Self Attenuation Correction Factor for Compton Scattering Imaging System

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1. Introduction

After the September 11th terror on the World Trade Center in New York city, USA, security concerns on all aspects have been multiplied and researches on new inspection modalities, which can compliment the current conventional inspection devises, have been increased. Recently, devices based on X-ray Compton scattering are studied for many practical uses such as NDT, agricultural engineering and scanning systems [1,2,3]. In the light of potential material identification without costly rotating scanning systems, 3-D Compton scattering imaging systems with a photon source, a transmission detector and side detectors are under development [2,3,4]. One of the major assumptions used in the system is that all the scattering events occur in the center of a voxel and incoming and outgoing photons are attenuated by the half-thickness of the voxel in the direction of the photon movement. This half-thickness attenuation assumption is investigated with MCNP simulations and self attenuation correction factor is newly introduced.

2. Materials and Methods

The general layout of the 90° Compton scattering system as modeled in MCNP simulation calculations is shown in Fig.1.

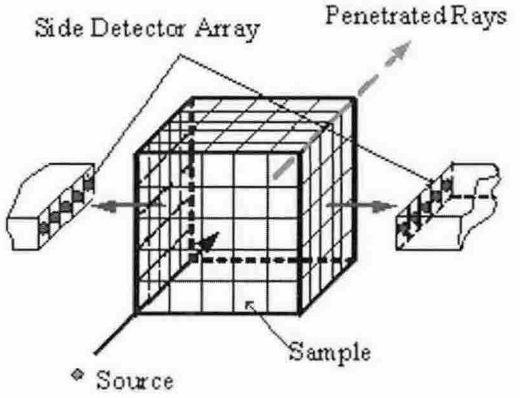

Fig.1 The geometry of 90° Compton scattering imaging system

It is assumed that the sample is made of a number of cubic voxels and all photon scatterings occur at the center of voxels. From the responses of a transmission detector and side detectors three voxel parameters (• and μ_1, total linear attenuation coefficients at incident and scattering photon energy, and μ_C Compton scattering coefficient at incident photon energy) are reconstructed.

In more realistic case shown in Fig. 2 with a narrow pencil source beam and a collimated perfect point detector, the actual scattering can occur anywhere in the interacting volume. Usually, the voxel size is equal 2a and the radius b of the parallel source beam is not larger than a half of the voxel size. The distance between the point detector and the voxel center is h.

With the assumption that Compton scattering occurs at the center of the voxel with a volume V, the volume-attenuation factors P_{in} and P_{out} corresponding to incoming and outgoing pathways of photons can be written as follows by half-thickness attenuation.

$$P_{in} = V\exp(-\mu a) = 2\pi ab^2 \exp(-\mu a) \quad (1)$$

$$P_{out} = V\exp(-\mu_1 b) = 2\pi ab^2 \exp(-\mu_1 b) \quad (2)$$

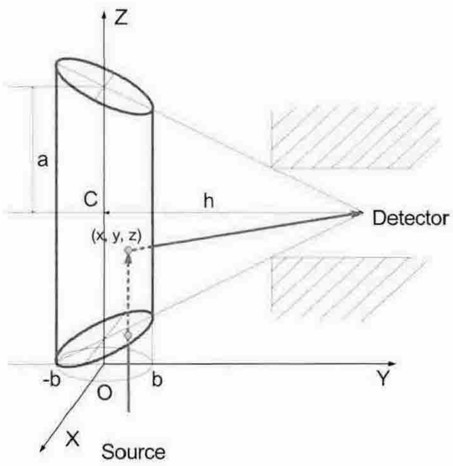

Fig.2 The geometry of the interaction volume in a voxel

In fact Compton scattering can occur anywhere in the interaction volume, the volume-attenuation factors are different from the Eqs. (1) and (2), and should be averaged for the whole interaction volume. Therefore, a self attenuation correction factors can be defined as follows

$$\alpha = \alpha_{in}\alpha_{out} = \frac{P'_{in}\,P'_{out}}{P_{in}\,P_{out}} \quad (3)$$

where P'_{in} and P'_{out} are the volume-attenuation factors calculated for the whole voxel and given by

$$P'_{in} = \int_{x_1}^{x_2} dx \int_{y_1}^{y_2} dy \int_{z_1}^{z_2} dz \exp(-\mu r) \quad (4)$$

$$P'_{out} = \int_{x_1}^{x_2} dx \int_{y_1}^{y_2} dy \int_{z_1}^{z_2} dz \exp(-\mu_1 r) \quad (5)$$

where x_1, x_2, y_1, y_2, z_1 and z_2 are the limits of the interaction volume corresponding to the coordinate axes, r is the position of point (x,y,z) where Compton scatter occurs.

For simplification, let assume that the cross section between the parallel beam and the surface of the detector-see-voxel cone is a plane. Then P'_{in} can be written as

$$P'_{in} = \int_{-b}^{b} dy \int_{0}^{\sqrt{b^2-y^2}} dx \int_{\frac{a}{h}y+\frac{ab}{h}}^{-\frac{a}{h}y+2a+\frac{ab}{h}} dz \exp(-\mu z)$$

$$= \frac{\exp\left(-\frac{ab\mu}{h}\right)}{\mu} \int_{-b}^{b} dy \sqrt{b^2-y^2} \left\{ e^{-\frac{a\mu}{h}y} - e^{\frac{a\mu}{h}y} e^{-2a\mu} \right\} \quad (6)$$

Applying the Taylor series into the integral, one obtains:

$$\alpha_{in} = \frac{1}{2a\mu}\left(1+\frac{a^2b^2\mu^2}{8h^2}\right)\left(e^{a\mu}-e^{-a\mu}\right)\exp\left(-\frac{\mu ab}{h}\right) \quad (7)$$

By the same way for P'_{out} with the note that

$$\exp(-\mu_1 r) \approx 1 - \mu_1 \frac{(a-y)}{h}\sqrt{x^2+h^2+(a-z)^2} + \mu_1^2 \frac{(a-y)^2}{2h^2}\left[x^2+h^2+(a-z)^2\right] \quad (8)$$

the formula of P'_{out} can be derived.

$$P'_{out} = 2\pi ab^2 + \frac{\pi ab^2}{h}\mu_1\left\{h^2 \ln\frac{h}{a+\sqrt{a^2+h^2}} - a\sqrt{a^2+h^2}\right\} + \frac{\pi ab^2}{12h^2}\mu_1^2\left(a^2+3h^2\right)\left(4a^2+b^2\right) \quad (9)$$

For the limiting cases of a<<h, that is usually correct in practical systems, the factor can be simplified as follows.

$$\alpha_{out} = e^{\mu_1 a}\left\{1+\frac{\mu_1}{2}\left(h\ln\frac{h}{a+h}-a\right)+\frac{\mu_1^2}{8}\left(4a^2+b^2\right)\right\} \quad (10)$$

Here, the self attenuation correction factors are introduced analytically for a simple parallel source. The same process can be applied to more sophisticated ones like cone beam source cases either analytically or numerically. Inside the reconstruction software, the factors are incorporated so that three voxel parameters and the factors converge to a solution iteratively.

3. Results and Discussion

Using MCNP simulated detector responses and the algorithm described in [2,3], voxel parameters were reconstructed and compared with true values. The energy of incident photons in parallel beam with the radius of 0.1cm was 0.122MeV. The MCNP simulation calculations showed that the self attenuation effect is so small that it can be neglected in calculation of the total and Compton attenuation coefficients for the cases that multiplicity of the total linear attenuation coefficient □ and the size of voxel is small.

For cases with voxel size a=0.5cm, the self attenuation correction factors were very close to unity for materials which have the atomic number Z less than 20, such as aluminum and polyethylene. For high Z materials, the self attenuation effects were large and required corrections. The use of self attenuation correction factors reduced the relative errors of attenuation coefficients significantly. For example, the error of μ_C for steel sample (Z_{eff} = 25.706) was reduced from 8.35% to 4.22%, zinc sample (Z = 30) – from 27.77% to 5.24%. For brass (Z_{eff} = 29.715) the error of μ_C was reduced from 50.68% to 17.17% and the error of μ – from 9.65% to 3.24%.

Because of the reconstruction algorithm in which μ_1 is calculated using the ratio of the responses of any two side detectors, the self attenuation effect always cancels out and the use of self attenuation correction factors does not change the accuracy of μ_1 calculations even when α_{out} are considered.

4. Conclusion

The half-thickness attenuation assumption is applicable when the total linear attenuation coefficient multiplied by the voxel size is small, such cases that a high-energy source is employed or the voxel is made of low-Z low-density material.

For 90° Compton scattering systems which employ a low-energy source, voxels of large size or highly attenuating samples, self attenuation effects in interacting volume should be considered to get better accuracy. The self attenuation correction factors defined in Eq. (3) can be applied not only for 90° Compton scattering imaging systems but also for any system using Compton scattering effects in a given control volume.

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