Variable Structure Model Reference Adaptive Control, for SIMO Systems

Ardeshir Karami mohammadi

* Department of Mechanical Engineering, Shahrood University of technology, Shahrood, Iran
(E-mail: akaramim@yahoo.com)

Abstract: A Variable Structure Model Reference Adaptive Controller (VS-MRAC) using state Variables is proposed for single input multi output systems. . The structure of the switching functions is designed based on stability requirements, and global exponential stability is proved. Transient behavior is analyzed using sliding mode control and shows perfect model following at a finite time. The effect of input disturbances on stability and transients is investigated and shows preference to the conventional MRAC schemes with integral adaptation law. Sliding surfaces are independent of system parameters and therefore VS-MRAC is insensitive to system parameter variations. Simulation is presented to clear the theoretical results.

Keywords: Variable Structure, Adaptive Control, Model Reference.

1. INTRODUCTION

Model Reference Adaptive Controllers with conventional continuous adaptation laws has been extensively investigated in the literature in two main branches: one assuming full state accessibility [7,8,9], and the others assuming accessibility of only input and output [3,4]. Continuous adaptation laws are in the form of pure integral actions and have some problems such as: I. Transient behavior is difficult to analysis.

II. Only global (but not asymptotic) stability has been guaranteed.

III. Undesirable transient responses and tracking performance.

IV. Lack of robustness.

The variable structure systems (VSS) have been studied in great details in the literatures [6, 10, 11]. The basic concept of the variable structure control is that of sliding mode control. switching control functions are generally designed to generate sliding surfaces, or sliding modes, in the state space [10]. When this is attained, the switching functions keep the trajectory on the sliding surfaces and the closed loop system becomes insensitive, to a certain extent, to parameter variations and disturbances.

Some authors have applied the variable structure control (VSC) to adaptive control, for full states accessible systems [2], and for only input – output measurable systems [1, 3, 4]. A tutorial account of VSC is presented by Hung [5].

In this article, we proposed a design and analyze a variable structure model reference adaptive controller for single input multi variable systems. The control law and the general structure of switching functions (adaptation laws) are designed based on stability criterion. Exponential stability is assured and exponential attractiveness to the origin is independent of sliding mode be reached. Then the magnitudes of switching functions are chosen based on reaching conditions for sliding mode. Realization procedure will be easy to perform and the controller has good transient behavior and is insensitive with respect to input disturbances and parameter variations. With the proposed controller, eventually sliding mode always occurs.

2. PROBLEM DEFINITION

Consider a linear time invariant plant with unknown parameters, which their bounds are known. Let the plant be of n-th order with accessible states and described by the differential equation

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{b}u \quad (1)$$

where $n \times n$ matrix **A** and vector **b** are unknown, and (**A**,**b**) is controllable.

The reference model is characterized by the linear time invariant differential equation

$$\dot{\mathbf{X}}_{\mathbf{m}} = \mathbf{A}_{\mathbf{m}}\mathbf{X}_{\mathbf{m}} + \mathbf{b}_{\mathbf{m}} r \quad (2)$$

where is $\mathbf{A_m}$ a $n \times n$ asymptotically stable matrix, $\mathbf{b_m}$ is a known vector, and r is a bounded reference input. The purpose is to find control u such that the state error

$$\mathbf{e} = \mathbf{X} - \mathbf{X}_{\mathbf{m}} \quad (3)$$

exponential tends to zero in a finite time.

3.VARIABLE STRUCTURE ADAPTIVE CONTROLLER

The solution can be attempted under different assumptions regarding the prior information available concerning the plant. We consider the following two cases:

Case I.

The matrix **A** is unknown, while the vector **b** is assumed to be known. In this case the vector $\mathbf{b_m}$ of the reference model can be chosen as

$$\mathbf{b}_{\mathbf{m}} = \mathbf{b}q^{*} \quad (4)$$

where q^{*} is a known scalar. It is further assumed that an unknown $m \times n$ matrix θ^{*} exists such that

$$\mathbf{A} + \mathbf{b}\theta^{*} = \mathbf{A}_{\mathbf{m}} \quad (5)$$

I.1 Stability, and switching function design

The control u to the plant, is generated introducing control law

$$u = \beta\,\mathbf{X} + q^{*} r \quad (6)$$

where n dimensional raw feedback vector β, with the elements β_i are adjusted using VS approach by designing switching functions β_i as described in the followings.

Subtracting equation (2) from (1), and using equations (4), (5) and (6), the error equation is obtained as

$$\dot{\mathbf{e}} = \mathbf{A}_{\mathbf{m}}\mathbf{e} + \mathbf{b}\,(\beta - \theta^{*})\mathbf{X} \quad (7)$$

Consider a Lyapunov function of the form

$$\mathbf{V} = \mathbf{e}^{\mathbf{T}}\mathbf{Pe} \quad \textbf{(8)}$$
where **P** is a positive definite symmetric matrix which satisfies the Lyapunov equation

$$\mathbf{A}_{\mathbf{m}}^{\mathbf{T}}\mathbf{P} + \mathbf{PA}_{\mathbf{m}} = -\mathbf{Q}_0 \quad \textbf{(9)}$$

where $\mathbf{Q_0}$ is a positive definite symmetric matrix.

Differentiating (8) with respect to time along the trajectory (7) yields

$$\dot{\mathbf{V}} = -\mathbf{e}^{\mathbf{T}}\mathbf{Q}_0\mathbf{e} + 2\,\mathbf{b}^{\mathbf{T}}\mathbf{Pe}\,(\beta - \theta^*)\,\mathbf{X}$$
$$= -\mathbf{e}^{\mathbf{T}}\mathbf{Q}_0\mathbf{e} + 2\,\mathbf{b}^{\mathbf{T}}\mathbf{Pe}\sum_{i=1}^{\mathbf{n}} (\beta_{\mathbf{i}} - \theta_{\mathbf{i}}^*)\,\mathbf{x_i} \quad \textbf{(10)}$$

Now, introducing the switching functions $\beta_{\mathbf{i}}$ as

$$\beta_{\mathbf{i}} = -\overline{\theta_{\mathrm{i}}}\ \mathrm{sgn}\,(\mathbf{b}^{\mathbf{T}}\mathbf{Pe}\,\mathbf{x}_{\mathrm{i}}), \qquad \overline{\theta_{\mathrm{i}}} > \left|\theta_{\mathbf{i}}^*\right| \quad \textbf{(11)}$$

and substitute into (10), yields

$$\dot{\mathbf{V}} = -\mathbf{e}^{\mathbf{T}}\mathbf{Q}_0\mathbf{e} + 2\,\mathbf{b}^{\mathbf{T}}\mathbf{Pe}\sum_{i=1}^{\mathbf{n}} [-\overline{\theta_{\mathbf{i}}}\mathrm{sgn}\,(\mathbf{b}^{\mathbf{T}}\mathbf{Pe}\,\mathbf{x_i}) - \theta_{\mathbf{i}}^*]\,\mathbf{x_i}$$
$$= -\mathbf{e}^{\mathbf{T}}\mathbf{Q}_0\mathbf{e} - 2\sum_{i=1}^{\mathbf{n}} [\overline{\theta_{\mathrm{i}}}\left|\mathbf{b}^{\mathbf{T}}\mathbf{Pe}\,\mathbf{x_i}\right| + \theta_{\mathbf{i}}^*\mathbf{b}^{\mathbf{T}}\mathbf{Pe}\,\mathbf{x_i}] \quad \textbf{(12)}$$

The terms in the summation are always positive, therefore $\dot{\mathbf{V}} < 0$ and regarding (8) it can be concluded that $\|\mathbf{e}\|$ decreases at least exponentially.

I.2 Existence of sliding mode

Here it is shown that the surface

$$\mathbf{S} = \mathbf{b}^{\mathbf{T}}\mathbf{Pe} = 0 \quad \textbf{(13)}$$

is always a sliding surface for the system. Let examine the following reaching conditions [15, 9].

$$\mathbf{S}\dot{\mathbf{S}} < 0, \qquad |\dot{\mathbf{S}}| > 0 \quad \textbf{(14)}$$

Regarding equations (7) and (11), we have

$$\mathbf{S}\dot{\mathbf{S}} = \mathbf{S}\,\mathbf{b}^{\mathbf{T}}\mathbf{PA}_{\mathbf{m}}\mathbf{e} + \mathbf{S}\,\mathbf{b}^{\mathbf{T}}\mathbf{Pb}\,(\beta - \theta^*)\,\mathbf{X}$$
$$= \mathbf{S}\,\mathbf{b}^{\mathbf{T}}\mathbf{PA}_{\mathbf{m}}\mathbf{e} + \mathbf{S}\,\mathbf{b}^{\mathbf{T}}\mathbf{Pb}\sum_{i=1}^{\mathbf{n}} (\beta_{\mathbf{i}} - \theta_{\mathbf{i}}^*)\,\mathbf{x_i}$$
$$= \mathbf{S}\,(\mathbf{b}^{\mathbf{T}}\mathbf{PA}_{\mathbf{m}})\mathbf{e} + \mathbf{S}\,\mathbf{b}^{\mathbf{T}}\mathbf{Pb}\sum_{\mathbf{i}=1}^{\mathbf{n}}\left[-\overline{\theta_{\mathbf{i}}}\ \mathrm{sgn}\,(\mathbf{b}^{\mathbf{T}}\mathbf{Pe}\,\mathbf{x_i}) - \theta_{\mathbf{i}}^*\right]\mathbf{x_i}$$
$$= \mathbf{S}(\mathbf{b}^{\mathbf{T}}\mathbf{PA}_{\mathbf{m}})\mathbf{e} - |\mathbf{S}|\,\mathbf{b}^{\mathbf{T}}\mathbf{Pb}\sum_{\mathbf{i}=1}^{\mathbf{n}}\left[\overline{\theta_{\mathrm{i}}} + \theta_{\mathrm{i}}^*\ \mathrm{sgn}(\mathbf{b}^{\mathbf{T}}\mathbf{Pe}\,\mathbf{x_i})\right]|\mathbf{x_j}|$$
$$< |\mathbf{S}|\left\{\varepsilon_1\|\mathbf{e}\| - \mathbf{b}^{\mathbf{T}}\mathbf{Pb}\sum_{\mathbf{i}=1}^{\mathbf{n}}\left[(\overline{\theta_{\mathbf{i}}} - |\theta_{\mathbf{i}}^*|)\right]|\mathbf{x_i}|\right\} \quad \textbf{(15)}$$

where ε_1 is a positive constant. Since $\|\mathbf{e}\|$ exponentially tends to zero, it can be concluded that if we have

$$\|\mathbf{X}(\mathrm{t})\| > \lambda > 0, \quad \forall \mathrm{t} > \mathrm{t}_0 \quad \textbf{(16)}$$

then there exists $\mathrm{T} > \mathrm{t}_0$ such that

$$\mathbf{S}\dot{\mathbf{S}} < 0, \quad \forall \mathrm{t} \geq \mathrm{T} \quad \textbf{(17)}$$

Condition $|\dot{\mathbf{S}}| > 0$ in the relation (14) is considered here. One can writes

$$\dot{\mathbf{S}} = \mathbf{b}^{\mathbf{T}}\mathbf{P}\dot{\mathbf{e}} = \mathbf{b}^{\mathbf{T}}\mathbf{PA}_{\mathbf{m}}\mathbf{e} + \mathbf{b}^{\mathbf{T}}\mathbf{Pb}\,(\beta - \theta^*)\,\mathbf{X}$$
$$= \mathbf{b}^{\mathbf{T}}\mathbf{PA}_{\mathbf{m}}\mathbf{e} - \mathbf{b}^{\mathbf{T}}\mathbf{Pb}\sum_{i=1}^{\mathbf{n}} (\beta_{\mathbf{i}} - \theta_{\mathbf{i}}^*)\,\mathbf{x_i}$$
$$= \mathbf{b}^{\mathbf{T}}\mathbf{PA}_{\mathbf{m}}\mathbf{e} - \mathbf{b}^{\mathbf{T}}\mathbf{Pb}\sum_{\mathbf{i}=1}^{\mathbf{n}}\left[\overline{\theta_{\mathrm{i}}}\ \mathrm{sgn}\,(\mathbf{b}^{\mathbf{T}}\mathbf{Pe}\,\mathbf{x_i}) - \theta_{\mathbf{i}}^*\right]\mathbf{x_i} \quad \textbf{(18)}$$

since $\|\mathbf{e}\|$ exponentially tends to zero, and consider to equation (16), it can be seen at least one of the terms in the summation is nonzero and we have

$$|\dot{\mathbf{S}}| > 0, \quad \text{for } \mathbf{t} > \mathbf{T} \quad \textbf{(19)}$$

Relations (17) and (19) mean that for all t >T, the surface $\mathbf{S} = \mathbf{b}^{\mathbf{T}}\mathbf{Pe} = 0$ is a sliding surface.

I.3 stability in the presence of bounded disturbance

If a bounded disturbance w(t), acts on the plant input, the error equation (7) becomes

$$\dot{\mathbf{e}} = \mathbf{A}_{\mathbf{m}}\mathbf{e} + \mathbf{b}\,(\beta - \theta^*)\mathbf{X} + \mathbf{b}\ \mathrm{w} \quad \textbf{(20)}$$

Consider the Lyapunov function (8), we have

$$\dot{\mathbf{V}} = -\mathbf{e}^{\mathbf{T}}\mathbf{Q}_0\mathbf{e} + 2\,\mathbf{b}^{\mathbf{T}}\mathbf{Pe}\sum_{i=1}^{\mathbf{n}} [(\beta_{\mathbf{i}} - \theta_{\mathbf{i}}^*)\,\mathbf{x_i}] + 2\mathbf{e}^{\mathbf{T}}\mathbf{Pb}\ \mathrm{w} \quad \textbf{(21)}$$

and regard to switching functions $\beta_{\mathbf{i}}$ as defined by equation (11), one can write

$$\dot{\mathbf{V}} = -\mathbf{e}^{\mathbf{T}}\mathbf{Q}_0\mathbf{e} - 2\sum_{i=1}^{\mathbf{n}} [\overline{\theta_{\mathrm{i}}}\left|\mathbf{b}^{\mathbf{T}}\mathbf{Pe}\,\mathbf{x_i}\right| + \theta_{\mathrm{i}}^*\,\mathbf{b}^{\mathbf{T}}\mathbf{Pe}\,\mathbf{x_i}]$$
$$+ 2\mathbf{e}^{\mathbf{T}}\mathbf{Pb}\ \mathrm{w} \quad \textbf{(22)}$$

Now, defining $\Delta\theta_{\mathbf{m}}$ and w as

$$\Delta\theta_{\mathbf{m}} = \min_{\mathrm{i}}\,(\overline{\theta_{\mathrm{i}}} - |\theta_{\mathbf{i}}^*|), \quad \overline{\mathrm{w}} = \sup^{t>0}|\mathrm{w}(\mathbf{t})| \quad \textbf{(23)}$$

one can concluded that

$$\dot{\mathbf{V}} < -\rho_1\|\mathbf{e}\|^2 - \rho_2\,\Delta\theta_{\mathbf{m}}\,\|\mathbf{X}\|\left|\mathbf{b}^{\mathbf{T}}\mathbf{Pe}\right| + \rho_3\|\mathbf{e}\|\,\overline{\mathrm{w}} \quad \textbf{(24)}$$

where ρ_1, ρ_2, and ρ_3 are positive constants.

From relation (24), it is understandable that e should have a residual set as

$$\|\mathbf{e}\| < \rho_4\,\overline{\mathrm{w}} \quad \textbf{(25)}$$

where ρ_4 is a positive constant.

I.4 Existence of sliding mode in the presence of bounded disturbance

As mentioned in section (I.3), the error equation in the presence of bounded disturbance w(t), will be in the form of equation (20).

In the same manner as followed in section (I.2) one can writes

$$\mathbf{S}\dot{\mathbf{S}} = \mathbf{S}\,\mathbf{b}^{\mathbf{T}}\mathbf{PA}_{\mathbf{m}}\mathbf{e} + \mathbf{S}\,\mathbf{b}^{\mathbf{T}}\mathbf{Pb}\,(\beta - \theta^*)\,\mathbf{X} + \mathbf{S}\,\mathbf{b}^{\mathbf{T}}\mathbf{pb}\ \mathrm{w}$$
$$< |\mathbf{S}|\left\{\varepsilon_1\|\mathbf{e}\| - \mathbf{b}^{\mathbf{T}}\mathbf{Pb}\sum_{\mathbf{i}=1}^{\mathbf{n}}\left[(\overline{\theta_{\mathbf{i}}} - |\theta_{\mathbf{i}}^*|)\right]|\mathbf{x_i}| + \mathbf{b}^{\mathbf{T}}\mathbf{pb}\ \mathrm{w}\right\}$$
$$< |\mathbf{S}|\left\{\varepsilon_1\|\mathbf{e}\| - \mathbf{b}^{\mathbf{T}}\mathbf{Pb}\,\Delta\theta_{\mathbf{m}}\,|\mathbf{x_i}| + \mathbf{b}^{\mathbf{T}}\mathbf{pb}\ \mathrm{w}\right\} \quad \textbf{(26)}$$

since $\|\mathbf{e}\|$ exponentially tends to a residual domain specified by relation (25), it can be concluded that if we have

$$\|\mathbf{X}(\mathbf{t})\| > \lambda' > \frac{\delta}{\mathbf{b}^{\mathbf{T}}\mathbf{pb}}\frac{\mathrm{w}}{\Delta\theta_{\mathbf{m}}}, \quad \forall \mathrm{t} > \mathrm{t}_0 \quad \textbf{(27)}$$

where δ is a positive constant, then there exists $T > t_0$ such that

$$\mathbf{S}\dot{\mathbf{S}} < 0, \quad \forall \mathrm{t} \geq \mathrm{T} \quad \textbf{(28)}$$

Condition $|\dot{\mathbf{S}}| > 0$ can be proved in the same manner as applied for equations (18) and (19).

I.5 Average control

If control signal in equation (6)is written as

$$\mathrm{u} = \mathrm{u}^{\mathrm{o}} + \mathrm{q}^*\mathrm{r}, \quad \mathrm{u}^{\mathrm{o}} = \beta\,\mathbf{X} \quad \textbf{(29)}$$

where u^{o} and $\mathrm{q}^*\mathrm{r}$ are variable structure and continuous parts of controller, respectively. When sliding mode is occurred, actually the average control u^o can be used [ ], which is the output of first order filter

$$\tau \dot{u}_{av}^o + u_{av}^o = u^o = \sum_{i=1}^{n} (\beta_i \, x_i) \quad \textbf{(30)}$$

where time constant τ is sufficiently small. Then the average control u_{av}^o can be obtained as

$$u_{av} = u_{av}^o + q^* \, r \quad \textbf{(31)}$$

The block diagram is presented in figure 1.

Case II.

The matrix A and vector b are assumed to be unknown. It is further assumed that there exist an unknown n dimensional raw vector θ^*, and a scalar q^* such that

$$\mathbf{A} + \mathbf{b}\theta^* = \mathbf{A_m} \quad \textbf{(32)}$$

$$\mathbf{b_m} = \mathbf{b}q^* \quad \textbf{(33)}$$

Furthermore, it is assumed that q^* may be varied such that the sign of q^* is not altered.

II.1 Stability, and switching function design

The control signal u to the plant is introduced as

$$u = \sigma (\beta \, \mathbf{X} + r) \quad \textbf{(34)}$$

where n dimensional raw feedback vector β, with the elements β_i and scalar σ are adjusted using VS approach by designing switching functions β_i and σ as described in the followings.

Subtracting equation (2) from (1), and using equations (4), (5) and (6), the error equation is obtained as

$$\dot{\mathbf{e}} = \mathbf{A_m e} + \mathbf{b_m} \, (\beta - \theta^*)\mathbf{X} + \mathbf{b_m} (q^{*-1} - \sigma^{-1}) \, u \quad \textbf{(35)}$$

Consider a Lyapunov function of the form as equation (8). Differentiating (8) with respect to time along the trajectory (35) yields

$$\dot{\mathbf{V}} = -\mathbf{e^T Q}_0\mathbf{e} + 2\,\mathbf{e^T P b_m} \, (\beta - \theta^*) \, \mathbf{X} + 2\,\mathbf{e^T P b_m} \, (q^{*-1} - \sigma^{-1}) \, u \quad \textbf{(36)}$$

If the control u in equation (34) be written as

$$u^o = \beta \, \mathbf{X} + r \;, \qquad u = \sigma \, u^o \quad \textbf{(37)}$$

and substituting into (36) one can write

$$\dot{\mathbf{V}} = -\mathbf{e^T Q}_0\mathbf{e} + 2\,\mathbf{e^T P b_m} \, (\beta - \theta^*) \, \mathbf{X} + 2\,\mathbf{e^T P b_m} \, (\sigma \, q^{*-1} - 1) \, u^o$$
$$= -\mathbf{e^T Q}_0\mathbf{e} + 2\,\mathbf{b^T P e} \sum_{i=1}^{n} (\beta_i - \theta_i^*) \, \mathbf{x_i} + 2\,\mathbf{e^T P b_m} \, (\sigma \, q^{*-1} - 1) \, u^o \quad \textbf{(38)}$$

Now, introducing the switching functions β_i as in (11), and the switching function σ as

$$\sigma = -\bar{q} \, \mathrm{sgn} \, (\mathbf{b_m^T P e} \, u^o) \, \mathrm{sgn}(q^*) , \qquad \bar{q} > \left| q^* \right| \quad \textbf{(39)}$$

and substitute into (38), yields

$$\dot{\mathbf{V}} = -\mathbf{e^T Q}_0\mathbf{e} - 2 \sum_{i=1}^{n} [\bar{\theta}_i \left| \mathbf{b_m}^\mathbf{T} \mathbf{Pe \, x_i} \right| + \theta_i^* \mathbf{b_m}^\mathbf{T} \mathbf{Pe \, x_i}]$$
$$- 2 \; [\, \bar{q} \left| q^* \right|^{-1} \left| \mathbf{b_m^T P e} \, u^o \right| + \mathbf{b_m^T P e} \, u^o \, \mathrm{sgn}(q^*)] \quad \textbf{(40)}$$

The terms in the summation are always positive, and the last term is negative, therefore $\dot{\mathbf{V}} < 0$ and regarding (8) it can be concluded that $\|\mathbf{e}\|$ decreases at least exponentially.

II.2 Existence of sliding mode

In the followings it is shown that the surface

$$\mathbf{S} = \mathbf{b_m^T P e} = 0 \quad \textbf{(41)}$$

is always a sliding surface for the system. Let examine the following reaching conditions [15, 9].

$$\mathbf{S\dot{S}} < 0 , \qquad \left| \dot{\mathbf{S}} \right| > 0 \quad \textbf{(42)}$$

Considering equations (35) and (37), we have

$$\mathbf{S\dot{S}} = \mathbf{S \, b_m^T P A_m e} + \mathbf{S \, b_m^T P b} \, (\beta - \theta^*) \, \mathbf{X} + \mathbf{S \, b_m^T p b_m} (q^{*-1} - \sigma^{-1}) \, u$$
$$= \mathbf{S \, b_m^T P A_m e} + \mathbf{S \, b_m^T P b} \, (\beta - \theta^*) \, \mathbf{X} + \mathbf{S \, b_m^T p b_m} (\sigma \, q^{*-1} - 1) \, u^o$$
$$= \mathbf{S \, b^T P A_m e} + \mathbf{S \, b^T P b} \sum_{i=1}^{n} (\beta_i - \theta_i^*) \, \mathbf{x_i} + \mathbf{S \, b_m^T p b_m} (\sigma \, q^{*-1} - 1) \, u^o$$

and consider to equations (11)and (39), one can write

$$\mathbf{S\dot{S}} = \mathbf{S \, (b_m^T P A_m) e}$$
$$+ \mathbf{S \, b_m^T P b_m} \sum_{i=1}^{n} \left[-\bar{\theta}_i \, \mathrm{sgn} \, (\mathbf{b_m^T P e \, x_i}) - \theta_i^* \right] \mathbf{x_i}$$
$$+ 2\,\mathbf{b_m^T p b_m} \, \{[-\bar{q} \, q^{*-1} \, \mathrm{sgn}(\mathbf{b_m^T P e} \, u^o) \, \mathrm{sgn}(q^*) - 1] \, u^o\}$$
$$= \mathbf{S \, (b_m^T P A_m) e}$$
$$- \left| \mathbf{S} \right| \mathbf{b_m^T P b_m} \sum_{i=1}^{n} \left[\left| \bar{\theta}_i + \theta_i^* \, \mathrm{sgn}(\mathbf{b_m^T P e \, x_i}) \right| \right] \left| \mathbf{x_i} \right|$$
$$- \left| \mathbf{S} \right| \mathbf{b_m^T P b_m} \sum_{i=1}^{n} \left[\left| \bar{q} \, q^{*-1} \right| + \mathrm{sgn}(\mathbf{b_m^T P e}) \right] \left| u^o \right|$$
$$< \left| \mathbf{S} \right| \left\{ \varepsilon_1 \|\mathbf{e}\| - \mathbf{b_m^T P b_m} \sum_{i=1}^{n} \left[(\bar{\theta}_i - \left| \theta_i^* \right|) \right] \left| \mathbf{x_i} \right| \right\}$$
$$- \left| \mathbf{S} \right| [\mathbf{b_m^T P b_m} (\left| \bar{q} \, q^{*-1} \right| - 1) \left| u^o \right|] \quad \textbf{(43)}$$

where ε_1 is a positive constant. Defining a new vector

$$\mathbf{Z^T} = [\mathbf{X^T} \;\; u^o] \quad \textbf{(44)}$$

Since $\|\mathbf{e}\|$ exponentially tends to zero, it can be concluded that if we have

$$\|\mathbf{Z}(t)\| > \lambda > 0 , \quad \forall t > t_0 \quad \textbf{(45)}$$

then there exists $T > t_0$ such that

$$\mathbf{S\dot{S}} < 0 , \quad \forall t \geq T \quad \textbf{(46)}$$

Condition $\left| \dot{\mathbf{S}} \right| > 0$ in the relation (42) is considered here. One can writes

$$\dot{\mathbf{S}} = \mathbf{b_m^T P \dot{e}} = \mathbf{b_m^T P A_m e} + \mathbf{b_m^T P b_m} \, (\beta - \theta^*) \, \mathbf{X} + \mathbf{b_m^T P b_m} \, (q^{*-1} - \sigma^{-1}) \, u$$
$$= \mathbf{b_m^T P A_m e} + \mathbf{b_m^T P b_m} \, (\beta - \theta^*) \, \mathbf{X} + \mathbf{b_m^T P b_m} \, (\sigma \, q^{*-1} - 1) \, u^o$$
$$= \mathbf{b_m^T P A_m e} - \mathbf{b_m^T P b_m} \sum_{i=1}^{n} \left[\bar{\theta}_i \, \mathrm{sgn} \, (\mathbf{b_m^T P e \, x_i}) - \theta_i^* \right] \mathbf{x_i} + \mathbf{b_m^T P b_m} \, (\sigma \, q^{*-1} - 1) \, u^o \quad \textbf{(47)}$$

since $\|\mathbf{e}\|$ exponentially tends to zero, and consider to equation (45), it can be seen at least one of the terms in the summation or the last term is nonzero and we have

$$|\dot{\mathbf{S}}| > 0, \quad \text{for } \mathbf{t} > \mathbf{T} \tag{48}$$

Relations (42) and (48) mean that for all t >T, the surface $\mathbf{S} = \mathbf{b}_\mathbf{m}^\mathbf{T}\mathbf{Pe} = 0$ is a sliding surface.

II.3 stability in the presence of bounded disturbance

If a bounded disturbance w(t), acts on the plant input, the error equation (35) becomes

$$\dot{\mathbf{e}} = \mathbf{A_m e} + \mathbf{b_m}\,(\beta - \theta^*)\mathbf{X} + \mathbf{b_m}\,(q^{*-1} - \sigma^{-1})\,u + \mathbf{b}\,w \tag{49}$$

In the same manner as used in section (II.1) one can writes

$$\dot{\mathbf{V}} = -\mathbf{e^T Q_0 e} - 2\ \textstyle\sum_{i=1}^{n} [\overline{\theta_i}\ \left|\mathbf{b_m^T Pe\, x_i}\right| + \theta_i^*\ \mathbf{b_m^T Pe\, x_i}]$$
$$-2\ [\ \overline{q}\,|q^*|^{-1}\ \left|\mathbf{b_m^T Pe}\,u^o\right| + \mathbf{b_m^T Pe}\,u^o\ \mathrm{sgn}(q^*)]$$
$$+2\mathbf{e^T Pb}\,w \tag{50}$$

Consider to (23), and defining $\Delta\sigma_\mathbf{m}$ as

$$\Delta\sigma_\mathbf{m} = \min(\overline{\sigma} - |\sigma^*|)$$

(51)

one can concluded that

$$\dot{\mathbf{V}} < -\rho_1\,\|\mathbf{e}\|^2 - \rho_2\,\Delta\theta_\mathbf{m}\,\|\mathbf{X}\|\left|\mathbf{b_m^T Pe}\right| + \rho_3\,\|\mathbf{e}\|\,\overline{w}$$
$$-\rho_4\,\Delta\sigma_\mathbf{m}\,|u|\left|\mathbf{b_m^T Pe}\right| \tag{52}$$

where ρ_1 , ρ_2 , ρ_3 , and ρ_4 are positive constants.

Relation (52) means that e should has a residual set as

$$\|\mathbf{e}\| < \rho_5\,\overline{w} \tag{53}$$

where ρ_5 is a positive constant.

II.4 Existence of sliding mode in the presence of bounded disturbance

As mentioned in section (II.3), the error equation in the presence of bounded disturbance w(t), will be in the form of equation (49).

In the same manner as followed in section (II.2) one can writes

$$\mathbf{S\dot{S}} = \mathbf{S\,b_m^T PA_m e} + \mathbf{S\,b_m^T Pb}\,(\beta - \theta^*)\,\mathbf{X} + \mathbf{S\,b_m^T pb}\,w$$
$$+\mathbf{Sb_m^T Pb_m}\,(q^{*-1} - \sigma^{-1})\,u + \mathbf{S\,b_m^T pb}\,w$$
$$< |\mathbf{S}|\left\{\varepsilon_1\|\mathbf{e}\| - \mathbf{b_m^T Pb_m}\sum_{i=1}^{n}\left[(\overline{\theta_i} - |\theta_i^*|)\right]|\mathbf{x_i}| + \mathbf{b_m^T pb}\,w\right\}$$
$$-|\mathbf{S}|\,[\mathbf{b_m^T Pb_m}\,(\left|\overline{q}\,q^{*-1}\right| - 1)|u^o|] \tag{54}$$

where ε_1 is a positive constant. Consider to equation (44), and defining $\Delta\xi_\mathbf{m}$ as

$$\Delta\xi_\mathbf{m} = \min[\Delta\theta_\mathbf{m}, (\left|\overline{q}\,q^{*-1}\right| - 1)] \tag{55}$$

where defined by (23). Hence one can writes

$$\mathbf{S\dot{S}} < |\mathbf{S}|\left\{\varepsilon_1\|\mathbf{e}\| - \mathbf{b_m^T Pb_m}\,\Delta\xi_\mathbf{m}\,\|Z\| + \mathbf{b_m^T pb}\,w\right\} \tag{56}$$

since $\|\mathbf{e}\|$ exponentially tends to a residual domain specified by relation (53), it can be concluded that if we have

$$\|\mathbf{Z(t)}\| > \lambda' > \frac{\delta'}{\mathbf{b_m^T pb_m}}\,\frac{w}{\Delta\xi_\mathbf{m}}, \quad \forall t > t_0 \tag{57}$$

where δ' is a positive constant, then there exists $T>t_0$ such that

$$\mathbf{S\dot{S}} < 0, \quad \forall t \ge T \tag{58}$$

Condition $|\dot{\mathbf{S}}| > 0$ can be proved in the same manner as applied for equations (47) and (48).

II.5 Average control

When sliding mode is occurred, actually the average control u_{av} can be used [ ], which is the output of first order filter

$$\tau\,u_{av} + u_{av} = u \tag{59}$$

where time constant τ is sufficiently small.

The block diagram is presented in figure 2.

4. SIMULATION RESULTS

In this section, simulation results are presented to show the performance of the proposed schemes and comparing these with the conventional schemes. The example consists of the cases I and II, with parameter variations and input disturbance.

Example

Consider a single input time-varying system described by the equation

$$\ddot{\mathbf{x}} - \dot{\mathbf{x}}\cos(t) - \mathbf{x}\sin(t) = g\,u \tag{60}$$

or in state space

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{b}u \tag{61}$$

where $\mathbf{X} = [\mathbf{x_1}\ \ \mathbf{x_2}]^\mathbf{T},\ \mathbf{x_1} = \mathbf{x},\ \mathbf{x_2} = \dot{\mathbf{x}}_1$,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ \sin(t) & \cos(t) \end{bmatrix},\quad \mathbf{b} = \begin{bmatrix} 0 \\ g \end{bmatrix} \tag{62}$$

but actually we know that

$$a_{11} = 0,\ a_{12} = 1,\ -1 \le a_{21} \le +1,\ -1 \le a_{22} \le +1 \tag{63}$$

Case I. Parameter g is known (g = 2, e.g. vector b is known).

Choose a reference model described by equation

$$\ddot{\mathbf{x}}_\mathbf{m} + 2\,\dot{\mathbf{x}}_\mathbf{m} + 2\,\mathbf{x} = r \tag{64}$$

or in the state space

$$\dot{\mathbf{X}}_\mathbf{m} = \mathbf{A_m X_m} + \mathbf{b_m}r \tag{65}$$

where $\mathbf{X_m} = [\mathbf{x_{m1}}\ \ \mathbf{x_{m2}}]^\mathbf{T},\ \mathbf{x_{m1}} = \mathbf{x_m},\ \mathbf{x_{m2}} = \dot{\mathbf{x}}_\mathbf{m1}$,

$$\mathbf{A_m} = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix},\quad \mathbf{b_m} = \begin{bmatrix} 0 \\ g \end{bmatrix} \tag{66}$$

hence, from (4) and (5) it can be concluded that

$$\theta_1^* = \frac{-2 - \sin(t)}{g},\quad \theta_2^* = \frac{-2 - \cos(t)}{g},\quad q^* = \frac{1}{g} \tag{67}$$

therefore, $\mathrm{Max}\,|\theta_1^*| = \frac{3}{2}$, $\mathrm{Max}\,|\theta_2^*| = \frac{3}{2}$, and with regard to equation (11) we can choose $\overline{\theta} = [\overline{\theta}_1\ \ \overline{\theta}_2] = [1.6\ \ 1.6]$. Thus the VS-MRA Controller can be designed using equations (11) and (6) for unfiltered, or equations (11) and (46-47) for filtered control. System was simulated, responses was compared with the responses of system with conventional adaptation law as $\dot{\theta} = -\mathbf{b^T pe\,X^T}$, and control law $u = \theta\,\mathbf{X} + q^* r$. The input signal was $r = 1 + 0.5\sin(4\pi\,t)$, and initial conditions was $\mathbf{X}(0) = [1 \;\; 1]^{\mathbf{T}}$, and $\mathbf{X_m}(0) = [0 \;\; 0]^{\mathbf{T}}$. Responses are presented in figure 3, for filtered control without disturbance, and in figure 4, for filtered control in the presence of a disturbance as $d = 1 + \sin(\pi\, t)$.

Case II. Parameter g is unknown (actually $g = 2 + \sin(t)$, but we only know that $1 \le g \le 3$).

Choose a reference model described by equations (65) and (66), hence from equations (32) and (33) it can be concluded that $\theta_1^* = -2 - \sin(t)$, $\theta_2^* = -2 - \cos(t)$, $q^* = \frac{1}{2 + \sin(t)}$ therefore $\mathrm{Max}\left|\theta_1^*\right| = 3$, $\mathrm{Max}\left|\theta_2^*\right| = 3$, and $\mathrm{Max}\left|q^*\right| = 1$, and consider to equations (11) and (39) we can choose $\overline{\theta} = [\overline{\theta}_1 \;\; \overline{\theta}_2] = [3.1 \;\; 3.1]$ and $\overline{q} = 1.1$. Thus the VS-MRA Controller can be designed using equations (11) and (6) for unfiltered, or equations (11), (39), (34) and (59) for filtered control. System was simulated, and responses was compared with the responses of system with conventional adaptation laws as $\dot{\theta} = -\mathbf{b_m^T pe\, X^T}$, and $q = -q\, \mathbf{b_m^T pe}\, u\, q$, and control law $u = q\,(\theta\, \mathbf{X} + r)$. The input signal was $r = 1 + 0.5\sin(4\pi\, t)$, and initial conditions was $\mathbf{X}(0) = [1 \;\; 1]^{\mathbf{T}}$, and $\mathbf{X_m}(0) = [0 \;\; 0]^{\mathbf{T}}$. Responses are presented in figure 5, for filtered control without disturbance, and in figure 6, for filtered control in the presence of a disturbance as $d = 1 + \sin(\pi\, t)$. In this case, the conventional controller is unstable.

5. CONCLUSIONS

A variable structure model reference adaptive controller has been proposed for SIMO systems. Structure of the switching functions was designed based on exponential stability requirements. Magnitude of the switching functions then can be determined using reaching conditions of sliding mode. For the case I, relation (5) is used to determine the bounds on elements $\theta_{\mathbf{i}}^*$, and these elements must be limited. For the case II, relation (31) and (32) are used to determine the bounds on elements $\theta_{\mathbf{i}}^*$ and q^*, and these elements must be limited. Then the elements $\theta_{\mathbf{i}}^*$ can be chosen regarding the requirements for existence the sliding mode.

This controller has some significant advantages compared to the conventional model reference adaptive controller. Global exponential stability is proved without requirements on persistence of excitation. Specially in the case II, stability of the conventional controller is unreliable because related Lyapunov function is not unbounded in this case, and only uniform stability (not in the large) is implied, (in the example, this controller was unstable), whereas the proposed controller has excellent performance similar to the case I. Then it was shown that, it is always possible to introduce a sliding mode into the system. Transient behavior was analyzed and showed perfect model following at a finite time. Insensitivity with respect to input disturbances was investigated and showed preference to the conventional schemes. Simulation was presented to clear the theoretical results.

REFERENCES

[1] Ambrosino, G., Celentano, G., and Garofalo, F., (1984), " Variable structure model reference adaptive control systems," Int. J. Cont., Vol/ 39, No. 6.

[2] Chiang-ju, C., King-Chuan, S., A-Chen, W., and Li-Chen, F., (1996), " A Robust MRAC Using Variable Structure Design for Multi variable plants," Automatica, 32,no. 6, pp. 833-848.

[3] Hsu, L., Araujo, A.D., and Costa, R.R., (1994), " Analysis and design of I/O based variable structure adaptive control," IEEE Trans. Automat. Control, pp. 4-21.

[4]] Hsu, L., (1990), " Variable structure model reference adaptive control using only I/O measurement: General case," IEEE Trans. Automat. Control, pp. 1238-1243.

[5] Hung, J.Y., Gao, W., Hung, J.C., (1993), " Variable structure control: A survey," IEEE Trans. On Indust. Elect. , Vol. 40, no. 1, pp. 2-22.

[6] Itkis, U., (1976)," control systems of variable structure," John Wiley & sons Inc., New york.

[7] Narandra, K.S., Annaswamy, A.M., (1989), " Stable Adaptive Systems," Prentice-Hall Int. Inc..

[8] Narandra, K.S., Lin, Y.H., Valavani, L.S., (1980), " Stable Adaptive controller design, part 2: proof of stability," IEEE Trans. Automat. Control, Vol. AC-25, pp. 440-448.

[9] Narandra, K.S., Kudva, P., (1974), " Stable Adaptive Schemes for system identification and control, part I & II," IEEE Trans. On sys., Man and cyber., SMC-4: pp. 542-560.

[10] Utkin, V.I., (1978), " Sliding modes and their application in variable structure systems," New york: MIR.

[11] Utkin, V.I., (1977), " Variable structure systems with sliding modes," IEEE Trans. Automat. Control, Vol. AC-22, pp. 212.

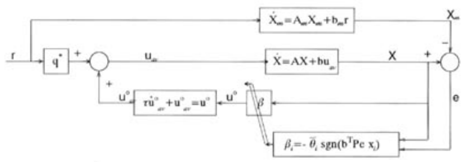

Fig. 1 The block diagram of the system in the case I .

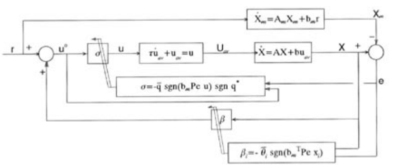

Fig. 2 The block diagram of the system in the case II .

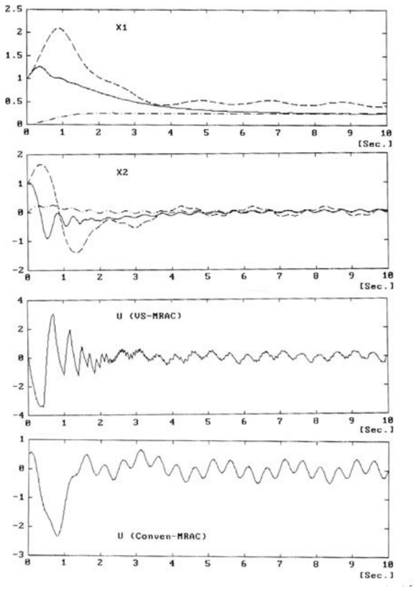

Fig. 3 case I, filtered control without disturbance.

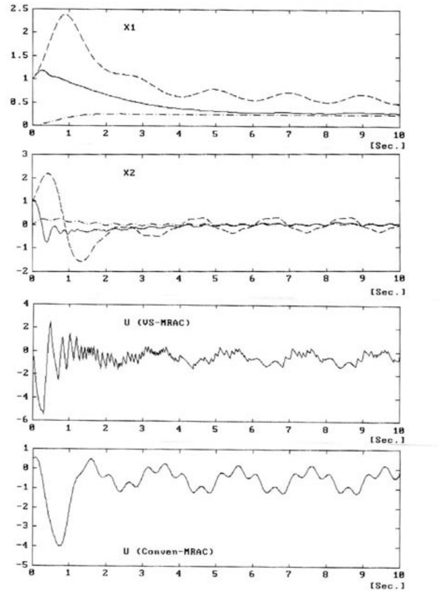

Fig. 4 case I, filtered control in the presence of a disturbance as $d = 1 + \sin(\pi t)$.

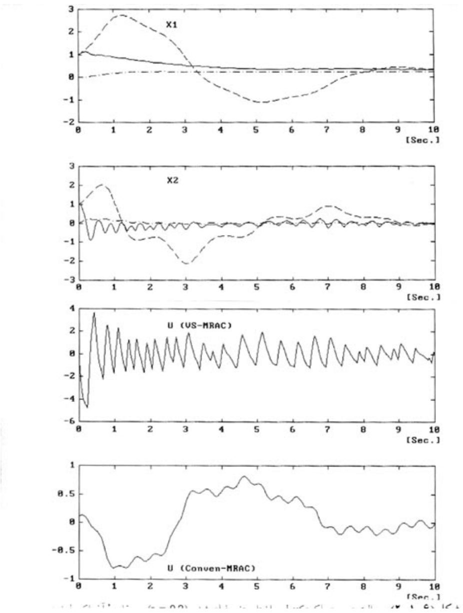

Fig. 5 case II, filtered control without disturbance.

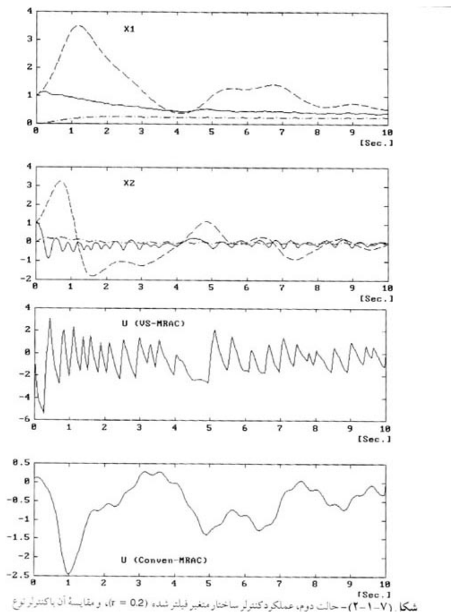

Fig. 6 case II, filtered control in the presence of a disturbance as $d = 1 + \sin(\pi t)$.