Robust observer-based H_∞ controller design for descriptor systems using an LMI

Jong Hae Kim*, Joong Jae Lee*, Seong Joon Ahn*, and Seung Joon Ahn**

* Division of Electronics, Information, and Communication Engineering, Sunmoon University, Asan-si, Chungnam, Korea
(Tel : +82-41-530-2352; E-mail: kjhae@sunmoon.ac.kr)

**Department of Physics/Center for Next Generation Semiconductor Technology, Sunmoon University, Chungnam, Korea

Abstract: This paper considers a robust observer-based H_∞ controller design method for descriptor systems with parameter uncertainties using just one LMI condition. The sufficient condition for the existence of controller and the controller design method are presented by a perfect LMI condition in terms of all variables using singular value decomposition, Schur complement, and change of variables. Therefore, one of the main advantages is that a robust observer-based H_∞ controller is found by solving one LMI condition compared with existing results. Numerical example is given to illustrate the effectiveness of the proposed controller design method.

Keywords: Descriptor systems; observer-based H_∞ controller; parameter uncertainty; linear matrix inequality

1. INTRODUCTION

The problem of H_∞ control for standard state-space has received considerable interest over the last decade. Although H_∞ control theory has been perfectly developed, most of works have been developed based on state space equations. Recently, much attention has been given to the extensions of the results of H_∞ control theory for state-space systems to descriptor systems. State space models are very useful, but the state variables thus introduced do not provide a physical meaning. Hence, the descriptor form is a natural representation of linear dynamical systems, and makes it possible to analyze a larger class of systems than state space equations do[1,2], because state space equations cannot represent algebraic restrictions between state variables and some physical phenomena, like impulse and hysterisis which are important in circuit theory, cannot be treated properly. Wang *et al.*[3] presented necessary and sufficient condition of H_∞ control for singular systems using bounded real lemma based on two generalized Riccati equations. Cobb[4] dealt with the problem of controllability, observability, and duality in singular systems. Lewis[5] treated the optimal control for singular systems.

Recently, the singular H_∞ control problem has been considered by many researchers. Especially, Masubuchi *et al.* [1] considered the H_∞ control problem for singular systems that possibly have impulsive modes and/or *jw* axis zeros in order to eliminate the assumptions. Lin[6] considered on the stability of uncertain linear descriptor systems with state feedback approach. Rehm and Allgöwer[7] treated the H_∞ control problem of high index or even non-regular linear singular systems with norm-bounded uncertainties in the system matrices. Much recently, Fridman and Shaked[8] proposed a descriptor system approach to H_∞ control of linear time-delay system. Although the approach[8] had the effective method to solve H_∞ output feedback problem, their approach was based on the solvability of two LMIs for special dynamic forms. Therefore, there are no papers considering robust observer-based H_∞ control design methods for descriptor systems with parameter uncertainties by a strict LMI technique. If we do not know all states in descriptor systems, the state feedback controller design methods cannot be applied. Although some papers treated H_∞ control problem of descriptor systems, it was not easy to calculate the solutions because of non-convexity of sufficient conditions.
In this paper, one LMI sufficient condition for the existence of robust observer-based H_∞ controller and robust observer-based H_∞ controller design algorithm for descriptor systems with parameter uncertainties are presented using singular value decomposition, change of variable, and LMI approach, Therefore, all solutions including feedback gain and observer gain can be obtained at the same time, because the presented sufficient condition is an LMI form regarding all variables. Compared with existing results, one of the main advantages is that the proposed robust observer-based H_∞ controller is found by solving one LMI condition.

The following notations will be used in this paper.$(\cdot)^T$, $(\cdot)^{-1}$, $deg(\cdot)$, $det(\cdot)$, and $rank(\cdot)$ denote the transpose, inverse, degree, determinant, and rank of a matrix. An identity matrix with proper dimensions is denoted as I. I_r, $x_r(t)$, and $\mathbf{R}^r$ denote an identity matrix with $r\times r$ dimension, $r\times 1$ dimensional vector, and $r\times 1$ dimensional real vector, respectively. * represents the transposed elements in the symmetric positions.

2. PROBLEM FORMULATION

Let us a linear time invariant singular system with parameter uncertainties

$$\begin{aligned} E\dot{x}(t) &= [A+\Delta A(t)]x(t)+[B_1+\Delta B_1(t)]u(t)+B_2w(t) \\ z(t) &= C_1x(t)+D_1u(t) \\ y(t) &= [C_2+\Delta C_2(t)]x(t)+D_2\omega(t) \end{aligned} \tag{1}$$

where, $x(t)\in\mathbf{R}^n$ is the descriptor variable, $z(t)\in\mathbf{R}^l$ is the controlled output variable, $y(t)\in\mathbf{R}^q$ is the measurement output variable, $u(t)\in\mathbf{R}^m$ is the control input variable, $w(t)\in\mathbf{R}^p$ is the disturbance input variable, E is singular matrix with $rank(E)=r\le n$, and all matrices have proper dimensions. And we assume that D_1 has full column rank. Here, parameter uncertainties are defined as follows:

$$\begin{aligned} \Delta A(t) &= N_1F_1(t)H_1 \\ \Delta B_1(t) &= N_2F_2(t)H_2 \\ \Delta C_2(t) &= N_3F_3(t)H_3 \end{aligned} \tag{2}$$

where, N_i, $(i=1,2,3)$ and H_i, $(i=1,2,3)$ are known constant real matrices with proper dimensions and $F_i(t)$, $(i=1,2,3)$ are unknown matrix functions which are bounded by

$$F_i(t)^T F_i(t) \le I. \tag{3}$$

Here, we summarize some definitions and useful properties[1] for descriptor systems in the following.

Definition 1. For the descriptor system $E\dot{x}(t) = Ax(t)$, if *det(sE-A)* is not identically zero, a pencil *sE-A* (or a pair (*E,A*)) is regular. The property of regularity guarantees the existence and uniqueness of solution for any specified initial condition. The singular system has no impulsive mode (or impulse free) if and only if *rank(E)=degdet(sE-A)*. The condition of impulse free ensures that singular system has no infinite poles.

Associated with the uncertain descriptor system (1), we propose the following robust observer-based H_∞ control law

$$\begin{aligned} E\zeta(t) &= A\zeta(t) + B_1 u(t) + L[y(t) - C_2\zeta(t)] \\ u(t) &= K\zeta(t) \end{aligned} \tag{4}$$

where $\zeta(t) \in \mathbf{R}^n$ is the observer state, L is an observer gain matrix, and K is a feedback gain matrix with proper dimensions. When we take error state vector $e(t) = x(t) - \zeta(t)$, the error dynamics can be obtained as

$$\begin{aligned} E\dot{e}(t) &= [A - LC_2 - \Delta B_1(t)K]e(t) \\ &\quad + [\Delta A(t) - \Delta B_1(t)K - L\Delta C_2(t)]x(t) + [B_2 - LD_2]\omega(t). \end{aligned} \tag{5}$$

Moreover, the state equation for the closed-loop system can be obtained as

$$\begin{aligned} E\dot{x}(t) &= [A + \Delta A(t) + (B_1 + \Delta B_1(t))K]x(t) \\ &\quad - [B_1 + \Delta B_1(t)]Ke(t) + B_2\omega(t), \end{aligned} \tag{6}$$

and controlled output, *z(t)*, can be expressed as

$$z(t) = (C_1 + D_1 K)x(t) - D_1 Ke(t) = C_{1K}x(t) - D_1 Ke(t) \tag{7}$$

where, $C_{1K} = C_1 + D_1 K$. Also, we introduce H_∞ performance measure as follows:

$$\int_0^\infty \left[z(t)^T z(t) - \gamma^2 w(t)^T w(t)\right] dt \tag{8}$$

Hence, the objective of this paper is to determine observer gain L and feedback gain K satisfying not only regular, impulse-free, and asymptotic stability of the closed-loop system but also H_∞ norm bound within a prescribed level in (8).

3. MAIN RESULTS

The sufficient condition in terms of an LMI form and robust observer-based H_∞ controller design method are presented in this section. In the following, we present sufficient conditions for the existence of a robust observer-based H_∞ controller for uncertain descriptor systems.

Theorem 1. For a given positive real number γ, if there exist invertible symmetric matrices P_c, P_o, observer gain L, and feedback gain K satisfying

$$E^T P_c = P_c^T E \ge 0 \tag{9}$$

$$E^T P_o = P_o^T E \ge 0 \tag{10}$$

$$\begin{bmatrix} \Gamma_1 & -P_c^T B_1 K - C_{1K} D_1 K & P_c^T B_2 \\ * & \Gamma_2 & P_o^T (B_2 - LD_2) \\ * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{11}$$

then, (4) is a robust observer-based H_∞ controller such that the closed-loop system in (5), (6), and (7) is regular, impulse free, and asymptotic stability. Also, the closed-loop system is satisfied with γ bound. Here, some notations are defined as follows:

$$\begin{aligned} \Gamma_1 &= A^T P_c + P_c^T A + K^T B_1^T P_c + P_c^T B_1 K + C_{1K}^T C_{1K} + P_c^T N_1 N_1^T P_c \\ &\quad + 2P_c^T N_2 N_2^T P_c + 2H_1^T H_1 + 2K^T H_2^T H_2 K + H_3^T H_3 \\ \Gamma_2 &= A^T P_o + P_o^T A - C_2^T L^T P_o - P_o^T LC_2 + K^T D_1^T D_1 K + P_o^T N_1 N_1^T P_o \\ &\quad + 2P_o^T N_2 N_2^T P_o + P_o^T LN_3 N_3^T L^T P_o + 2K^T H_2^T H_2 K. \end{aligned}$$

Proof . For asymptotic stability of system in (5) and (6), if we take a Lyapunov functional

$$V(x(t)) = x(t)^T E^T P_c x(t) + e(t)^T E^T P_o e(t) \tag{12}$$

with $E^T P_c = P_c^T E \ge 0$ and $E^T P_o = P_o^T E \ge 0$, then the time derivative of (12) is given by

$$\begin{aligned} \dot{V}(x(t)) &= \dot{x}(t)^T E^T P_c x(t) + x(t)^T P_c^T E\dot{x}(t) \\ &\quad + \dot{e}(t)^T E^T P_o e(t) + e(t)^T P_o^T E\dot{e}(t). \end{aligned} \tag{13}$$

Therefore, $\dot{V}(x(t)) < 0$ with zero input, $w(t) = 0$, implies that the closed-loop systems in (5) and (6) are asymptotically stable. To satisfy asymptotic stability and H_∞ norm bound in the closed-loop system from the (8), (12), and (13), we can obtain the following relations

$$z(t)^T z(t) - \gamma^2 w(t)^T w(t) + \dot{V}(x(t)) < 0. \tag{14}$$

Therefore, we have

$$\begin{aligned} & z(t)^T z(t) - \gamma^2 w(t)^T w(t) + \dot{V}(x(t)) \\ &= [C_{1K}x(t) - D_1 Ke(t)]^T [C_{1K}x(t) - D_1 Ke(t)] - \gamma^2 w(t)^T w(t) \\ &\quad + \{[A + \Delta A(t) + (B_1 + \Delta B_1(t))K]x(t) \\ &\quad - [B_1 + \Delta B_1(t)]Ke(t) + B_2\omega(t)\}^T P_c x(t) \\ &\quad + x(t)^T P_c^T \{[A + \Delta A(t) + (B_1 + \Delta B_1(t))K]x(t) \\ &\quad - [B_1 + \Delta B_1(t)]Ke(t) + B_2 w(t)\} + \{[A - LC_2 - \Delta B_1(t)K]e(t) \\ &\quad + [\Delta A(t) + \Delta B_1(t)K - L\Delta C_2(t)]x(t) + [B_2 - LD_2]\omega(t)\}^T P_o e(t) \\ &\quad + e(t)^T P_o^T \{[A - LC_2 - \Delta B_1(t)K]e(t) \\ &\quad + [\Delta A(t) + \Delta B_1(t)K - L\Delta C_2(t)]x(t) + [B_2 - LD_2]\omega(t)\} < 0. \end{aligned} \tag{15}$$

And, using the following Lemma

$$2x(t)^T PNF(t)Hx(t) \le x(t)^T PNN^T Px(t) + x(t)^T H^T Hx(t) \tag{16}$$

(15) is transformed into

$$\begin{bmatrix} x(t) \\ e(t) \\ \omega(t) \end{bmatrix}^T \begin{bmatrix} \Gamma_1 & -P_c^T B_1 K - C_{1K} D_1 K & P_c^T B_2 \\ * & \Gamma_2 & P_o^T (B_2 - LD_2) \\ * & * & -\gamma^2 I \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \\ \omega(t) \end{bmatrix} < 0. \quad (17)$$

Therefore, the proof is completed.

However, it is not easy to solve Theorem 1, because the sufficient condition of (11) is not an LMI form and the equality conditions are included in (9) and (10). In order to make a perfect LMI condition in terms of finding all variables and eliminate equality condition, the obtained sufficient conditions are changed in the following Theorem 2 by proper manipulations. Moreover, the robust observer-based H_∞ controller design method for descriptor systems with parameter uncertainties is proposed.

Theorem 2. For a given positive real number γ, if there exist positive definite matrices P_1, P_4, an invertible symmetric matrices P_3, P_6, and matrices P_2, P_5, M_1, M_2 satisfying

$$\begin{bmatrix} \Sigma_1 & 0 & \Sigma_2 & \Pi_6 & 0 \\ * & \Sigma_3 & \Sigma_4 & 0 & \Pi_7 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \quad (18)$$

then the matrices expressed by

$$\begin{aligned} K &= -(D_1^T D_1)^{-1}(B_1^T P_c + D_1^T C_1) \\ L &= (P_o^T)^{-1} M \end{aligned} \quad (19)$$

are controller gain and observer gain in robust observer-based H_∞ controller of (4) satisfying asymptotic stability and H_∞ norm bound within γ. Here,

$$\begin{aligned}
\Sigma_1 &= \begin{bmatrix} \Pi_1 & \Pi_2 \\ * & \Pi_3 \end{bmatrix}, \\
\Sigma_2 &= \begin{bmatrix} P_1 B_{21} + P_2^T B_{22} \\ P_3 B_{22} \end{bmatrix}, \\
\Sigma_3 &= \begin{bmatrix} \Pi_4 & P_5^T A_4 - M_1 C_{22} - C_{21}^T M_2^T \\ * & \Pi_5 \end{bmatrix}, \\
\Sigma_4 &= \begin{bmatrix} P_4 B_{21} + P_5^T B_{22} - M_1 D_2 \\ P_6 B_{22} - M_2 D_2 \end{bmatrix} \\
\Pi_1 &= A_1^T P_1 + P_1 A_1 + C_{11}^T C_{11} + 2H_{11}^T H_{11} + H_{31}^T H_{31}, \\
\Pi_2 &= P_2^T A_4 + C_{11}^T C_{12} + 2H_{11}^T H_{12} + H_{31}^T H_{32}, \\
\Pi_3 &= A_4^T P_3 + P_3 A_4 + C_{12}^T C_{12} + 2H_{12}^T H_{12} + H_{32}^T H_{32}, \\
\Pi_4 &= A_1^T P_4 + P_4 A_1 - M_1 C_{21} - C_{21}^T M_1^T, \\
\Pi_5 &= A_4^T P_6 + P_6 A_4 - M_2 C_{22} - C_{22}^T M_2^T,
\end{aligned} \quad (20)$$

$$\Pi_6 = \begin{bmatrix} P_1 N_{11} + P_2^T N_{12} & \sqrt{2}(P_1 N_{21} + P_2^T N_{22}) & -\sqrt{2}(P_1 B_{11} + P_2^T B_{12} + C_{11}^T D_1)(D_1^T D_1)^{-1} H_2^T \\ P_3 N_{12} & \sqrt{2} P_3 N_{22} & -\sqrt{2}(P_3 B_{12} + C_{12}^T D_1)(D_1^T D_1)^{-1} H_2^T \end{bmatrix}$$

$$\Pi_7 = \begin{bmatrix} -(P_1 B_{11} + P_2^T B_{12} + C_{11}^T D_1)(D_1^T D_1)^{-1} D_1^T & P_4 N_{11} + P_5^T N_{12} & \sqrt{2}(P_4 N_{21} + P_5^T N_{22}) & -\sqrt{2}(P_1 B_{11} + P_2^T B_{12} + C_{11}^T D_1)(D_1^T D_1)^{-1} H_2^T \\ -(P_3 B_{12} + C_{12}^T D_1)(D_1^T D_1)^{-1} D_1^T & P_6 N_{12} & \sqrt{2} P_6 N_{22} & -\sqrt{2}(P_3 B_{12} + C_{12}^T D_1)(D_1^T D_1)^{-1} H_2^T \end{bmatrix}$$

Proof. The matrix inequality of (11) can be changed to

$$\begin{bmatrix} \Lambda_1 & 0 & P_c^T B_2 \\ * & \Lambda_2 & P_o^T B_2 - MD_2 \\ * & * & -\gamma^2 i \end{bmatrix} < 0 \quad (21)$$

with controller gain and observer gain matrices of (19). Here, some notation are represented as

$$\begin{aligned}
\Lambda_1 &= A^T P_c + P_c^T A + C_1^T C_1 - K^T D_1^T D_1 K + P_c^T N_1 N_1^T P_c \\
&\quad + 2P_c^T N_2 N_2^T P_c + 2H_1^T H_1 + 2K^T H_2^T H_2 K + H_3^T H_3 \\
\Lambda_2 &= A^T P_o + P_o^T A - MC_2 - C_2^T M^T + K^T D_1^T D_1 K + \\
&\quad P_o^T N_1 N_1^T P_o + 2P_o^T N_2 N_2^T P_o + MN_3 N_3^T M + 2K^T H_2^T H_2 K, \\
M &= P_o^T L.
\end{aligned}$$

Moreover, the matrix inequality of (21) is negative-definite when the following LMI

$$\begin{bmatrix} \psi_1 & 0 & P_c^T B_2 & \psi_3 & 0 \\ * & \psi_2 & P_o^T B_2 - MD_2 & 0 & \psi_4 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \quad (22)$$

is negative-definite using Schur complements, change of variable, and proper manipulations. Here, some notations are defined as follows:

$$\begin{aligned}
\psi_1 &= A^T P_c + P_c^T A + C_1^T C_1 + 2H_1^T H_1 + H_3^T H_3, \\
\psi_2 &= A^T P_o + P_o^T A - MC_2 - C_2^T M^T, \\
\psi_3 &= \begin{bmatrix} P_c^T N_1 & \sqrt{2} P_c^T N_2 & \sqrt{2} K^T H_2^T \end{bmatrix} \\
\psi_4 &= \begin{bmatrix} K^T D_1^T & P_o^T N_1 & \sqrt{2} P_o^T N_2 & MN_3 & \sqrt{2} K^T H_2^T \end{bmatrix}
\end{aligned}$$

To obtain an LMI sufficient condition in terms of finding all variables and eliminate the equalities in (9) and (10), we make use of singular value decomposition and changes of variables. Without loss of generality, we assume that the systems matrices of (1) have the following singular decomposition form[1,2]

$$E = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} A_1 & 0 \\ 0 & A_4 \end{bmatrix}, B_1 = \begin{bmatrix} B_{11} \\ B_{12} \end{bmatrix}, B_2 = \begin{bmatrix} B_{21} \\ B_{22} \end{bmatrix},$$

$$C_1 = \begin{bmatrix} C_{11} & C_{12} \end{bmatrix},\ C_2 = \begin{bmatrix} C_{21} & C_{22} \end{bmatrix},\ D_1 = D_1,\ D_2 = D_2,$$

$$\Delta A(t) = \begin{bmatrix} N_{11} \\ N_{12} \end{bmatrix} F_1(t) \begin{bmatrix} H_{11} & H_{12} \end{bmatrix}, \tag{23}$$

$$\Delta B_1(t) = \begin{bmatrix} N_{21} \\ N_{22} \end{bmatrix} F_2(t) H_2,\ \Delta C_2(t) = N_3 F_3(t) \begin{bmatrix} H_{31} & H_{32} \end{bmatrix}$$

where, all matrices have appropriate dimensions. Also, if we set

$$P_c = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix},\ P_o = \begin{bmatrix} P_4 & 0 \\ P_5 & P_6 \end{bmatrix} \tag{24}$$

in order to satisfy (9) and (10), and if other solution has the following structure

$$M = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \tag{25}$$

and we apply (23), (24), and (25) to (22), then (22) is equivalent to (18). Therefore, (18) is an LMI form in terms of all variables, P_1, P_2, P_3, P_4, P_5, P_6, M_1, and M_2.

Remark 1: In the case of $E = I$, the problem can be solvable directly from an LMI condition in (22) with positive definite matrices, P_c and P_o. Therefore, the proposed design algorithm can be solvable in non-singular systems with an LMI condition. Thus, the result is general design method. In most existing results (Masubuchi *et al*. 1997 and references therein), observer-based controller including output feedback controller can be constructed from two sufficient conditions of control Riccati equation(or inequality), and filter(or observer) Riccati equation(or inequality), while the proposed robust observer-based H_∞ controller can be obtained by one LMI sufficient condition, which can be solved efficiently by convex optimization.

Example 1. Consider a descriptor system with parameter uncertainties

$$\begin{aligned} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \dot{x}(t) &= \left\{ \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} F_1(t) \begin{bmatrix} 0.1 & 0.1 \end{bmatrix} \right\} x(t) \\ &+ \left\{ \begin{bmatrix} -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} F_2(t) 0.1 \right\} u(t) + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} \omega(t) \\ z(t) &= \begin{bmatrix} 0.2 & 0.1 \end{bmatrix} x(t) + u(t) \\ y(t) &= \left\{ \begin{bmatrix} 0.5 & 0.2 \end{bmatrix} + F_3(t) \begin{bmatrix} 0.1 & 0.1 \end{bmatrix} \right\} x(t) + \omega(t) \end{aligned} \tag{26}$$

All solutions can be calculated at the same time from the LMI Toolbox[9] because the proposed optimization problem of Theorem 2 is an LMI form in terms of finding all variables as follows;

$$\begin{aligned} P_c &= \begin{bmatrix} 0.2888 & 0 \\ -0.2490 & -0.1605 \end{bmatrix}, P_o = \begin{bmatrix} 0.3058 & 0 \\ -0.2412 & -0.2088 \end{bmatrix} \\ M &= \begin{bmatrix} -0.1010 \\ -0.0209 \end{bmatrix} \end{aligned} \tag{27}$$

The gain matrices can be obtained from (19) as follows:

$$\begin{aligned} K &= \begin{bmatrix} 0.1286 & -0.2605 \end{bmatrix}, \\ L &= \begin{bmatrix} -0.2512 \\ 0.1001 \end{bmatrix}. \end{aligned} \tag{28}$$

Therefore, the robust observer-based H_∞ controller can be constructed as follows:

$$\begin{aligned} E\dot{\hat{x}}(t) &= \begin{bmatrix} -2.1316 & 0.5712 \\ -0.1787 & 1.2405 \end{bmatrix} \hat{x}(t) + \begin{bmatrix} -0.2512 \\ 0.1001 \end{bmatrix} y(t) \\ u(t) &= \begin{bmatrix} 0.1286 & -0.2605 \end{bmatrix} \hat{x}(t). \end{aligned} \tag{29}$$

Thus, the controller guarantees not only asymptotic stability including regularity and the property of impulse free but also the H_∞ norm bound, $(\gamma < 1)$, of the closed-loop system in spite of disturbance input and parameter uncertainties.

4. CONCLUSIONS

This paper considered the design problem of robust observer-based H_∞ controller for descriptor system with parameter uncertainties by an LMI approach. One of main advantages was that a robust observer-based H_∞ controller could be constructed from the solutions of an LMI sufficient condition. The presented robust observer-based H_∞ controller guaranteed asymptotic stability, regularity, the property of impulse free, and H_∞ norm bound in the closed-loop system.

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