A Full Order Sliding Mode Tracking Controller For A Class of Uncertain Dynamical System

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Abstract

This paper presents the development of a full order sliding mode controller for tracking problem of a class of uncertain dynamical system, in particular, the direct drive robot manipulators. By treating the arm as an uncertain system represented by its nominal and bounded parametric uncertainties, a new robust full-order sliding mode tracking controller is derived such that the actual trajectory tracks the desired trajectory as closely as possible despite the non-linearities and input couplings present in the system. A proportional-integral sliding surface is chosen to ensure the stability of overall dynamics during the entire period i.e. the reaching phase and the sliding phase. Application to a three DOF direct drive robot manipulator is considered.

1 Introduction

The concept of robot directly driven by electrical motors eliminate the problems associated with gear backlash as well as reducing the friction significantly. The construction is much stiffer than the conventional robot manipulator with gearing, wear and tear is not a problem, and the arm is more reliable and easy to maintain due to its simplicity. In direct-drive arm, the complex dynamics of the arm are directly reflected to the motor axes. Therefore, the varying inertia effect and the effects of the coupling and non-linear torques will have a substantial dynamical effect. Moreover, large inductance in typically used direct-drive actuators, such as Brush-less DC Motors (BLDCM) and Variable Reluctance Motors (VRM), will have a direct influence on the overall dynamics of the direct drive arm.

Variable structure control with Sliding Mode Control (SMC) is a powerful technique that has been successfully applied for the control of the numerous nonlinear systems [1], [2], [3]. The design philosophy behind the SMC is to design a switching surface and followed by the design of a high-speed switching control law to drive the nonlinear plant's state trajectory onto the surface such that the system dynamics is strictly determined by the dynamics of the sliding surfaces and hence insensitive to parameter variations and system disturbances.

In this paper, a robust tracking controller capable of withstanding the expected variations and uncertainties in the direct-drive robot system is presented. A complete model of the direct-drive robot manipulator is used in designing the controller. It is assumed that the upper bounds on the non-linearities and uncertainties present in the system are available. On the basis of the SMC theory, a Full-Order Sliding Mode Control (FOSMC) controller for robust tracking of direct-drive robot manipulators is proposed. The performance of the proposed control law is evaluated by means of computer simulation studies using a three DOF revolute direct-drive robot manipulator actuated by the BLDCMs.

2 Problem Formulation

The integrated dynamic model of an N DOF direct-drive revolute manipulator can be represented in state-space form as [4]:

$$\dot{X}(t) = A(X,\xi,t)X(t) + B(X,\xi,t)U(t) \tag{1}$$

where

$$\left.\begin{aligned} &X(t) = [X_1^T(t), \quad X_2^T(t), \quad \ldots, \quad X_N^T(t)]^T \\ &X_i(t) = [\theta_i(t), \dot{\theta}_i(t), \ddot{\theta}_i(t)]^T \\ &U(t) = [U_1(t), \quad U_2(t), \quad \ldots, \quad U_m(t)]^T \\ &X_i(t) \in \Re^3;\ i \in \Im \end{aligned}\right\} \tag{2}$$

ξ is a vector of the parameters of the mechanism, such as payload, which belong to the finite region of allowable parameter values Ξ, that is $\xi \in \Xi$. $\theta, \dot{\theta}$ and $\ddot{\theta}$ are the joint angle, velocity and acceleration, respectively.

The dynamics (1) can be transformed into an uncertain dynamical system as follows:

$$\dot{X}(t) = [A + \Delta A(*)]X(t) + [B + \Delta B(*)]U(t) \tag{3}$$

where * represents the term (X,ξ,t) for simplicity, while A and B are nominal constant matrices. The elements of the $\Delta A(*)$ and $\Delta B(*)$ matrices, denoted by $\Delta a_{ij}(*)$ and $\Delta b_{ij}(*)$, may be considered as uncertainties that belong to uncertainty bounding sets □ and □. The uncertainty bounding sets may be defined as follows

$$\square \triangleq \{\Delta a_{ij}(*); \forall i,j \in \Im, \forall j \in \Im \mid -r_{ij} \le \Delta a_{ij}(*) \le r_{ij}\} \tag{4}$$

$$\square \triangleq \{\Delta b_{ij}(*); \forall i,j \in \Im, \forall j \in \Im \mid -w_{ij} \le \Delta b_{ij}(*) \le w_{ij}\} \tag{5}$$

where the values of the constants r_{ij} and w_{ij} are assumed known.

Let a continuous function $X_d(t) \in \Re^{3N}$ be the desired state trajectory, where $X_d(t)$ is defined as:

$$\left.\begin{aligned} X_d(t) &= [X_{d1}^T(t),\ X_{d2}^T(t),\ \ldots,\ X_{dN}^T(t)]^T \\ X_{di}(t) &= [\theta_{di}(t), \dot{\theta}_{di}(t), \ddot{\theta}_{di}(t)]^T \\ X_{di}(t) &\in \Re^3;\ i \in \Im \end{aligned}\right\} \quad (6)$$

and define the tracking error, $Z(t)$ as

$$Z(t) = X(t) - X_d(t) \quad (7)$$

In this study, the following assumptions are made:

1. There exist continuous functions $H(*) \in \Re^{3\times 3N}$ and $E(*) \in \Re^{3\times 3N}$ such that for all $X \in \Re^{3N}$ and all t:

$$\Delta A(*) = BH(*)\ ; \quad \|H(*)\| \le \alpha \quad (8)$$

$$\Delta B(*) = BE(*)\ ; \quad \|E(*)\| \le \beta \quad (9)$$

2. There exist a Lebesgue function $\Omega(t) \in \Re^{m\times 1}$, which is integrable on bounded interval such that

$$\dot{X}_d(t) = AX_d(t) + B\Omega(t) \quad (10)$$

Assumption 1) assures that all uncertain portions $\Delta A(*)$ and $\Delta B(*)$ are contained in the range space of the nominal input matrix B. This structural condition on the uncertainty is termed matching condition [5]. The continuous functions $H(*)$ and $E(*)$ exist if and only if the following rank conditions are satisfied:

$$rank\ [B] = rank\ [B, \Delta A(*)] \quad (11)$$

$$rank\ [B] = rank\ [B, \Delta B(*)] \quad (12)$$

The rank conditions (11) and (12) are essentially related to the structure of the matrices B, $\Delta A(*)$ and $\Delta B(*)$, and not to the values of their elements. These conditions impose constraints on the structure of the system matrix uncertainty $\Delta A(*)$, and the input matrix uncertainty $\Delta B(*)$ to lie within the range space of the input matrix B. This assumption is needed so that the control, $U(t)$, which enters the system through B may compensate the uncertainty in the system. On the other hand, assumption 2) is needed to ensure asymptotic tracking of controlled plant.

In view of (6) - (9), equation (3) can be written as

$$\dot{Z}(t) = [A + BH(t)]Z(t) + BH(t)X_d(t) - B\Omega(t) + [B + BE(t)]U(t) \quad (13)$$

Define the sliding surface $s(t) \in \Re^{m\times 1}$ as

$$s(t) = CZ(t) - \int_0^t [CA + CBK]Z(\tau)d\tau \quad (14)$$

where $C \in R^{m\times 3N}$ and $K \in R^{m\times 3N}$ are constant matrices. The structure of matrix C is as follows:

$$C = diag[c_1\ \ c_2\ \ \cdots\ \ c_{nj}] \quad (15)$$

The matrix C is chosen such that $CB \in R^{m\times m}$ is non-singular and the matrix K satisfies

$$\lambda_{\max}(A + BK) < 0 \quad (16)$$

The condition (16) guarantees that all the desired poles are located in the left half of the s-plane to ensure stability. The gain matrix K may be computed using the conventional pole placement technique with the pre-specified poles location.

The control problem then is to design a controller using the sliding surface (14) such that the system state trajectory $X(t)$ tracks the desired state trajectory $X_d(t)$ as closely as possible for all t in spite of the uncertainties and non-linearities present in the system. In view of the error space, the tracking problem has become the problem of stabilizing the error system (13).

3 Tracking Controller Design

Differentiating (13) gives

$$\dot{s}(t) = C\dot{Z}(t) - [CA + CBK]Z(t) \quad (17)$$

Substituting (12) into (16) and equating it to zero gives the equivalent control, $U_{eq}(t)$, a mathematically derived tool for the analysis of a sliding motion. It can be shown that the equivalent control, $U_{eq}(t)$, is given by

$$U_{eq}(t) = -[I_n + E(t)]^{-1}\{(H(t) - K)Z(t) - \Omega(t) + H(t)X_d(t)\} \quad (18)$$

The system dynamics during sliding mode can be found by substituting the equivalent control (18) into the system error dynamics (13):

$$\dot{Z}(t) = [A + BK]Z(t) \quad (19)$$

Hence if the matching condition is satisfied (conditions (11) and (12) hold), the system error dynamics during sliding mode are independent of the system uncertainties and couplings between the inputs, and, insensitive to the parameter variations, and may be shaped up through a proper selection of the desired closed loop poles locations.

The manifold (13) is asymptotically stable in the large, if the following hitting condition is held [6]:

$$((s^T(t)\dot{s}(t)) / \|s(t)\|) < 0 \quad (20)$$

As a proof, let the positive definite function be

$$V(t) = \|s(t)\| \quad (21)$$

Differentiating (21) with respect to time, t yields

$$\dot{V}(t) = (s^T(t)\dot{s}(t)) / \|s(t)\| \quad (22)$$

Following the Lyapunov stability theory, if (20) holds, then $s(t)$ is asymptotically stable in the large.

Theorem 4.1: The hitting condition (20) of the manifold (14) is satisfied if the control $U(t)$ of system (3) is given by :

$$U(t) = -(CB)^{-1}[\gamma_1\|Z(t)\| + \gamma_2\|X_d(t)\| + \gamma_3\|\Omega(t)\|]SGN(s(t)) + \Omega(t) \quad (23)$$

where

$$\gamma_1 > (\alpha\|CB\| + \|CBK\|)/(1+\beta) \quad (24)$$

$$\gamma_2 > (\alpha\|CB\|)/(1+\beta) \quad (25)$$

$$\gamma_3 > (\beta\|CB\|)/(1+\beta) \quad (26)$$

Proof: See [7].

The conditions imposed by (24), (25) and (26) not only guarantee that the hitting condition (20) is met, but it also assure that based on the Lyapunov theory, the system dynamics is stable in the large.

The sign function $SGN(s(t))$ in (23) is an $m\times 1$ vector of discontinuous functions and may gives rise to the input chattering and direct application of such control to the plant may be impractical. To eliminate the control input chattering, each element of the discontinuous function vector $SGN(s(t))$ may be replaced by a proper continuous function [8] - [10] as follows:

$$S_{\delta_i}(t) = \frac{s_i(t)}{|s_i(t)| + \delta_i} \tag{28}$$

where δ_i is a positive constant.

4 Simulation Results

Consider a three DOF revolute direct-drive robot manipulator actuated by BLDCM motors shown in Fig. 1. An integrated model comprising the mechanical part of the robot and the actuator dynamics have been derived and used in the simulations. The model is highly non-linear and coupled, taking into account the contributions of the actuator dynamics, as well as the inertias, the Coriolis forces, the centrifugal forces and the gravitational forces present in the mechanical part of the robot arm [4]. These equations were used in the simulation to represent a real direct-drive robot manipulator without any approximation and simplification of the highly non-linear and coupled system. For the purposes of deriving the FOSMC tracking controller, the nominal matrices A and B, as well as the bounds on the non-zero elements of the matrices $\Delta A(*)$ and $\Delta B(*)$ in (3) have been calculated based on the given range of the payload, joint angles and velocities. The nominal matrices are as in equation (29), while the bounds on the non-zero elements are listed in the Appendix.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -21.08 & -8.78 & 15.19 & 28.05 & 3.23 & 0.03 & 13.78 & 0.99 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 49.56 & 4.82 & -112.51 & -6.52 & -8.57 & 2.60 & 2.27 & -0.62 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -0.18 & -0.14 & 132.64 & -32.64 & 0.99 & -11.44 & -18.08 & -7.30 \end{bmatrix} ; \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 20.91 & -4.56 & 0.16 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -5.70 & 37.58 & -39.11 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.23 & -44.67 & 175.98 \end{bmatrix} \tag{29}$$

The controller is required to track a reference trajectory:

$$\theta_{di}(t) = \begin{cases} \theta_i(0) + \dfrac{\Delta_i}{2\pi}[\dfrac{2\pi t}{\tau} - \sin(\dfrac{2\pi t}{\tau})], & 0 \le t \le \tau \\ \theta_i(\tau), & \tau \le t \end{cases} \tag{30}$$

where $\Delta_i = \theta_i(\tau) - \theta_i(0), i = 1,2,3.$ The input trajectory is set to start from $[-0.8 \;\; -1.5 \;\; -0.5]^T$ to $[1.0 \;\; 0.2 \;\; 1.2]^T$ radians in 2 seconds.

Using (8) and (9), the bounds of $H(*)$ and $E(*)$ may be computed as follows:

$$\|H(*)\| \le \alpha = 5.9874 \; ; \; \|E(*)\| \le \beta = 0.6200 \tag{31}$$

Define the gain K as follows:

$$K = \begin{bmatrix} 0.01 & -0.59 & -0.26 & 0.07 & 1.27 & 0.07 & 0 & 0.67 & -0.01 \\ 0 & 1.65 & 0.09 & -2.99 & -0.21 & -0.21 & 0 & 0.06 & -0.11 \\ 0 & 0.42 & 0.02 & 0 & -0.24 & -0.06 & -0.07 & -0.08 & -0.06 \end{bmatrix} \tag{32}$$

such that the closed loop poles are:

$$\begin{aligned} &\text{Joint 1:} \quad \lambda_1 = \{-0.3, -0.31, -3.0) \\ &\text{Joint 2:} \quad \lambda_2 = \{-0.3, -0.31, -3.0) \\ &\text{Joint 3:} \quad \lambda_3 = \{-0.03, -0.031, -0.3) \end{aligned} \tag{33}$$

Let the matrix C be:

$$C = \left[\begin{array}{ccc:ccc:ccc} 2 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 30 & 20 & 1 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 & 0 & 0 & 30 & 20 & 1 \end{array}\right] \tag{34}$$

Using (24), (25) and (26), the controller parameter γ may be computed as follows:

$$\gamma_1 > 76.7962 \; ; \; \gamma_2 > 66.2157 \; ; \; \gamma_3 > 6.8566 \tag{35}$$

The simulation was then carried out using the controller as defined by (23) with the direct drive robot load fixed at its extremity i.e. no load (0 kg) and maximum load (10 kg) with the controller parameters set as follows:

$$\gamma_1 = 350 \quad ; \quad \gamma_2 = 300 \quad ; \quad \gamma_3 = 30 \quad . \tag{36}$$

Fig. 2 shows the tracking responses of each joint of the robot. The tracking performances are good for all joints indicating that the controller is capable of withstanding the non-linearities and uncertainties present in the system. The control input generated switches indiscriminately very fast to ensure all states are directed toward the sliding surfaces as shown in Fig. 3. To eliminate the input chattering, the simulation was carried again but using the proper continuous functions as defined by (28) with the constants δ_i's are as in Table 1.

Table 1. Continuous Function Constants

	δ_i		
Joint	**1**	**2**	**3**
Set 1	200	100	60
Set 2	1000	550	250
Set 3	2000	800	600

The simulation results for the control inputs and the tracking errors for each joint are shown in Fig. 4 and Fig. 5, respectively. It can be seen from the graphs that the chattering in the control input may be suppressed with a suitable choice of constant δ_i. The value of δ_i should be properly selected since too large values will only make the control input chattering reappear but with a lower frequency. Besides, larger tracking errors may also be noticed at every joints of the robot as can be seen in Fig. 5.

5 Conclusions

In this paper, a full-order Sliding Mode tracking controller is proposed for a three DOF direct drive robot manipulator. It is shown mathematically that the error dynamics during sliding mode is stable and can easily be shaped-up using the conventional pole-placement technique. Beside during the sliding phase, the system stability is also guaranteed during the reaching phase. Results from the simulation shows that the proposed controller is effective and feasible since the tracking error is guaranteed to decrease asymptotically to zero if certain conditions pertaining to the controller parameter are satisfied.

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$-102.79 \quad a_{32}^{1}(*) \le 60.63$; $-14.33 \le a_{33}^{1}(*) \le -3.22$; $-26.86 \le a_{32}^{3}(*) \le -9.31$; $-8.70 \le a_{33}^{3}(*) \le -5.89$

$-129.62 \quad a_{31}^{2}(*) \le -95.39$; $-64.26 \le a_{32}^{2}(*) \le 51.22$ $\quad -4.61 \le a_{31}^{12}(*) \le 34.99$; $17.63 \le a_{32}^{12}(*) \le 38.48$

$-10.40 \le a_{33}^{2}(*) \le -6.74$; $-31.88 \le a_{31}^{3}(*) \le 9.00$; $0.22 \le a_{33}^{12}(*) \le 6.23$; $0.03 \le a_{31}^{13}(*) \le 0.03$

$-0.64 \le a_{32}^{13}(*) \le 28.19$; $-0.52 \le a_{33}^{13}(*) \le 2.50$; $-1.77 \le a_{33}^{23}(*) \le 0.53$; $-0.91 \le a_{32}^{31}(*) \le 0.54$

$8.27 \le \ {}_{32}^{21}(*) \le 90.85$ $\quad 3.37 \le a_{33}^{21}(*) \le 6.26$ $\quad -6.36 \le a_{33}^{31}(*) \le 6.08$; $125.08 \le a_{31}^{32}(*) \le 140.20$

$-1.98 \le a_{31}^{23}(*) \le 7.17$; $-5.82 \le a_{32}^{23}(*) \le 10.36$; $-91.77 \le a_{32}^{32}(*) \le 26.50$; $-1.18 \le a_{33}^{32}(*) \le 3.17$

$15.54 \le \ {}_{3}^{1}(*) \le 26.28$; $35.08 \le b_{3}^{2}(*) \le 40.07$; $-40.27 \le b_{3}^{23}(*) \le -37.95$

$.95 \le b_{3}^{3}(*) \le 179.00$; $-10.82 \le b_{3}^{12}(*) \le 1.70$ $\quad -46.00 \le b_{3}^{32}(*) \le -43.34$

$-0.16 \le b_{3}^{13}(*) \le 0.49$; $-13.52 \le b_{3}^{21}(*) \le 2.19$; $-0.23 \le b_{3}^{31}(*) \le 0.69$

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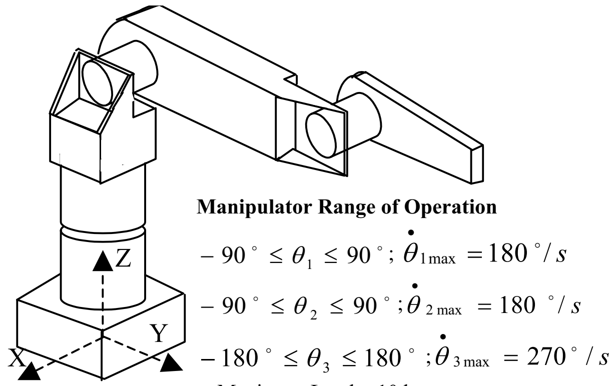
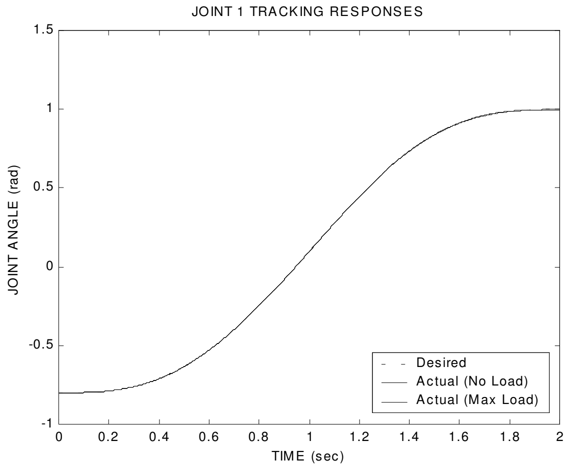
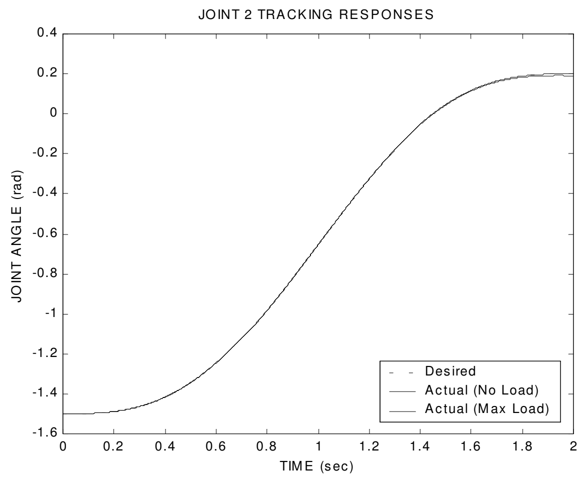


Fig. 1: A Three DOF Revolute Direct Drive Robot Manipulator



a) Joint 1



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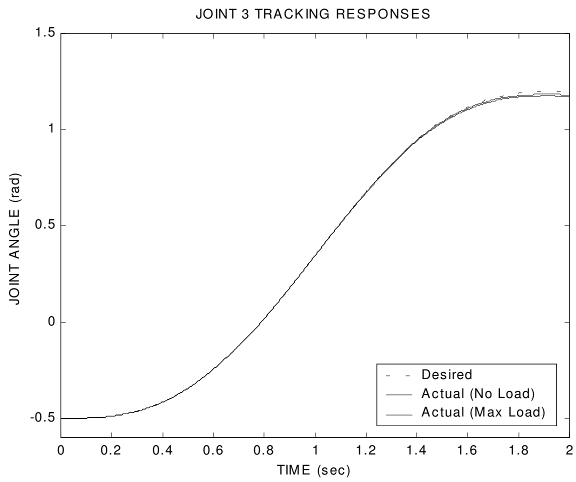
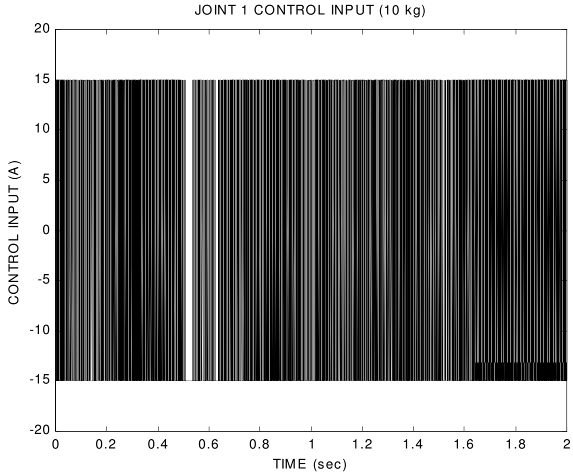
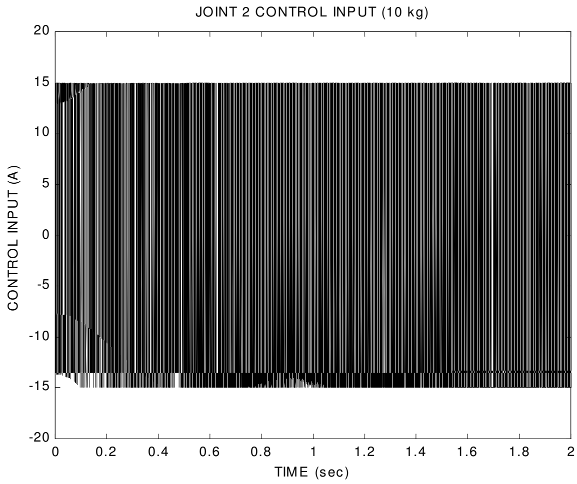


c) Joint 3

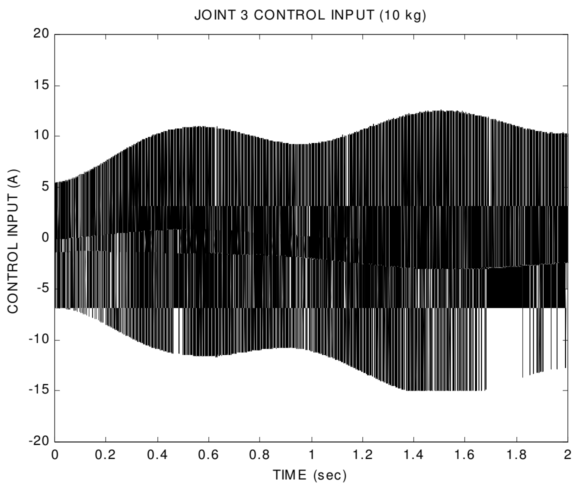
Fig. 2: Tracking Responses.



a) Joint 1

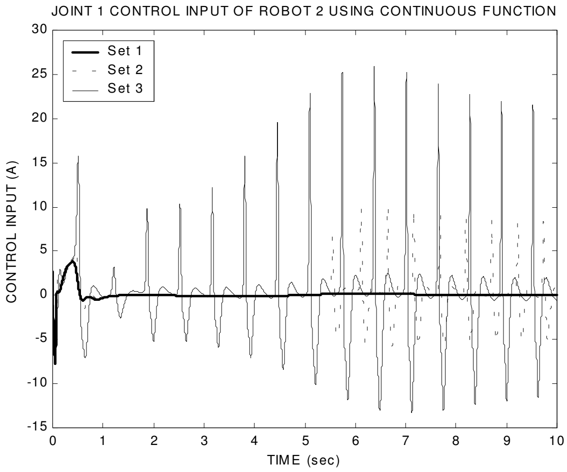


b) Joint 2

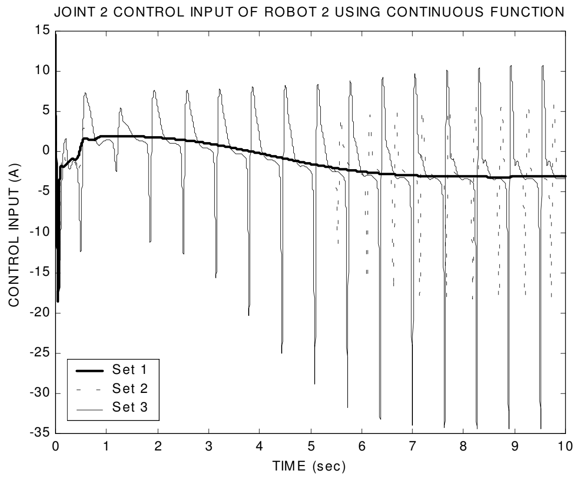


c) Joint 3

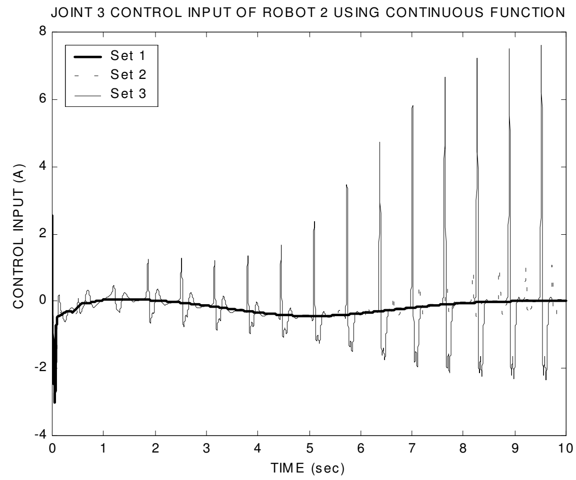
Fig. 3: Control Inputs when the Robot Handling a Maximum Load



a) Joint 1

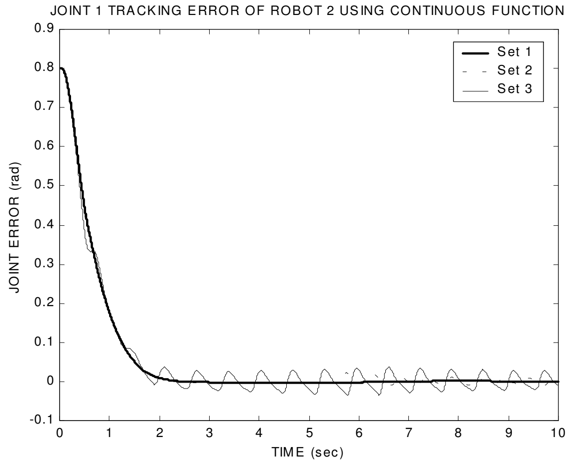


a) Joint 2

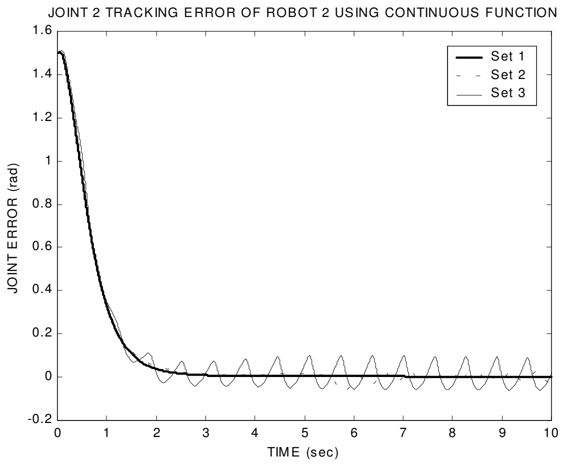


a) Joint 3

Fig. 4: Control Inputs with Continuous Function



a) Joint 1



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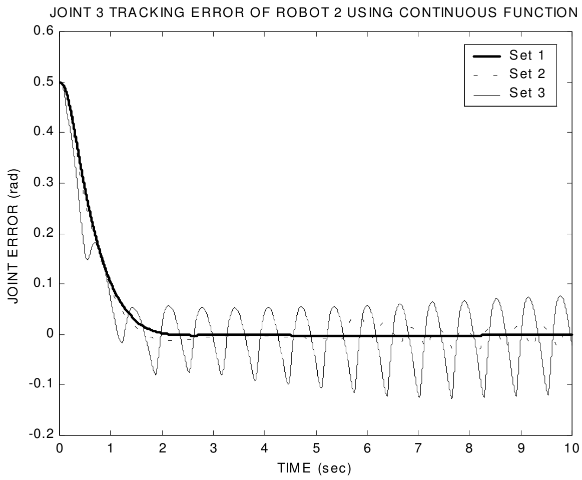

a) Joint 3

Fig. 5: Tracking Errors