Gyroless Yaw Angle Compassing of Earth-Pointing Spacecraft Using Magnetic Sensor

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Abstract: This paper formulates a yaw angle determination algorithm for earth-point satellite. The algorithm based on vector observation, is implemented with the limited vector measurements. The proposed algorithm doesn't require gyro measurement data but magnetic sensor measurement data. In order to confirm the usefulness of the proposed method, we investigate the simulated telemetry data of the KOMPSAT-2, a satellite that is scheduled to be launched into a 685km altitude sun synchronous circular orbit in 2005.

Keywords: Yaw Angle Compassing, Earth-Pointing Spacecraft, Magnetic Sensor, GPS, IGRF

1. Introduction

In spacecraft, a magnetic sensor is one of the key sensors, whose primary usage is to monitor the near-earth space environmental magnetic field. The geomagnetic field naturally interacts with disturbances from the sun, i.e., a solar storm which travels through space and reaches the earth. It extends further into space beyond the atmosphere and constructs the magnetosphere, which acts as a shield from the normal solar wind. The strong solar storm compresses the magnetosphere to make communications satellites directly exposed to the solar wind. Although LEO(Low-Earth-Orbit) satellites is less vulnerable to the solar storm than GEO/MEO (Geosynchronous/Medium-Earth-Orbit) satellites, the solar storm may affect the estimation performance. Nevertheless, the geomagnetic field is considered to be stationary or almost time-invariant for LEO satellites in many cases. The knowledge of the local space environmental magnetic field is applied to the spacecraft attitude determination and control, and the momentum dumping of momentum exchanging devices such as reaction wheels and control moment gyros. There are well-known effective attitude determination methods from two or more vector observations such as TRIAD and QUEST algorithms [1-9]. The TRIAD algorithm provides a deterministic but not-optimal solution for the spacecraft attitude based on two vector observations. Since the algorithm is very simple, it has become the most popular method for determining three-axis attitude for spacecraft that provide complete vector information. The QUEST algorithm which is an optimal algorithm, determines the spacecraft attitude by achieving the best weighted overlap of an arbitrary number of reference and observation vectors. The Kalman filter is the well known optimal algorithm to achieve more precision determination. However, due to merit of the simplicity and low computation burden, the vector observation based algorithm has been in existence and has been in many recent spacecraft missions. In this paper, we propose a yaw angle determination algorithm for earth-point satellite. The algorithm based on vector observation, is implemented with the limited vector measurements. The well known yaw angle determination algorithm is the yaw gyro compassing using gyro measurement. The proposed algorithm doesn't require gyro measurement data but magnetic sensor measurement data. Particularly, in order to confirm the usefulness of the proposed method, we investigate the simulated telemetry data of the KOMPSAT-2, a satellite that is scheduled to be launched into a 685km altitude sun synchronous circular orbit in 2005 with an inclination angle of 98.13 degrees.

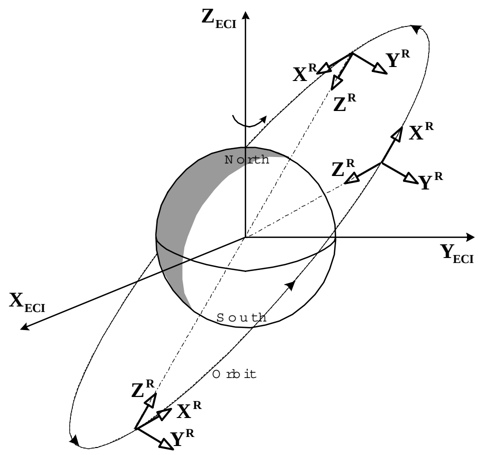

Fig. 1. Normal attitude of earth-pointing spacecraft.

2. Earth-Pointing Spacecraft

The spacecraft considered in this paper is assumed to satisfy the following properties:

(i) The magnetic sensor is mounted in the inside of the satellite body. Particularly, the measurement coordinate axes of the magnetic sensor are aligned with the spacecraft body coordinate axes (X^B, Y^B, Z^B).

(ii) The spacecraft normally operates in the earth-pointing

mode with the given mission requirements. In the earth-pointing mode, the reference attitude frame is (X^R, Y^R, Z^R) such that the Z^R axis points to the center of the earth, the X^R axis points to the direction of the satellite velocity, and the Y^R axis becomes normal to the orbit plane as shown in Fig. 1. The earth-pointing spacecraft is controlled to align (X^B, Y^B, Z^B) to (X^R, Y^R, Z^R) with available sensor information. In this case we assume that a earth sensor is mounted in the spacecraft as like KOMPSAT-2.
(iii) The earth sensor inherently provides the attitude information along roll(X^B) and pitch(Y^B) axes. Thus, in order to obtain the attitude information along yaw(Z^B) axis we need another attitude sensor in the spacecraft. If this is the case, yaw angle can be estimated using the yaw gyro compassing based on the gyro measurement along roll and pitch axes. However, in this paper we assume that the gyro measurement is not available due to the gyro failure.
(iv) The GPS receiver is mounted to provide the current location information of the spacecraft on ECEF frame.

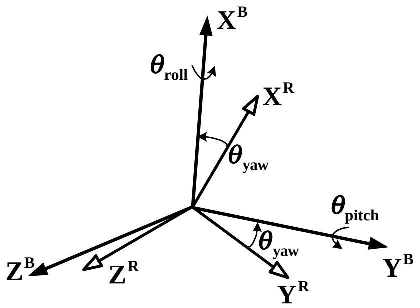

Fig. 2. Spacecraft body frame with respect to orbit reference frame.

3. Gyroless Yaw Angle Compassing

In order to determine the yaw angle, it is essential to determine the X^B and Y^B in ECI frame at first. We denote the magnetic field vector measured by the magnetic sensor in the spacecraft as

$$B^B = [B^B_x, B^B_y, B^B_z]^\top \tag{1}$$

Moreover, we denote the reference magnetic field vector calculated by the IGRF model in the ECEF frame as

$$M^E = [M^E_x, M^E_y, M^E_z]^\top \tag{2}$$

and denote the corresponding vector transformed into the ECI frame as

$$M^I = [M^I_x, M^I_y, M^I_z]^\top \tag{3}$$

The GPS receiver provides the spacecraft position in ECEF frame denoted by

$$P^E = [P^E_x, P^E_y, P^E_z]^\top \tag{4}$$

and the transformation quaternion from ECI frame to the orbit reference frame is denoted by Q^R_I.

Then, we introduce the following three conditions for the formulation of the gyroless yaw angle compassing.
First, since both X^B and Y^B are unit vectors, we obtain

$$\|X^B\| = \|Y^B\| = 1 \tag{5}$$

Second, since Z^R is normally controlled to be perpendicular to X^B and Y^B respectively, we obtain

$$Z^R \cdot X^B = Z^R \cdot Y^B = 0 \tag{6}$$

Third, since the projection of M^I into the spacecraft is equal to B^B, we obtain

$$M^I \cdot X^B = B^B_x \tag{7}$$

$$M^I \cdot Y^B = B^B_y \tag{8}$$

From the above three conditions, we solve the associated nonlinear equations to obtain the four solution candidates. The magnetic field of the current attitude of the satellite is measured by the magnetic sensor and the reference magnetic field is obtained by the IGRF model for the given satellite position given by GPS. In order to obtain true solution, we apply the four candidates to the cost function for minimization. Then, the yaw angle of satellite body axis is derived using the obtained true solution. The proposed algorithm is a conceptually simple but effective to determine the satellite's yaw angle. Furthermore, the satellite's roll and pitch angle error can be combined to obtain precision determination. The proposed algorithm can be applied to a gyroless attitude determination or to the calculation of an initial condition for the Kalman Filter based determination method.

4. Algorithm Procedure

The procedure of the gyroless yaw angle determination is described as follows:
1. Compute the spherical-coordinated spacecraft position in ECEF frame, $[r, \theta, \phi]^\top$

$$r = \sqrt{P^{E^2}_x + P^{E^2}_y + P^{E^2}_y} \tag{9}$$

$$\theta = \text{atan2}(\sqrt{P^{E^2}_x + P^{E^2}_y}, P^E_z) \tag{10}$$

$$\phi = \text{atan2}(P^E_y, P^E_x) \tag{11}$$

2. Compute the spherical-coordinated reference magnetic field in ECEF frame, $[M^E_r, M^E_\theta, M^E_\phi]^\top$ by the derivative of the scalar potential of the IGRF model

$$V = (R_e)\{\sum_{n=1}^{N}\sum_{m=0}^{n}(g^m_n \cos m\phi + h^m_n \sin m\phi) \cdot (\frac{R_e}{r})^{n+1} P^m_n(\cos\theta)\} \tag{12}$$

$$[M^E_r, M^E_\theta, M^E_\phi]^\top = -\nabla V \tag{13}$$

3. Compute the cartesian-coordinated reference magnetic field in ECEF frame, $[M^E_x, M^E_y, M^E_z]^\top$ using quaternion multiplication

$$Q_\phi = [\sin\frac{-\phi}{2}, 0, 0, \cos\frac{-\phi}{2}]^\top \tag{14}$$

$$Q_\theta = [0, 0, \sin\frac{-\theta}{2}, \cos\frac{-\theta}{2}]^\top \quad (15)$$

$$Q_{\phi\theta} = Q_\phi \otimes Q_\theta \quad (16)$$

$$\begin{bmatrix} M_z^E \\ M_x^E \\ M_y^E \\ 0 \end{bmatrix} = (Q_{\phi\theta})^{-1} \otimes \begin{bmatrix} M_r^E \\ M_\theta^E \\ M_\phi^E \\ 0 \end{bmatrix} \otimes Q_{\phi\theta} \quad (17)$$

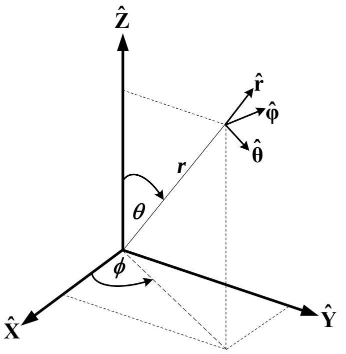

Fig. 3. Trasformation from spherical coordinate to cartesian coordinate.

4. Compute the cartesian-coordinated reference magnetic field in ECI frame, $[M_x^I, M_y^I, M_z^I]^\top$

$$Q_g = [0, 0, \sin\frac{A_g}{2}, \cos\frac{A_g}{2}]^\top \quad (18)$$

$$\begin{bmatrix} M_x^I \\ M_y^I \\ M_z^I \\ 0 \end{bmatrix} = (Q_g)^{-1} \otimes \begin{bmatrix} M_x^E \\ M_y^E \\ M_z^E \\ 0 \end{bmatrix} \otimes Q_g \quad (19)$$

where A_g is the Greenwich angle in ECI frame.

5. Compute the X-axis of the reference orbit frame represented in ECI frame, $[X_x^R, X_y^R, X_z^R]^\top$ using Q_I^R

$$\begin{bmatrix} X_x^R \\ X_y^R \\ X_z^R \\ 0 \end{bmatrix} = (Q_I^R)^{-1} \otimes \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \otimes Q_I^R \quad (20)$$

6. Compute the Z-axis of the reference orbit frame represented in ECI frame, $[Z_x^R, Z_y^R, Z_z^R]^\top$ using Q_I^R

$$\begin{bmatrix} Z_x^R \\ Z_y^R \\ Z_z^R \\ 0 \end{bmatrix} = (Q_I^R)^{-1} \otimes \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \otimes Q_I^R \quad (21)$$

7. Find the solutions $X^B \in \{X_1^B, X_2^B\}$ and $Y^B \in \{Y_1^B, Y_2^B\}$ of the following two simultaneous equations

$$\begin{bmatrix} (X^B)^\top \\ (Z^R)^\top \\ (M^I)^\top \end{bmatrix} X^B = \begin{bmatrix} 1 \\ 0 \\ B_x^B \end{bmatrix} \quad (22)$$

$$\begin{bmatrix} (Y^B)^\top \\ (Z^R)^\top \\ (M^I)^\top \end{bmatrix} Y^B = \begin{bmatrix} 1 \\ 0 \\ B_y^B \end{bmatrix} \quad (23)$$

8. Find X_i^B and Y_j^B that minimizes for $1 \le i, j \le 2$

$$J = \gamma |X_i^B \cdot Y_j^B| + (1-\gamma)\|X_i^B \times Y_j^B - Z^R\| \quad (24)$$

9. Compute the estimated yaw angle of θ_{yaw}^{est}

$$\theta_{yaw}^{est} = \text{sign}\{(X^R \times X_i^B) \cdot Z^R\}\text{acos}(X^R \cdot X_i^B) \quad (25)$$

5. Simulation Results

For the verification of the proposed method, we consider the spacecraft of KOMPSAT-2 where the yaw gyro compassing algorithm is implemented to determine the yaw angle for the reaction wheel based earth-pointing. The yaw gyro compassing computes the yaw angle using the filtered gyro measurement. However, in the case of the gyro failure the yaw gyro compassing can not be used any more until the recovery of the gyro. If this is the case, the proposed gyroless yaw angle compassing is applicable for the control of yaw axis for the coarse earth-pointing. For the simulation, we consider the measurement noise of the magnetic sensor such as

$$\tilde{B}^B = B^B + \Delta B \quad (26)$$

where B^B is the true magnetic field vector and ΔB is the measurement noise with a normal distribution of $N(0, \Delta_1/3)$. Moreover, the attitude control error along roll and pitch axes is considered to have a normal distribution of $N(0, \Delta_2/3)$. Table 1 shows that the estimation error θ_{yaw}^{err} from the simulation results of various Δ_1 and Δ_2. Moreover, Fig. 4 and Fig. 5 show the yaw angle estimation error and the cost function value with the respect to the weighting factor γ. We observe that the estimation error is increased depending on Δ_1 and Δ_2.

Table 1. Estimation error for various Δ_1 and Δ_2

$\Delta_1(3\sigma)$	$\Delta_2(3\sigma)$	$\theta_{yaw}^{err}(worst)$
0.1 nT	0.1 deg	1.0058 deg
0.1 nT	0.5 deg	4.2453 deg
0.1 nT	1 deg	8.2205 deg
1 nT	0.1 deg	1.2557 deg
1 nT	0.5 deg	4.1667 deg
1 nT	1 deg	6.7872 deg
10 nT	0.1 deg	1.2862 deg
10 nT	0.5 deg	5.9396 deg
10 nT	1 deg	7.7479 deg
100 nT	0.1 deg	3.3117 deg
100 nT	0.5 deg	4.5840 deg
100 nT	1 deg	8.7165 deg

6. Conclusions

This paper proposes a determination algorithm of the yaw angle of the earth-point satellite, which is based on vector observation. The proposed algorithm is usefully applicable to the satellites under gyro failure. Thus, it will be used as an effective substitute for the well known yaw gyro compassing.

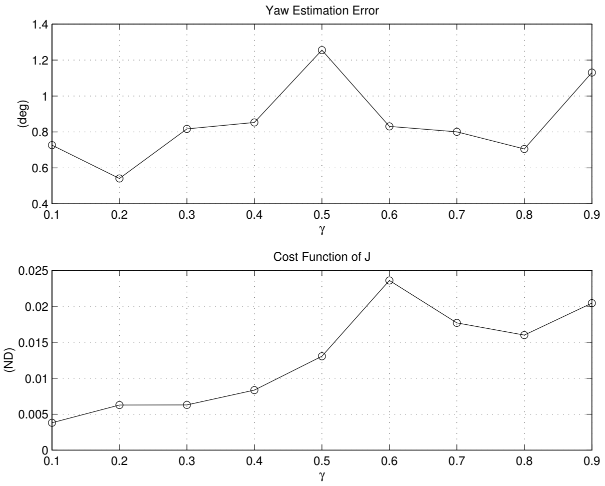

Fig. 4. Results for Δ_1 = 1nT and Δ_2=0.1deg (3σ).

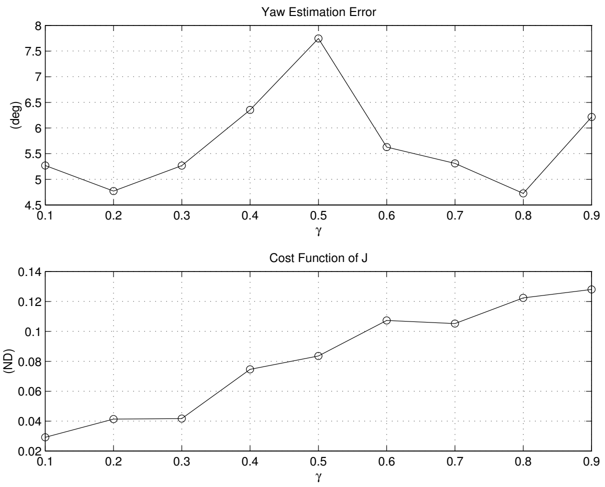

Fig. 5. Results for Δ_1 = 10nT and Δ_2=1deg (3σ).

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