Near-Optimal Collision Avoidance Maneuvers for UAV

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Abstract: Collision avoidance for the aircraft can be stated as a problem of maintaining a safe distance between aircrafts in conflict. Optimal collision avoidance problem seeks to minimize the given cost function while simultaneously satisfying the constraints. The cost function can be a function of time or input. This paper addresses the trajectory time-optimization problem for collision avoidance of the unmanned aerial vehicles. The problem is difficult to handle, because it is a two points boundary value problem with dynamic environment. Some simplifying algorithms are used for application in on-line operation. Although there are more complicated problems, by prediction of conflict time and some assumptions, we changed a dynamic environment problem into a static one.

Keywords: collision avoidance, UAV, near-time optimal, performance index, on-line application

1. INTRODUCTION

Collision avoidance for the aircraft can be stated as a problem of maintaining a safe distance between aircrafts in conflict. Optimal collision avoidance problem seeks to minimize the given cost function while simultaneously satisfying the constraints. The cost function can be a function of time or input.

Many studies have been conducted on UAV, mobile robots avoiding obstacles and arriving at destinations safely [1-4]. And there is an on-line application and optimal solution in the case of non-moving obstacles [3].

However, it is a much different problem in the case of a moving obstacle. The problem is difficult to handle and it is not easy to derive an optimal solution in on-line operation.

Piorini and Shiller studied optimal trajectory of a mobile robot in dynamic environment [4]. They assumed that the obstacle moves straight with a constant speed. Also since there is no limit on the robot speed, it is difficult to apply this method directly to the UAV. Hu, *et. al* studied collision avoidance between aircraft [5]. They did not include the aircraft dynamics and treated the problem as finding a middle waypoint.

In this paper, we consider the aircraft dynamics and make the performance index as a function of time. A solution to minimize the performance index and avoid obstacle simultaneously is found. And some simplifying algorithms are used in order to be able to apply it to a on-line operation. Although there may be more complicated problems by prediction of conflict time and some assumptions, we transform a dynamic environment problem into static one.

We derive the analytic solution of the time optimal trajectory for static obstacle and calculate the expected conflict time and distance for a moving obstacle. Then the algorithm for calculation of the desired heading angle for collision avoidance maneuver and on-line application technique is proposed. Finally simulation study is performed for various situations.

2. ANALYTIC SOLUTION OF TIME OPTIMAL TRAJECTORY

We briefly discuss the optimal trajectory in the static environment. We treat a dynamic obstacle which has a certain lateral acceleration as a static one by calculating expected conflict time and distance by assumptions.

Let us assume that aircraft is a point mass in 2-dimensional environment. Then the aircraft dynamics are described by

$$\begin{aligned} \dot{x} &= V\cos\psi \\ \dot{y} &= V\sin\psi \\ \dot{\psi} &= u \end{aligned} \tag{1}$$

where ψ is the heading angle of the aircraft, V is the velocity, and x, y represent 2-dimensional position of the aircraft, respectively.

A performance index is defined as a function of time.

$$J = \int_{t_0}^{t_f} 1dt \tag{2}$$

subject to

$$S(x,y) = R_P^2 - (x-O_x)^2 - (y-O_y)^2 \le 0$$
$$|u| \le C$$

where R_P is a safe distance, O_x, O_y corresponds to the position of the obstacle in the x-axis, y-axis respectively, u is the input to the aircraft, and C is the constant value that limits input.

This problem is an inequality constraint problem, so it can be divided into two positions. One is in the feasible region and the another is at the boundary region.

2.1 Feasible region

For the case of $S(x,y) < 0$, we define the Hamiltonian as

$$H = 1 + \lambda_x V\cos\psi + \lambda_y V\sin\psi + \lambda_\psi u \tag{3}$$

using

$$\dot{\lambda}^T = -\frac{\partial H}{\partial x}, \quad \frac{\partial H}{\partial u} = 0$$

it follows

$$\begin{aligned} &\dot{\lambda}_x = 0, \quad \dot{\lambda}_y = 0 \\ &\dot{\lambda}_\psi = -\lambda_x V\sin\psi + \lambda_y V\cos\psi \\ &\lambda_\psi = 0 \Rightarrow \dot{\lambda}_\psi = 0 \end{aligned} \tag{4}$$

then

$$\tan\psi = \frac{\lambda_y}{\lambda_x} = \text{constant} \quad (5)$$

Thus, one can see that

$\psi = \text{constant}$

2.2 Boundary region

For the case of $S(x, y) = 0$.

Since $S(x, y)$ has no input term, we differentiate $S(x, y)$ with respect to t until an expression explicitly dependent on input u is derived,

$$\begin{aligned} &\dot{S}(x,y) \\ &= -2(x-O_x)V\cos\psi - 2(y-O_y)V\sin\psi = 0 \\ &\ddot{S}(x,y) \\ &= -2V^2 + 2(x-O_x)V\sin\psi u - 2(y-O_y)V\cos\psi u \\ &= 0 \end{aligned} \quad (6)$$

Hamiltonian is

$$\begin{aligned} H &= 1 + \lambda_x V\cos\psi + \lambda_y V\sin\psi + \lambda_\psi u \\ &+\mu(-2V^2 + 2(x-O_x)V\sin\psi u \\ &\quad -2(y-O_y)V\cos\psi u) \end{aligned} \quad (7)$$

then, the solution is

$$\tan\psi = -\frac{x-O_x}{y-O_y}, \quad \mu = \frac{1}{4V^2} \quad (8)$$

differentiate the above equation with respect to time yields

$$\dot{\psi} = -\frac{\dot{x}(y-O_y) - \dot{y}(x-O_x)}{(y-O_y)^2}\cos^2\psi \quad (9)$$

with

$$\sin^2\psi + \cos^2\psi = 1 \quad (10)$$

substitute Eqs. (8), (10) and dynamic Eq. (1), then

$$\cos^2\psi = \frac{(y-O_y)^2}{R_P^2} \quad (11)$$

$$x - O_x = \pm R_P\sin\psi, \quad y - O_y = R_P\cos\psi \quad (12)$$

substitute Eqs.(1) and (11) into (9), results in

$$\dot{\psi} = \pm V / R_P \quad (13)$$

From the above solution, one find that the aircraft flies straight outside the safety distance, and at the boundary region the aircraft flies along the boundary. This is illustrated in Fig.1.

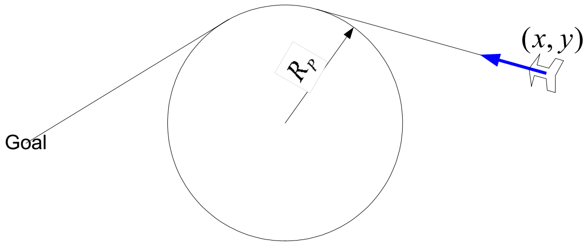

Fig. 1 Time optimal trajectory

3. EXPECTED CONFLICT TIME AND CONFLICT DISTANCE

3.1 Obstacle moves straight with constant speed

If an obstacle moves straight with constant speed, the solution is obtained easily. The position of the obstacle and the aircraft is defined with respect to time as below.

$$\begin{aligned} &X_T = (x_T(t), y_T(t)) \\ &x_T(t) = x_{T0} + \dot{x}_T t, \quad y_T(t) = y_{T0} + \dot{y}_T t \end{aligned} \quad (14)$$

and

$$\begin{aligned} &X_I = (x_I(t), y_I(t)) \\ &x_I(t) = x_{I0} + \dot{x}_I t, \quad y_I(t) = y_{I0} + \dot{y}_I t \end{aligned} \quad (15)$$

The range between the aircraft and the obstacle is given by

$$\begin{aligned} R(t) &= \| X_T(t) - X_I(t) \| \\ &= \sqrt{(x_I(t)-x_T(t))^2 + (y_I(t)-y_T(t))^2} \end{aligned} \quad (16)$$

The expected conflict time is obtained by minimizing the range function $R(t)$.

Since $R(t)$ is a monotonic function, we define $R'(t) = R(t)^2$. The expected conflict time t_c is the time minimizing the range function $R(t)$, such that

$$t_c = -\frac{(x_{I0}-x_{T0})(\dot{x}_I-\dot{x}_T) + (y_{I0}-y_{T0})(\dot{y}_I-\dot{y}_T)}{(\dot{x}_I-\dot{x}_T)^2 + (\dot{y}_I-\dot{y}_T)^2} \quad (17)$$

Furthermore, the conflict distance is $R(t_c)$. If $R(t_c)$ is greater than the safety distance, it is considered as no conflict and vice versa.

3.2 Obstacle has lateral acceleration

If the obstacle moves with a certain lateral acceleration, the problem becomes quite different.

First, the position of the obstacle is given as,

$$\begin{aligned} x_T(t) &= x_{T0} + V_T^2 / a_T \sin\psi_{T0} - V_T^2 / a_T \sin\psi_T(t) \\ y_T(t) &= y_{T0} - V_T^2 / a_T \cos\psi_{T0} + V_T^2 / a_T \cos\psi_T(t) \\ \psi_T(t) &= \psi_{T0} - V_T / a_T \, t \end{aligned} \quad (18)$$

The above function has nonlinear terms. So if we try to minimize $R(t)=\sqrt{(x_I(t)-x_T(t))^2+(y_I(t)-y_T(t))^2}$ directly to obtain the expected conflict time, there may exist many local minimums and it is hard to find the true solution.

Even if the lateral acceleration is constant, the result is same. So we propose a different approach to estimate the expected conflict time and distance.

There are two situations between the obstacle and the aircraft trajectory. Figures 2 and 3 show the two situations. One is when the trajectory of the aircraft crosses the trajectory of the obstacle, and the other is when the trajectory of the aircraft doesn't cross.

Case 1: The trajectory of the aircraft crosses the trajectory of the obstacle

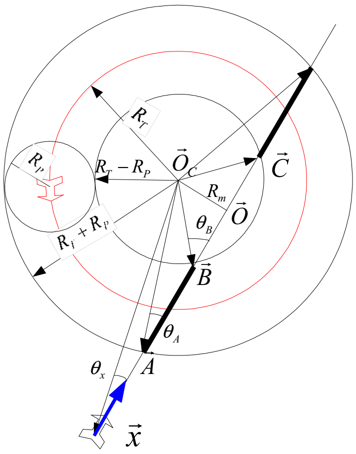

Fig. 2 Aircraft penetrates in the turn radius of obstacle

Case 2: The trajectory of the aircraft doesn't cross the trajectory of the obstacle

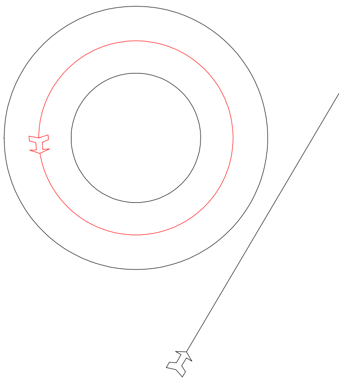

Fig. 3 Aircraft flies outside the turn radius of the obstacle

The second case could be ignored because collision situation does not occur. So we consider only the first case. In the first case the time region for search is restricted as bold lines($\vec{A}$ $\vec{B}$, $\vec{C}$ $\vec{D}$). This prevents us from calculating a wrong solution.

Let us calculate t_A, t_B, t_C, t_D, the time at $\vec{A}, \vec{B}, \vec{C}, \vec{D}$ in the Fig. 2.

The center of the turn circle of an obstacle $\vec{O}_C$ is

$$\vec{O}_C=[o_{Cx}, o_{Cy}] \tag{19}$$

where

$$o_{Cx}=x_{T0}+R_T\sin(\psi_{T0})$$
$$o_{Cy}=y_{T0}-R_T\cos(\psi_{T0})$$
$$R_T={V_T^2}/{|a_T|}$$

where R_T is the turn radius of the obstacle, and ψ_{T0} is the initial heading angle of the obstacle.

Using the trajectory function of the aircraft

$$\vec{x}(t)=[x_0+v_x t \quad y_0+v_y t] \tag{20}$$

where v_x, v_y represent the velocity of aircraft in the x and y axis. And the center of the turn circle of an obstacle $\vec{O}_C$ from Eq. (19), we can obtain time t_m which minimizes the range between the aircraft and the center of the turn circle $\vec{O}_C$.

$$t_m=\frac{(o_{Cx}-x_0)v_x+(o_{Cy}-y_0)v_y}{v_x^2+v_y^2} \tag{21}$$

So the minimum range R_m between the aircraft and the center of the turn circle of the obstacle becomes

$$R_m=\sqrt{(x_0+v_x t_m-o_{Cx})^2+(y_0+v_y t_m-o_{Cy})^2} \tag{22}$$

From Fig. 2, t_A represents the time to $\vec{A}$, and t_B the time to $\vec{B}$ are expressed in the form

$$t_A={R_{Ax}}/{V_I}={(R_{ox}-R_{oA})}/{V_I}$$
$$t_B={R_{Bx}}/{V_I}={(R_{ox}-R_{oB})}/{V_I} \tag{23}$$

where R_{oA}, R_{oB}, R_{ox} are given by

$$R_{oA}=|\vec{O}-\vec{A}|=\sqrt{(R_T+R_p)^2-R_m^2}$$
$$R_{oB}=|\vec{O}-\vec{B}|=\sqrt{(R_T-R_P)^2-R_m^2} \tag{24}$$
$$R_{ox}=|\vec{O}-\vec{x}|=\sqrt{|\vec{O}_C-\vec{x}|^2-R_m^2}$$

In the same way, we can obtain t_C, t_D to $\vec{C}, \vec{D}$, respectively. However when we reduce the search region, it is impossible to obtain a closed-form solution because the equation still possesses nonlinear terms.

3.3 On-line application technique

Two techniques for on-line application are proposed. First one is an obstacle velocity linearization method. It can be made possible by assuming that the turn angle of the obstacle in the search region is small. Second is a Hybrid gradient descent particle swarm optimization(HGPSO) [6]. It is a mixed method of the gradient descent and the particle swarm optimization method.

3.3.1 Obstacle velocity linearization

The path function of the obstacle is already defined in Eq. (18). By rewriting this equation from $\vec{A}$ to $\vec{B}$, it follows as

$$
\begin{aligned}
x_T(t) &= x_{T0} + R_T \sin\psi_{T0} \\
&\quad - R_T \sin\{\psi_0 - a_T/V_T t_A - a_T/V_T(t-t_A)\} \\
&= x_{T0} + R_T \sin\psi_{T0} \\
&\quad - R_T \begin{bmatrix} \sin(\psi_T(t_A))\cos(a_T/V_T(t-t_A)) \\ -\cos(\psi_T(t_A))\sin(a_T/V_T(t-t_A)) \end{bmatrix} \\
y_T(t) &= y_{T0} - R_T \cos\psi_{T0} \\
&\quad + R_T \begin{bmatrix} \cos(\psi_T(t_A))\cos(a_T/V_T(t-t_A)) \\ +\sin(\psi_T(t_A))\sin(a_T/V_T(t-t_A)) \end{bmatrix} \\
\psi_T(t_A) &= \psi_{T0} - a_T/V_T t_A
\end{aligned} \tag{25}
$$

If $a_T/V_T(t-t_A)$ is very small, we can rewrite Eq. (25) as

$$
\begin{aligned}
x_T(t) &= \alpha + \beta \bullet t \\
y_T(t) &= \gamma + \delta \bullet t
\end{aligned} \tag{26}
$$

where

$$
\begin{aligned}
\alpha &= x_{T0} + R_T \sin\psi_{T0} - R_T \sin(\psi_{T0} - a_T/V_T t_A) \\
&\quad - V_T t_A \cos(\psi_{T0} - a_T/V_T t_A) \\
\beta &= V_T \cos(\psi_{T0} - a_T/V_T t_A) \\
\gamma &= y_{T0} - R_T \cos\psi_{T0} + R_T \cos(\psi_{T0} - a_T/V_T t_A) \\
&\quad - V_T t_A \sin(\psi_{T0} - a_T/V_T t_A) \\
\delta &= V_T \sin(\psi_{T0} - a_T/V_T t_A)
\end{aligned}
$$

Substituting this equation into Eq. (17), the expected conflict time and conflict range could be calculated as follows;

$$
t_{c\_AB} = -\frac{(x_{I0}-\alpha)(\dot{x}_I-\beta) + (y_{I0}-\gamma)(\dot{y}_I-\delta)}{(\dot{x}_I-\beta)^2 + (\dot{y}_I-\delta)^2} \tag{27}
$$

From $\vec{C}$ to $\vec{D}$, the expected conflict time and conflict range are computed in an analogous way.

3.3.2 HGPSO method

PSO is the optimization method which is inspired by social behavior. It finds a solution by parallel search same as evolutionary algorithm. However it chooses next generation by cooperation method instead of competition [7].

Since the problem is only dependent on time, it is not useful to find solution by normal PSO algorithm. So in this paper we propose a HGPSO algorithm which is PSO mixed with gradient method [6].

4. NUMERICAL SOLUTION OF TIME OPTIMAL TRAJECTORY

In this section, we propose the algorithm to solve the time optimal trajectory problem numerically.

For this goal, we must calculate the desired heading angle in order to obtain the direction of collision avoidance maneuver and input lateral acceleration. If there is an obstacle, the desired heading angle can be derived using Fig. 4.

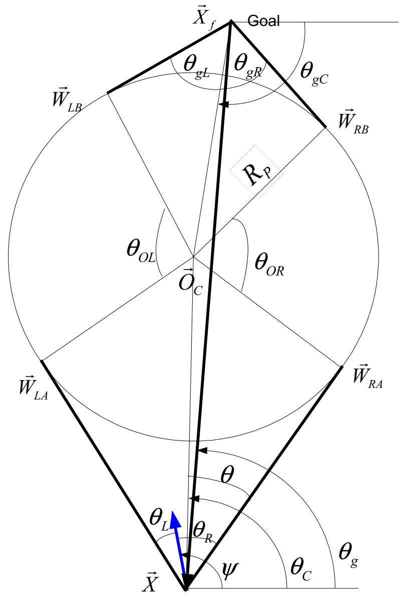

Fig.4 Calculating desired heading angle

$$
\begin{aligned}
\theta_c &= \tan_2^{-1}\left(\frac{o_{cy}-y}{\sqrt{(x-o_{cx})^2+(y-o_{cy})^2}}\right) \\
\theta &= \sin^{-1}\left(\frac{R_P}{\sqrt{(x-o_{cx})^2+(y-o_{cy})^2}}\right)
\end{aligned} \tag{28}
$$

Using Eq. (28) the desired heading angle from the present angle is given by

$$
\begin{aligned}
\theta_L &= \theta_C + \theta - \psi \\
\theta_R &= \theta_C - \theta - \psi
\end{aligned} \tag{29}
$$

and θ_{gL} θ_{gR} are obtained in the same way.

$$
\begin{aligned}
\theta_{gC} &= \tan_2^{-1}\left(\frac{o_{cy}-y_f}{\sqrt{(x_f-o_{cx})^2+(y_f-o_{cy})^2}}\right) \\
\theta_{gL} = \theta_{gR} &= \sin^{-1}\left(\frac{R_P}{\sqrt{(x_f-o_{cx})^2+(y_f-o_{cy})^2}}\right)
\end{aligned} \tag{30}
$$

using Eq. (30)

$$
\begin{aligned}
\theta_{OR} &= \pi - \theta_C + \theta - \theta_{gC} + \theta_{gR} \\
\theta_{OL} &= \theta_C + \theta + \theta_{gC} + \theta_{gL} - \pi
\end{aligned} \tag{31}
$$

Using the above equations, one can evaluate left and right maneuvering time from the present position to the goal as follows;

$$
\begin{aligned}
t_{R\_man} &= (|\vec{W}_{RA} - \vec{X}| + R_P\theta_{OR} + |\vec{X}_f - \vec{W}_{RB}|)/V_I \\
t_{L\_man} &= (|\vec{W}_{LA} - \vec{X}| + R_P\theta_{OL} + |\vec{X}_f - \vec{W}_{LB}|)/V_I
\end{aligned} \tag{32}
$$

From Eq. (32), the maneuvering direction whose time is smaller can be decided. For a static obstacle, the direction of the obstacle avoidance maneuver and desired heading angle are determined by the above algorithm, but for a dynamic obstacle it may not be quite sufficient. Since the obstacle moves toward the aircraft performing avoidance maneuver, the obstacle region changes during the maneuver. Fig. 5 shows this situation and possible solution.

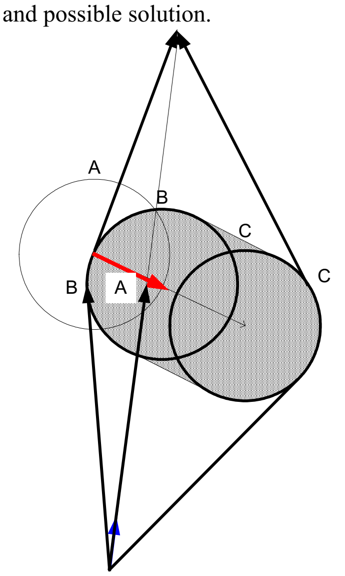

Fig.5 Calculation of the heading angle about dynamic obstacle

In the Fig. 5, the position of the obstacle is A, but during the left obstacle avoidance maneuver the obstacle moves to B, and during right maneuver the obstacle moves to C. So for the dynamic obstacle, the region B to C (the grey region) is regard as the obstacle.

In the above algorithm the desired heading angle as in the static obstacle case (29) can be constructed.

5. SIMULATION RESULT

It is assume that the aircraft and the obstacle are two-dimensional point mass objects. The equations of motion are simply given in the form.

$$\begin{aligned} \dot{x} &= V\cos\psi \\ \dot{y} &= V\sin\psi \\ \dot{\psi} &= -\tfrac{1}{V}u \end{aligned} \qquad (33)$$

Safety range is 6000ft and initial position of the main aircraft is (0,0) and the goal location is (0,60760 ft). The speed of aircraft and obstacle is 337 ft/sec.

The simulation was performed in four cases. First case is that obstacle is not moving, second case is obstacle moves straight with a constant forward speed. Third case is that obstacle moves with a constant lateral acceleration and constant forward speed. The last case is obstacle moves with changing lateral acceleration and constant forward speed.

5.1 Collision avoidance maneuver for non-moving obstacle

Two cases of simulation for static obstacle were examined. First case is one obstacle and the second case is multi-obstacles case. Figures 6 and 7 shows the simulation results.

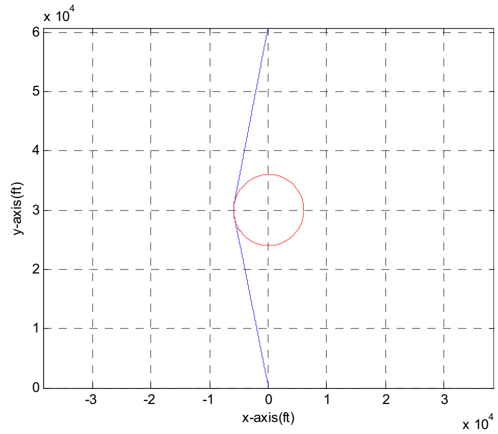

Fig.6 One obstacle case

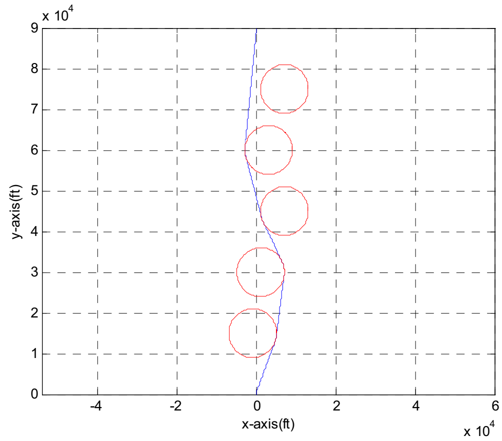

Fig.7 Multi-obstacles case

It can be shown that the aircraft maneuvers as expected, and collision avoidance is performed in a satisfactory manner.

5.2 Collision avoidance maneuver for moving obstacle

For dynamic obstacle case, we assume that obstacle is a two-dimensional point mass, and equations of motion are already presented in Eq. (33).

5.2.1 Straight and constant speed obstacle

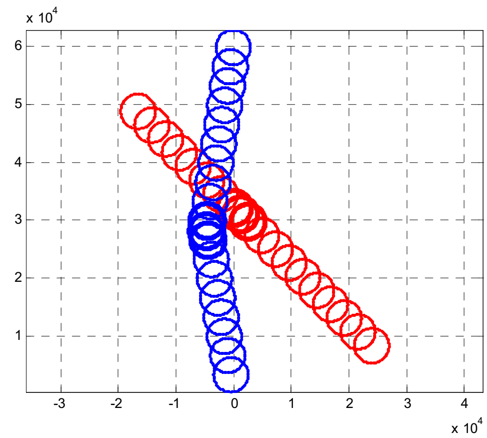

Fig.8 Collision avoidance for straight and constant speed obstacle

5.2.2 Moving obstacle with a constant lateral acceleration

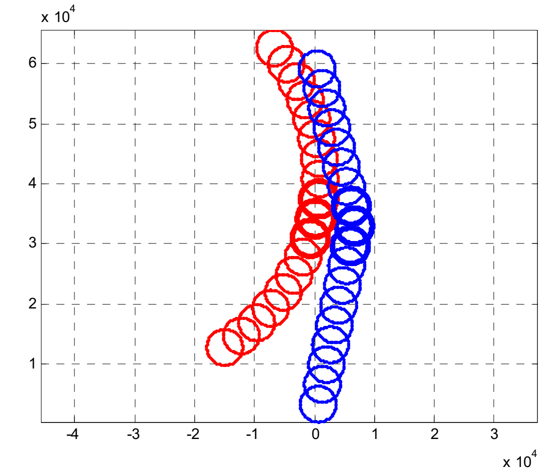

Fig.9 Collision avoidance for obstacle with constant acceleration

5.2.3 Moving obstacle with a random lateral acceleration

In this simulation, the obstacle moves with changing lateral acceleration and constant speed. For the simulation, the lateral acceleration of the obstacle is settled into a cosine wave.

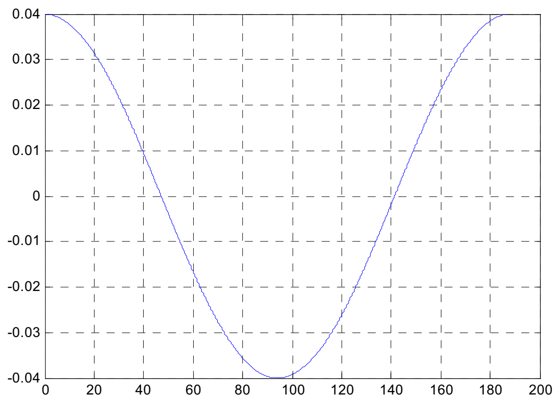

Fig.10 Applied acceleration of obstacle

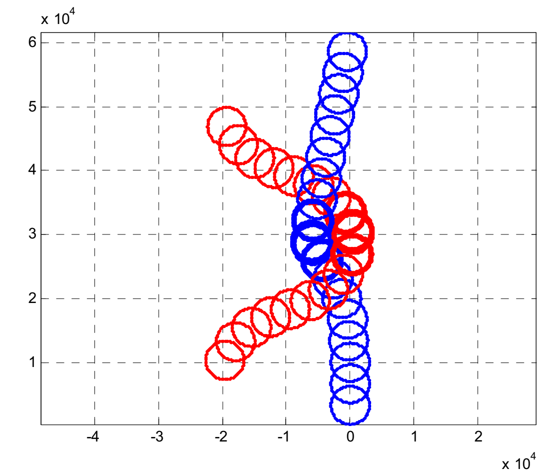

Fig.11 Collision avoidance for obstacle with cosine wave acceleration

6. CONCLUSION

The near-minimum-time optimal collision avoidance for UAV was studied. The solution that minimizes the performance index and avoids obstacle simultaneously was derived. Obstacle velocity linearization and HGPSO are proposed for application into on-line operation.

We derived the analytic solution of the time optimal trajectory for static obstacle and calculate the expected conflict time and distance for a moving obstacle. The moving obstacle was treated as the static one, and the numerical solution for collision avoidance for moving obstacle was thought. Simulations in the static and dynamic environment were conducted.

While it shows desirable results, there remains more work to be done. First, the chattering problem in aircraft input should be resolved. And the case of multi-moving-obstacle case should be investigated with extension into three -dimensional cases.

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