\beta}\left(e^{3\beta} - 1\right)\left(X_{t-1}\right)^{2\gamma} = m_{tt}^2$.

To get the estimation of parameters, MLE can be used because the above discussion assumes Gaussian estimation. Log likelihood function is defined as follows.

$$L(\theta) = \sum_{t=1}^{T} \left[-\ln m_{tt} - \frac{\left\{ X_t - e^{\beta} X(t-1) - \frac{\alpha}{\beta}\left(e^{\beta} - 1\right) \right\}^2}{2 m_{tt}^2} \right]^{11} \qquad (2.10)$$

where θ is the parameter vector with $\alpha, \beta, \sigma^2, \gamma$.

The goodness of fit can be also obtained by comparing the value of likelihood function. The Gaussian estimation approached by Nowman makes the MLE possible with theoretical grounds. However, it has a limit that the process is not an exact Gaussian but an approximate Gaussian one.

2.3.3. Gaussian Estimation by Yu-Phillips[12]

In the study of Yu-Phillips an exact discrete model with Gaussian errors is proposed. The model is possible through exploiting the martingale property and using time-change technique.

Yu-Phillips have their result from the DDS theorem[13] that after suitable time change, any continuous time martingale can be expressed as a Brownian motion. Since the changes of Brownian motion follow a normal distribution, they have closer Gaussian estimation approach.

The general stochastic process equation (2.3) can be express like equation (2.11) with given initial value.

$$X_t = \left(X_0 + \frac{\alpha}{\beta} \right) e^{\beta t} - \frac{\alpha}{\beta} + \int_0^t e^{\beta(t-s)} \sigma X_s^{\gamma} dZ_s \qquad (2.11)$$

Also for any $h > 0$

[11] In Nowman(1997), $L(\theta)$ is defined as multiplying by -2 to equation (2.12). However this difference doesn't affect the result of estimation.

[12] Yu, J. and Peter C.B. Phillips (2001)

$$X_{t+h} = \frac{\alpha}{\beta}\left(e^{\beta h} - 1\right) + e^{\beta h} X_t + \int_0^h \sigma e^{\beta(h-\tau)} X_{t+\tau}^{\gamma} dZ_\tau . \tag{2.12}$$

Let the third term of right hand side of equation (2.12) be M_h with a continuous martingale variation

$$[M]_h = \sigma^2 \int_0^h e^{2\beta(h-\tau)} X_{t+\tau}^{2\gamma} d\tau \tag{2.13}$$

To express M_h as a Brownian motion, the sequence of positive number (h_j) should be generated.

$$h_{j+1} = \inf\{s \mid [M_j]_s \geq a\} = \inf\left\{s \mid \sigma^2 \int_0^s e^{2\beta(s-\tau)} X_{t_j+\tau}^{2\gamma} d\tau \geq a\right\} \tag{2.14}$$

After generating (h_j), time sequence also should be modified as $t_{j+1} = t_j + h_{j+1}$ [14] with $t_1 = 0$.

Now equation (2.12) can be written by

$$X_{t_{j+1}} = \frac{\alpha}{\beta}\left(e^{\beta h_{j+1}} - 1\right) + e^{\beta h_{j+1}} X_t + M_{h_{j+1}} \tag{2.15}$$

where $M_{h_{j+1}}$ follows $N \sim (0, a)$.

The advantage of Gaussian error of equation (2.15) is that the exact MLE can be applied. Log likelihood function is defined as follows.

$$L(\theta) = \sum_{t=1}^{T} \left[-\frac{1}{2}\ln a - \frac{\left\{X_{t_{j+1}} - e^{\beta h_{j+1}} X_{t_j} - \frac{\alpha}{\beta}\left(e^{\beta h_{j+1}} - 1\right)\right\}^2}{2a} \right] \tag{2.16}$$

where θ is the parameter vector with $\alpha, \beta, \sigma^2, \gamma$.

In the case of practical data with discrete observation, h_{j+1} is obtained by following formula.

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[13] Dambis, Durbins-Schwarz Theorem

[14] By this new time sequence of different interval, the model does not have equi-spaced observation. This is somewhat

$$h_{j+1} = \Delta \min\left\{ s \mid \sum \sigma^2 e^{2\beta(s-i)\Delta} X_{t_j+i\Delta}^{2\gamma} \ge a \right\} \qquad (2.17)$$

where a is recommended to be an estimated σ^2 in Vasicek model[15].

3. SIMULATION

The section deals with the issues of the identification of preferred estimation methods based on simulation.

Sample paths for the Vasicek model and CIRSR model are generated 1000 times. The various data frequencies are applied to ensure the validity of the conclusion. That is, data are generated daily (dt=1/240), monthly (dt=1/12), quarterly (dt=1/4) and semi annually (dt=1/2). These frequencies are somewhat different from other studies dealing with financial asset movement[16]. It is because the underlying asset in ROA is usually a daily stock price or quarterly/semi annual total sales. Table 2 and Table 3 show the parameter settings. The parameters in drift term and diffusion term are determined in consideration of real data.[17]. All the sample paths are estimated under GMM and MLE proposed by Nowman and Yu-Phillips[18]. However, since there is no difference between the methods by Nowman and Yu-Phillips in Vasicek model, the Yu-Phillips' estimations under Vasicek model were deleted.

Table 2 Parameter set in CIRSR model

	α	β	σ	N	dt
Daily	3	-1.2	1.5	400	1/250
Monthly	1.5	-2	0.9	180	1/12
Quarterly	1.8	-0.8	0.2	60	1/4
Semi-annually	2.25	-0.5	1.1	30	1/2

different from Nowman's unit interval period to implement constant volatility.

15 In Yu-Phillips paper, a is recommended to be a variance of residuals in Vasicek model. This inconsistence will be discussed in Section 3.

16 Usually in financial study, daily, weekly, monthly frequencies are examined.

17 The parameter set in daily data is determined according to the construction industry stock index from January 1990 to November 2003. In the case of monthly, quarterly and semi annual data, the parameter set puts its ground on real sales of overall manufacturing industry in Korea from 1990 to 2002.

18 The program coding is done by GAUSS. Every code to estimate parameters via various methods and evaluate time changes in the paper of Yu-Phillips is obtainable through personal contact.

Note: N is the number of data generated.

Table 3 Parameter set in Vasicek model

	α	β	σ	N	dt
Daily	1.35	-0.5	1.7	400	1/250
Monthly	3	-4	2	180	1/12
Quarterly	1.8	-0.8	2	60	1/4
Semi-annually	2.25	-0.5	2	30	1/2

Note: N is the number of data generated

Table 4 Comparison between various estimation methods in CIRSR model for daily data

GMM			
	α	β	σ
Est.	2.8517523†	-1.5069788†	1.4949727
Std.	0.9698243	0.6268197	0.0564524
RMSE	0.9811007	0.6980209	0.0566760
Nowman			
	α	β	σ
Est.	11.2547748	-4.6520937	1.5063038†
Std.	8.3514642	3.1180155	0.0540790
RMSE	11.7454872	4.6530528	0.0544456
Yu-Phillips			
	α	β	σ
Est.	11.1715431	-4.6219349	1.5063038
Std.	8.1707739	3.0931630	0.0540790
RMSE	11.5586549	4.6140023	0.0544456

Note:† the most appropriate estimation

Table 5 Comparison between various estimation methods in CIRSR model for monthly data

GMM			
	α	β	σ
Est.	1.6052185†	-2.1707836†	0.8286076
Std.	0.3990201	0.5764454	0.0488325
RMSE	0.4126731	0.6012367	0.0865250
Nowman			
	α	β	σ
Est.	1.7188984	-2.3272601	0.9237539†
Std.	0.4377897	0.6428898	0.0566266
RMSE	0.4895143	0.7214664	0.0614116
Yu-Phillips			
	α	β	σ
Est.	1.3127430	-1.7958332	0.9237539
Std.	0.5848251	0.8670364	0.0566266
RMSE	0.6141015	0.8907737	0.0614116

Note:† the most appropriate estimation

Table 6 Comparison between various estimation methods in CIRSR model for quarterly data

GMM			
	α	β	σ
Est.	1.7833787†	-0.7929687†	0.1807378
Std.	0.4672312	0.2043860	0.0172188
RMSE	0.4675271	0.2045070	0.0258435
Nowman			
	α	β	σ
Est.	2.6082598	-1.1589613	0.2063206†
Std.	1.2322800	0.5474891	0.0230556
RMSE	1.4739239	0.6547721	0.0239071
Yu-Phillips			
	α	β	σ
Est.	1.9366349	-0.8628876	0.2063206
Std.	1.2141842	0.5483682	0.0230556
RMSE	1.2218555	0.5519660	0.0239071

Note:† the most appropriate estimation

Table 7 Comparison between various estimation methods in CIRSR model for semi-annual data

GMM			
	α	β	σ
Est.	3.1879194†	-0.7168177†	0.8773523
Std.	1.5164470	0.3185735	0.3352157
RMSE	1.7833073	0.3854167	0.4024813
Nowman			
	α	β	σ
Est.	4.1205324	-0.9377604	1.1768001†

Std.	3.0452879	0.7134045	0.2163613
RMSE	3.5743772	0.8371212	0.2296005
Yu-Phillips			
	α	β	σ
Est.	3.2345506	-0.7474071	1.1768001
Std.	3.1619095	0.7682879	0.2163613
RMSE	3.3117944	0.8071789	0.2296005

Note:† the most appropriate estimation

Additionally Aït-Shahalia's Hermite approach was applied to the simulation; however, it shows too many divergences to implement in practical use. In the view of the focus of this paper to show practical application of identifying diffusion process, the method is not likely to be accepted until the diverging property is greatly reduced[19]. Table 4 through Table 7 show the estimation results of the generated data set of CIRSR model.

Table 8 Comparison between various estimation methods in Vasicek model for the daily data

GMM			
	α	β	σ
Est.	1.5719180†	-0.9738015†	1.6930443
Std.	1.6373026	1.0835904	0.0608466
RMSE	1.6522884	1.1827429	0.0612432
Nowman			
	α	β	σ
Est.	2.3282335	-3.2106321	1.6993587†
Std.	3.8605026	2.7712415	0.0594981
RMSE	3.9826347	3.8774554	0.0595015

Note:† the most appropriate estimation

Table 9 Comparison between various estimation methods in Vasicek model for the monthly data

GMM			
	α	β	σ
Est.	2.6730690†	-3.5600843†	2.6730690†
Std.	0.5826352	0.5423444	0.5826352
RMSE	0.6681726	0.6984676	0.6681726
Nowman			
	α	β	σ
Est.	3.1879587	-4.2486753	2.0078193†
Std.	0.7638333	0.7859561	0.1180238
RMSE	0.7866417	0.8243957	0.1182828

Note:† the most appropriate estimation

Table 10 Comparison between various estimation methods in Vasicek model for the quarterly data

GMM			
	α	β	σ
Est.	2.0131564†	-0.9398867†	1.7781186
Std.	0.8774934	0.3448623	0.1700120
RMSE	0.9030370	0.3721799	0.2796153
Nowman			
	α	β	σ
Est.	2.3451429	-1.0990035	2.0265785†
Std.	1.1462396	0.4818096	0.2118679
RMSE	1.2693871	0.5671270	0.2135302

Note:† the most appropriate estimation

Table 11 Comparison between various estimation methods in Vasicek model for the semi-annual data

GMM			
	α	β	σ
Est.	2.5910976†	-0.6088688†	1.5811825
Std.	1.0086171	0.2446731	0.6338895
RMSE	1.0647875	0.2678231	0.7598681
Nowman			
	α	β	σ
Est.	3.2232323	-0.7656212	2.0166085†
Std.	1.6527522	0.4372786	0.3140816
RMSE	1.9182594	0.5117009	0.3145209

Note:† the most appropriate estimation

19 This diverging property is also mentioned in Lee (2001). In his paper, the modification of raw data should be implemented before applying Aït -Sahalia's approach especially in the case of monthly and quarterly data. Considering the frequencies of sales as underlying asset in ROA are likely to over quarter, this estimation method is undesirable to practical application.

The CIRSR process encompasses adjusted level effect by setting γ as 0.5. The original value of parameters used to generate data, estimated parameters, bias, standard deviation and RMSE is provided according to the estimation methods. As pointed above, the non-zero value of γ indicates that the equation (2.17) to obtain h can affect the time change. Therefore, unlike Vasicek model, three estimation methods are used to implement simulation. The results show that the performance of the estimation of drift term by GMM is better than that of MLE. On the other hand, diffusion term is estimated better by MLE. Notice that estimations of the diffusion term in Nowman's and Yu-Phillips' method are identical.

Table 8 through Table 11 give the estimation results of the generated data set of Vasicek model. Apparently, while the RMSE of α and β is very competitive in the result of GMM, that of σ is superior in Nowman's method.

As the results from Table 4 to Table 11 present, performance of estimation on drift term is much better in GMM. On the other hand, diffusion term is estimated more correctly by the MLE suggested by Nowman.

Table 12 Comparison on parameter set

	α	β	σ	N
Yu-Phillips	0.72	-0.12	0.6	500
This paper	0.72	-0.12	0.6	180

Here, we can find some different results from Yu-Phillips' paper. According to Yu-Phillips (2001), his exact Gaussian estimation improves the estimation of drift term. However, it is more sensitive to the data whether it really gives improved estimation or not. We can confirm this fact by the simulation under given parameter by Yu-Phillips and different parameter set in this paper provided in Table12. In Table 13 and Table 14, CIRSR model was used to compare and the best estimation values are provided. As the value shows, changing the number of data brought different outcome[20]. As given parameter data set, the same result as in the paper of Yu-Phillips has drawn (see Table 13) but sometimes, though it is not an overall trend, using different data with other parameter set shows Nowman's estimation is superior to Yu-Phillips' (see Table 14.) There is another point that should be considered in choosing "a" in equation (2.18). According to his paper, to use the residual's variance in the Vasicek model is recommended. However, this recommendation does not work in the simulation. Rather, using the variance of diffusion term in Vasicek model[21] itself makes time changes significant and finally improves estimation of parameters.

[20] The re-examination of the method of Yu-Phillips was also conducted by Kawai & Maekawa(2003). However, their focus on changing volatility with the same sample size is the difference from this paper.

[21] In Vasicek model $dX_t = (\alpha + \beta X_t)dt + \sigma dZ_t$, the variance of residual is $\sigma^2 dt$. The recommended value of a is σ^2 not σ.

As the simulation results, the estimation of drift term should follow GMM and that of diffusion go with Nowman. This conclusion is obtained from sample data based on real daily industry index and real sales. Therefore this conclusion of section 3 still holds in examining empirical study in section 4.

Table 13 Comparison of MLE estimations with the data of Yu-Phillips

Nowman			
	α	β	σ
Est.	1.3913977	-0.2435318	0.6015933
Std.	0.8126718	0.1394106	0.0195246
RMSE	1.0543536	0.1863081	0.0195895
Yu-Phillips			
	α	β	σ
Est.	1.1331829†	-0.2075963†	0.6015933
Std.	0.7078243	0.1439729	0.0195246
RMSE	0.8196989	0.1685497	0.0195895

Note:† the estimation value shows better performance

Table 14 Comparison of MLE estimations with the data suggested in this paper

Nowman			
	α	β	σ
Est.	2.9112050†	-0.4882104†	0.6056261
Std.	2.1811846	0.3260994	0.0338495
RMSE	3.0925316	0.4919913	0.0343143
Yu-Phillips			
	α	β	σ
Est.	2.9249242	-0.4894633	0.6056261
Std.	2.1968662	0.3266038	0.0338495
RMSE	3.1133227	0.4932645	0.0343143

Note:† the estimation value shows better performance

4. EMPIRICAL STUDIES

The main purpose of this section is to identify the significant stochastic models and parameters in practical underlying asset in ROA. In spite of the importance of technology and company valuation within every industry group, most work has been done without great consideration on the movement of underlying asset. However, to give exact valuation and optimal investment timing, the appropriate stochastic model with reliable parameters should describe the movement of underlying asset. In the first sector of this section the best stochastic model and its estimations of parameters of each industry are provided as a form of table and we call it as an industrial parametric table. The table is worth to study because it gives us much information which greatly improves the valuation of industry. The exact value of industry and appropriate information of investment timing is very important to frame, adopt, change and carry out the policy. Since the provided tables show the movement of overall industries in Korea, one can easily draw the valuation of each industry and it helps establishing national policy. To be more competitive country, Korea has to develop possible area of industry on time based on selection and concentration and the table gives clue to make a selection. The industrial parametric table has its meaning in not only the national level but also private level. Even though the table is made of overall data of each industry, a private company within an industry group can refer to the result of table and it will be a good benchmark to devise a plan for a company. It is because the performance of individual company is closely related to the performance of the industry group it belongs. However to get more reliable and realistic information of certain companies, some company groups specialized in certain interest area are studied in the second part of empirical study for examples.

4.1. Empirical study 1

4.1.1. The Data

The data used in this study are the semi-annual sales and the daily industry stock index of each industry group. The semi-annual sales obtained from Company Information TS2000[22] are semi-annually covering the period from 1990 to 2002[23] with 38 industry divisions. Daily industry indices used in this paper are obtained from Korea Stock Exchange and covering the period from January 1999 to November 2003[24]. The classification of industry group provided by Korea Listed Company Association (KLCA) and KSE[25] are shown in Table 15.

In the case of semi-annual sales, since the value of raw data itself is very large, normalization is applied to make the data unit concisely[26]. It can probably bias the estimated result; however obtaining the estimation of raw data is quite simple. If the raw data X_t is normalized by factor p, then the estimation is applied to $Y_t = pX_t$. Then the estimated parameter set of X_t is obtained by converting that of Y_t as[27]

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[22] Company Information TS 2000 is run by Korea Listed Company Association (KLCA) which provides financial and business statements of each listed companies in Kospi and Kosdaq.

[23] The number of data is different according to the division but to ensure the validity of the estimation, some industry groups which have less than 20 observations were deleted and it reduces the number of examined industry group to 33. All industry groups examined by semi-annual data have 26 observations. There exists a lot more annual information of each company; however, the semi annual information is limited from 1990.

[24] To remove the huge impact of financial crisis in the late 1990, the data is collected from 1999.

[25] Korean version of Table 15 is provided in Appendix 1.

[26] Normalizing is preferred because it gives more memories to computer which enables efficient estimation.

[27] Jeong (2003)

$$a' = a$$
$$\beta' = \beta$$
$$\sigma' = \alpha^{-\gamma+1}\sigma$$
$$\gamma' = \gamma$$

where parameters with prime is of X_t.

The drift term and elasticity of variance doesn't change though normalizing is applied[28].

Table 15 Industry Groups

KLCA	KSE
Manufacture of Coke, Refined Petroleum Products and Nuclear Fuel (Coke)	
Manufacture of Chemicals and Chemical Products(chemicals)	Chemicals
Manufacture of Rubber and Plastic Products(rubber)	
Post and Telecommunications (com)	Communication (Com)
General Construction (construct)	Construction (Construct)
Sale of Motor Vehicles and Motorcycles; Retail Sale of Automotive Fuel(auto_fuel)	
Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles(wholesale)	Distribution
Retail Trade, Except Motor Vehicles and Motorcycles (retail)	
Manufacture of Computers and Office Machinery(comput)	
Manufacture of Electrical Machinery and Apparatuseses n.e.c.(elec_m)	Electrical & Electronic Equipment (Electric)
Manufacture of Electronic Components, Radio, Television and Communication Equipment and Apparatuses (component)	
Electricity, Gas, Steam and Hot Water Supply(elec_gas)	Electricity & Gas (Elec_gas)
	Finance*
Manufacture of Food Products and Beverages(food)	Food & Beverages (Food)
Manufacture of Basic Metals(basic_m)	Iron & Metals
Manufacture of Fabricated Metal Products, Except Machinery and Furniture (fabric_m)	Machinery
Manufacture of Other Machinery and Equipment(other_m)	

KLCA	KSE
Manufacture of Medical, Precision and Optical Instruments, Watches and Clocks(medical)	Medical & Precision Machines (Med_p)
	Medical Supplies (Med_s)
Manufacture of Other Non-metallic Mineral Products (non-metal)	Non-metallic Minerals (Non_metal)
Manufacture of Pulp, Paper and Paper Products (pulp)	Paper & Wood
Hotels and Restaurants (hotel)	Services
Motion Picture, Broadcasting and Performing Arts Industries (m_pic)	
Manufacture of Textiles, Except Sewn Wearing apparel (textile)	Textile & Apparel (Textile)
Manufacture of Sewn Wearing Apparel and Fur Articles (apparel)	
Land Transport ; Transport Via Pipelines (land_t)	Transport & Storage(Trans_s)
Air Transport (air_t)	
Manufacture of Motor Vehicles, Trailers and Semitrailers (m_trail)	Transport Equipment(Trans_e)
Manufacture of Other Transport Equipment (other_t)	
Tanning and Dressing of Leather, Manufacture of Luggage and Footwear (leather)	
Publishing, Printing and Reproduction of Recorded Media (publish)	
Manufacture of Furniture; Manufacturing of Articles n.e.c. (furniture)	
Mining of Metal Ores (metal)	
Mining of Coal, Crude Petroleum and Natural Gas, Uranium and Thorium Ores(coal)	
Fishing	

Note: The words in parentheses at the end of each industry group are the abbreviation that will be used throughout the rest of the paper.

* Finance group by KSE includes "Bank" and "Insurance."

4.1.2. Industrial Parametric Table

To have a reliable parametric table, now the problem is narrowed to the issue of choosing best model among the alternative stochastic processes. As indicated in Table 1, there are 8 stochastic models well known

[28] In addition, though the values of likelihood functions changes as normalizing, the order of that values between alternative models do not change. Hence the order of goodness- to-fit also remains same regardless of normalizing.

and each model has been used to describe the movement of underlying asset numerically or analytically. To choose the best model, CKLS suggested very powerful way that enables us to compare the explanatory power of models under Chi-square statistics and Nowman also gave the way to choose the competitive model according to the value of log likelihood function.

The matter of choosing the best model in CKLS and Nowman are different problem from this paper since their comparison is done within the only one estimation method; either GMM or MLE. However, in this paper, one should select best model within both GMM and MLE. Therefore other approach should be implemented. According to the ROA, the movement of underlying asset is very important and especially the value of diffusion term has much more significant than drift term. We can confirm this by the Girsanov Theorem which provides the general transform framework from one probability measure to another. According to the theorem, the stochastic process with drift and diffusion term can be transformed to another stochastic process with diffusion term only by changing of probability measure[29]. Moreover, the well-known option pricing method, Black-Scholes formulas, also only employs volatility in stochastic model. Therefore the volatility in diffusion term has much more useful information to have option value than drift term and in this respect, the volatility can be a reasonable criterion to select the best model. The way to compare between models within MLE is likelihood ratio test which select the model of the nearest maximum of likelihood to that of unrestricted model.

However, one should not have the confidence on drift term because Nowman's method cannot ensure good performance of the estimation of drift term[30]. Therefore it is recommended that the drift term be improved by GMM. However, one should keep in mind that the data should follow the stationary process in order to apply GMM[31]. If this property is not assured, the result of parameter estimation of Nowman can be adopted. Four industry groups show non-stationary in semi-annul data[32] and all industry groups are stationary in daily industry indices and this result is assured by unit root test. Also even though the maximum of likelihood of certain model is closest to that of unrestricted model, t-statistics of γ in unrestricted model should be examined before fixing the model as the best. The selected stochastic process model and the

Table 16 Parametric Table: Selected model and estimated parameters

Industry (selected model)	α	β	σ	γ	t-value†
coke (CEV)	0	0.042291	0.079412 (0.742088)	1.06043 (5.48292)	7.816

[29] For example, by applying Girsanov theorem, stochastic process $dX_t = \mu dt + \sigma dZ_t$ is transformed to $dX_t = dZ_t$ with the change of probability measure.

[30] It is also examined in the short term interest rates studying done by Nowman (1997).

[31] Stationary and ergodicity are the assumption in GMM.

[32] Those are " Retail Trade, Except Motor Vehicles and Motorcycles" , "Motion Picture, Broadcasting and Performing Arts Industries" , " Manufacture of Sewn Wearing Apparel and Fur Articles" and " Manufacture of Motor Vehicles, Trailers and Semitrailers."

chemicals (CEV)	0	0.046781	2.305E-11 (0.537282)	2.46227 (4.37967)	11.88
rubber** (CIRSR)	0.719982 (0.019658)	0.081837 (3.90153)	151.089 (1508.72)	1/2	1.578
com (CEV)	0	0.036249	0.166993 (1.81452)	1.14478 (10.7133)	16.92
construct (Vasicek)	1.34982	0.024650	3570700 (13.4970)	0	0.167
auto_fuel (CEV)	0	-0.114032	0.294010 (1.00143)	1.02324 (4.87759)	7.979
wholesale (CEV)	0	0.032962	222.700 (0.610735)	0.591796 (1204.90)	3.671
retail* (Vasicek)	0.720110 (0.021978)	0.172534 (4.81268)	497952 (4958093)	0	1.224
comput** (CIRSR)	0.719925 (0.031072)	0.109002 (0.981028)	558.930 (5378.16)	1/2	1.941
elec_m (Vasicek)	1.35020	0.040871	457570 (13.5009)	0	1.901
component (CEV)	0	-0.124879	0.000047 (0.749702)	1.655488 (4.63073)	8.554
elec_gas (CEV)	0	0.123048	0.000005 (0.481806)	1.66845 (4.01419)	7.011
food (CEV)	0	0.069032	0.000029 (0.427220)	1.565516 (3.45782)	5.769
basic_m (CEV)	0	0.062392	0.043410 (0.446606)	1.05452 (3.55325)	4.339
fabric_m (CEV)	0	0.061246	0.037647 (0.585188)	1.086809 (4.85638)	8.467
other_m (Vasicek)	1.35107	0.026986	991771 (13.5096)	0	1.233
medical (CEV)	0	0.105678	2499.59 (1.52180)	0.365834 (23805.3)	3.510
non-metal (CEV)	0	0.021363	0.013171 (0.412717)	1.176324 (3.03923)	5.199
pulp (CEV)	0	0.042690	0.055668 (0.633570)	1.092667 (4.35119)	5.901
hotel** (CIRSR)	0.719992 (0.019655)	0.099177 (4.09199)	43.8343 (419.701)	1/2	0.719
m_pic* (Vasicek)	0.719773 (0.018221)	0.044745 (0.29691)	1628.63 (15236.1)	0	0.564
textile (Vasicek)	1.35032	-0.147505	322515 (13.5021)	0	1.204
apparel* (Vasicek)	0.719996 (0.019658)	-0.094464 (0.83977)	483993 (466417)	0	0.581
land_t** (CIRSR)	0.719927 (0.017024)	0.1239858 (2.38598)	507.223 (5028.03)	1/2	1.318
air_t (CEV)	0	0.095268	0.946142 (0.487463)	0.865591 (0.198097)	3.684
m_trail* (CEV)	0	0.162160 (4.73565)	6.22494 (1.84540)	0.779637 (5.93161)	3.794
other_t (CEV)	0	0.052166	359.745 (2.37413)	0.586972 (6187.73)	10.52
leather (CIRSR)	0.720210 (0.017005)	0.013217 (0.11809)	349.764 (3367.80)	1/2	2.301
publish (Vasicek)	1.34963	0.049186	89450.8 (13.4954)	0	1.578
furniture (Vasicek)	1.349647	0.035153	209878 (13.4955)	0	0.655
metal (Vasicek)	15409.6	-0.061251	28669.3 (154086)	0	0.549
coal (CEV)	0	-0.2410966	0.012389 (0.578325)	1.40348 (2.65049)	4.022
fishing (Vasicek)	1.35064	-0.551819	267231 (13.5054)	0	0.383

Note: Stochastic process models of 33 industry groups are examined through semi-annual sales with t-statistics in parenthesis. The name of industry follows the abbreviation listed in Table 15.

* Industry with non-stationary data at 1% confidence level.

** Industry diverging in GMM.

†The t-value of γ under null hypothesis of γ is significantly different from zero.

Table 17 Parametric Table: Selected model and estimated parameters

Industry (selected model)	α	β	σ	γ	t-value†
Banks (CEV)	0	-0.1205679	1.569937 (8.830914)	0.7678541 (11.94408)	16.13
Chemicals (CEV)	0	0.0602224	0.0814195 (4.849417)	1.226008 (14.91133)	19.95
Com (CEV)	0	-0.7254677	0.0041659 (7.812286)	1.766091 (17.19681)	30.62
Construct (BS)	50.63730	-0.8969336	0.5128578 (3515.578)	1	32.14
Distribution (CEV)	0	-0.1189173	0.0967686 (9.277076)	1.277674 (21.41783)	30.34
Electric (CEV)	0	0.0479837	1.677866 (4.808180)	0.8488693 (13.43541)	16.90
Elec_gas (CEV)	0	-0.1982684	0.0001704 (3.247811)	2.170101 (9.604683)	21.01
Finance (CEV)	0	0.0059218	1.185152 (8.299873)	0.8416846 (4.302014)	19.52
Food (CEV)	0	0.0059218	0.0317505 (4.866243)	1.344255 (16.2710)	22.64
Insurance (CEV)	0	-0.0437645	2.961068 (6.306831)	0.7849357 (49.69226)	19.93
Iron & Metals (CEV)	0	0.0215607	0.2308781 (4.321348)	1.0801900 (12.01210)	16.94
Machinery (CEV)	0	-0.1175668	0.2410925 (8.633937)	1.0997639 (17.61109)	25.56
Med_p (CEV)	0	-0.9958294	0.3092321 (14.00646)	1.108420 (18.46893)	29.99
Med_s (CEV)	0	-0.0507466	1.115997 (2.815246)	0.8555001 (1.103201)	9.011
Non-metal (CIRSR)	580.6044	-0.8701011	9.039904 (40308.66)	1/2	9.186
Paper & Wood (CEV)	0	-0.2912996	0.0613409 (11.26075)	1.332640 (25.77792)	36.54
Services (CEV)	0	-0.6815883	0.0254869 (10.58554)	1.460508 (23.06285)	34.84
Textile (CEV)	0	-0.2148170	0.1027911 (13.61301)	1.230432 (30.43561)	41.74
Trans_s (CEV)	0	-0.0612008	0.4779985 (9.986444)	1.0157625 (15.89643)	30.96
Trans_e (CEV)	0	0.0818249	1.129860 (6.900221)	0.8396044 (2.698857)	17.36

Note: Stochastic process models of 20 industry groups are examined through daily industry indices with t-statistics in parenthesis. The name of industry follows the abbreviation listed in Table 15.

†The t-value of γ under null hypothesis of γ is significantly different from zero.

updated estimation on parameters in the case of semi-annual sales data and the daily industry stock indices are shown as the parametric table in Table 16 and Table 17. The result is obtained after the t-test at 1% confidence level on γ is considered to select the best model. After updating the drift term by GMM, the diffusion term is estimated again under MLE with the fixed drift term obtained by GMM[33]. Interestingly

[33] This procedure is desirable to get the more exact estimation of diffusion term. Actually after assigning updated drift term parameters in Maximum likelihood Estimation alter the drift term and its t-statistics.

most selected models are limited to CEV, Vasicek and CIRSR. Only one group of construction using daily index shows BS process. That is, if the data has level effect, then the best model requires unrestricted γ rather than specified given value. The reason of frequent selection of CEV comes from the restriction property of the model; only one restriction on the drift term, α. Similarly, among the models Merton and Vasicek which have no level effect, Vasicek is freer from parameter restriction and that is why the model is frequently chosen.

Specifically, for detailed study on the value of γ, the results can be divided to four categories according to the value of γ.

Table 18 The division of Industry group in semi-annual sales data

Extent of γ	Industry Group
$\gamma < 0.5$	medical
$0.5 \le \gamma < 1.0$	rubber, wholesale, air_t, m_trail, other_t, comput, hotel, land_t, leather
$1.0 \le \gamma < 1.5$	coke, com, auto_fuel, basic_m, fabric_m, non-metal, pulp, coal
$1.5 \le \gamma$	Chemicals, component, elec_gas, food,

Table 19 The division of Industry group in daily Industry index

Extent of γ	Industry Group
$\gamma < 0.5$	
$0.5 \le \gamma < 1.0$	banks, electric, finance, insurance, med_s, trans_e, non-metal
$1.0 \le \gamma < 1.5$	chemicals, distribution, food, iron & metals, machinery, med_p, paper & wood, services, textile, trans_s, construct
$1.5 \le \gamma$	com, elec_gas

As seen in Table 18 for semi-annual sales data, cshows very low dependency to the level of semi-annual sales itself while “Manufacture of Chemicals and Chemical Products (chemicals)”, “Electricity, Gas, Steam and Hot Water Supply (elec_gas)”, “Manufacture of Electronic Components, Radio, Television and Communication Equipment and Apparatuses (component)” and “Manufacture of Food Products and Beverages(food)”' gives the high dependency to the level of semi-annual sales. Especially “Manufacture of Chemicals and Chemical Products (chemicals)” has very sensitive level effect and we can interpret it as the industry should keep its high sales as long as possible and the small reduction of its sales would affect whole sales negatively. On the other hand, “Manufacture of Medical, Precision and Optical Instruments, Watches and Clocks (medical)” group has low effect by the reduction of its sales. Also we cannot expect high return from the industry with its short-term increase because of the low level effect by the factor of 0.366.

The industry groups by KSE examined by daily industry index are more likely to follow CEV process than groups by KLCA examined by semi-annual sales. For the detailed discussion of level effect, the division according to the value of γ is provided in Table 19. Unlike the semi-annul sales data every industry depends on its level of index by more than factor of 0.5. The most sensitive industry group to its daily index is "Electricity & Gas (elec_gas)". It is likely to expect that the companies within this group are moving along with other companies since the index represents the overall tendency of the industry. Apparently the financial group of "Bank", "Insurance" and "Finance" depends on its index by less than the factor of 1. It means that the effect of overall finance companies is comparatively less dominant.

In the case of the absence of level effect, the process usually follows Vasicek model. It is noticeable that any industry group does not follow GBM which is commonly assumed process in many application studies. It gives us the lesson that emphasizes the importance of the exact identification on underlying asset.

4.2 Empirical study 2

In this section, rather than broad industry group provided by Korea Stock Exchange, alternative classification is adopted. We examine two industry categories. One is specialized in the Internet service and the other is focused on the mobile telecommunication service. The number of companies studied in the Internet service category is three[34]. In the case of mobile telecommunication service category, the existing three telecom companies are analyzed. To show the market movement of each category the daily stock index is calculated independently[35]. The base date is October 29, 2002 of level 100 for the Internet service group and September 21, 2000 for the mobile telecommunication service group. The studied period is from the base date to November 28, 2003. The data descriptions of daily stock index are shown in Table 20 and Table 21. It assures the stationary property in index changes and satisfies the stationary assumption of GMM.

Table 20 Data Description of Internet service group

Variable	N	Mean	Std D	$>_1$	$>_2$	$>_3$	$>_4$	$>_5$	$>_6$	ADF
Y_t	269	213.8	87.38	0.992	0.135	-0.272	0.103	-0.093	0.132	0.486
$Y_t - Y_{t-1}$	268	-0.5802	9.203	-0.004	0.132	-0.140	-0.037	-0.129	0.003	-10.04

Note: Number of data (N), means, standard deviation (Std D), six autocorrelations (>) and ADF statistics of daily calculated stock index and index changes of Internet service categories are provided.

Table 21 Data Description of mobile telecommunication service group

Variable	N	Mean	Std D	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ADF
Y_t	783	97.10	16.85	1.027	-0.077	-0.013	0.039	0.029	-0.018	-0.597
$Y_t - Y_{t-1}$	782	0.033	2.615	0.034	-0.043	-0.055	-0.015	-0.013	-0.013	-20.25

Note: Number of data (N), means, standard deviation (Std D), six autocorrelations (>) and ADF statistics of daily calculated stock index and index changes of mobile telecommunication categories are provided.

[34] Though there is a list of other companies in the Internet service category, some companies were deleted because of poor performance and too recent register.

Table 22 shows the result of estimated parameters by MLE in each model in the Internet group. The best-fitted model is colored dark according to the log likelihood test. Table 24 gives the revised estimated parameters of the best stochastic model. The maximum of likelihood of CEV model is the nearest value to that of unrestricted model and hence, CEV model is selected. It provides the evidence that the mean reverting property is not observed and the level effect is valid in the Internet service group through gamma within 10% up to one. The reason of positive parameter in drift term comes from prosper aspect of the industry. The Internet group has developed various contents such as on-line game, avatar, Internet advertisement or searching engines. Those attractive contents with loyal members[36] are relatively less affected by economic stability and low marginal cost of such industry mainly contributes to the on-going development. Also though the movement of underlying asset in the Internet group looks like unstable by disturbing diffusion term, the existence of drift term proves stable growth.

Table 22 Result of MLE estimation

	α	β	σ	γ	Likelihood
Unrestricted	1.003958 (0.068694)	1.122873 (2.577812)	0.3914226 (1.026472)	1.088274 (15.09018)	0.006362
Merton	145.0412 (1.464002)	0	145.2422 (4755.291)	0	0.004449
Vasicek	0.7232676 (0.0674993)	0.2821106 (0.6466825)	145.3880 (4758.198)	0	0.0044348
CIRSR	0.7301172 (0.007603)	0.6745990 (1.085309)	9.213907 (301.4277)	1/2	0.0056687
Dothan	0	0	0.6291708 (31.27155)	1	0.0062713
GBM	0	1.067293 (2.509181)	0.6241908 (20.42900)	1	0.0063444
BS	0.7285117 (0.027736)	1.063169 (2.350987)	0.6241875 (20.42863)	1	0.0063445
CIRVR	0	0	0.0466731 (1236.412)	3/2	0.0058666
CEV	0	1.128718 (2.645386)	0.3913869 (0.7641351)	1.088292 (12.64188)	0.0063619

Note: Parameters of t he daily stock index of the Internet service category in each model are estimated by MLE by Nowman with t-statistics in parenthesis.

Table 23 Result of MLE estimation

	α	β	σ	γ	Likelihood
Unrestricted	216.1715 (2.512869)	-2.306334 (2.474541)	0.3500024 (0.7581306)	1.039767 (10.27934)	0.0578650
Merton	-8.310282 (0.5029834)	0	41.32216 (2311.037)	0	0.0540629

[35] The calculation method is as same as the usual stock index calculation that uses the total market capitalization as weights.
[36] It is considerer that the Internet group has a large number of loyal members.

Vasicek	257.181 (2.636834)	-2.733868 (2.762181)	41.44638 (2304.001)	0	0.0543289
CIRSR	231.5798 (2.539043)	-2.470211 (2.588656)	4.140826 (230.2873)	1/2	0.0568562
Dothan	0	0	0.4185391 (567.6128)	1	0.0576253
GBM	0	-0.0089729 (0.0443064)	0.4185430 (23.40278)	1	0.0576254
BS	216.9715 (2.510330)	-2.314786 (2.478402)	0.4196233 (23.34273)	1	0.0578593
CIRVR	0	0	0.0430955 (3618.935)	3/2	0.0568399
CEV	0	-0.0029338 (0.0332341)	0.3504010 (0.5464166)	1.038954 (9.571107)	0.0576309

Note: Parameters of t he daily stock index of mobile telecommunication service category in each model are estimated by MLE by Nowman with t-statistics in parenthesis

Table 24 Parametric Table: Final revised result

model	CEV $dX_t = \beta X_t dt + \sigma X_t^{\gamma} dZ_t$			
parameters	α	β	σ	γ
Est.	0	0.2851472	0.4303914 (4.503065)	1.071270 (7.906888)

Table 25 Parametric Table: Final revised result

model	BS $dX_t = (\alpha + \beta X_t)dt + \sigma X_t dZ_t$			
parameters	α	β	σ	γ
Est.	0.9825013	-0.1720961	0.4187864 (54.94999)	1

Table 23 shows the result of estimated parameters by MLE in each model in the mobile telecommunication group. Table 25 gives the revised estimated parameters of the best stochastic model. Based on the comparison between the maximum of likelihood in each model, BS model is chosen. This model shows the evidence of the level effect whose elasticity of variance is 2. Also the index mean reverts toward approximately 5.71 with relatively low reverting speed of 0.172 in this group. Unlike the Internet group, mobile telecommunication service group has mean reverting property though both experience similar level effect close to 1. This mean reverting results does not follow the common expectation of remarkable growth in the industry. The reason why the best model describes the movement as mean reverting comes from two aspects. First, the industry is already saturated and faces long run average growth. Second, the industry meets the price regulations on telecommunication business and price reduction for consumers. Due to the number portability policy launching 2004, the movement of underlying asset in each company would be changed, however, overall movement is expected to move according to estimated results with its mean reverting and level effect.

5. CONCLUSION

In this paper, the parametric tables of industry groups are provided. These tables are made by the most appropriate and practical estimation method. Based on theoretical understanding and the simulation results of this paper, it can be concluded that the Gaussian estimation using MLE is very powerful to estimate σ in diffusion term while the performance of GMM on drift term is superior to that of MLE. The procedure of choosing most preferred model is as follow. First, estimate parameters of each alternative model by the Gaussian Estimation proposed by Nowman and select the model which has the most explanatory power according to the maximum of likelihood and the t-test of γ in unrestricted model. Second, when the data reject the null hypothesis of unit root, GMM is applied to update drift term within the chosen model. Third, setting the updated drift parameters as constant in Nowman's procedure, obtain the diffusion term once again. The final estimation of parameters is now ready for being put in parametric table.

Many broad industry groups provided by KLCA and KSE have tendency to follow CEV or Vasiek model. In the case of specialized group, the Internet group follows CEV while the mobile telecommunication group shows the BS movement.

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Appendix 1 산업분류표

상장기업분류기준	KSE 산업별지수기준
코크스,석유정제품 및 핵연료제조업	화학
화합물 및 화학제품제조업	
고무 및 플라스틱제조업	
통신업	통신업
종합건설업	건설업
자동차판매 및 차량연료소매업	유통업
도매 및 상품중개업	
소매업(자동차제외)	
컴퓨터 및 사무용기기제조업	전기, 전자
기타전기기계및전기변화장치제조업	
전자부품영상음향및통신장비제조업	
전기,가스 및 증기업	전기가스업
	금융업-은행보험포함
음식료품 제조업	음식료품
제 1 차 금속산업	철강,금속
조립금속제품제조업 (기계 및 기구제외)	기계

상장기업분류기준	KSE 산업별지수기준
기타기계 및 장비제조업	기계
	의약품
비금속광물제품제조업	비금속광물
펄프, 종이 및 종이제품제조업	종이, 목재
숙박 및 음식점업	서비스업
영화, 방송 및 공연산업	
섬유제품제조업	섬유,의복
봉제의복 및 모피제품제조업	
육상운수 및 파이프라인운송업	운수창고
항공운송업	
자동차 및 트레일러	운수장비
기타운송장비제조업	
가죽, 가방 및 신발제조업	기타 제조업
출판, 인쇄 및 기록 매체 복제업	
가구 및 기타제품제조업	
금속광업	
어업	