Adjusting GPC Control Parameters Based on Gain and Phase Margins

Mohammad Haeri

Advanced Control System Lab., Electrical Engineering Department of Electrical Engineering,
Sharif University of Technology, Tehran, IRAN
(Tel: +98-21-616-5964; E-mail: haeri@sina.sharif.edu)

Abstract: Gain and phase margins of a first order plus delayed time (FOPDT) process controlled by generalized predictive controller (GPC) are related to the control parameters λ (control move suppression parameter) and α (smoothing filter coefficient) and the normalized delay of the process. Variation ranges of gain and phase margins are determined. It is shown that the margins cannot be assigned independently for a wide range of variation and the range is narrowing by increase of the normalized delay of the process. And finally curves are given to use for adjustment of the controller parameters in order to obtain a specific pair of gain and phase margins.

Key words: Model predictive controller, Gain and phase margins, First order plus delayed time

1. INTRODUCTION

In model predictive controller, the control signal is determined by solving an optimization problem. For unconstrained and linear time invariant processes, the defined optimization problem has closed form solution. The given solution is a relation to calculate the control signal. By applying some mathematical manipulations, one can get a transfer function form of the controller which in general is in RST (output feedback) structure [1]. Since model predictive controllers are designed based on the time domain analysis, this structure has not been used in adjustment of the control parameters so far. To analyze the closed loop performance in the frequency domain and to design the controller based on this analysis, it is required to obtain a clear and explicit relation among the control parameters and its poles/zeros location in RST structure. Having the RST structure of the controller, one can simply realize that the existing relations are too complex to be usable in a frequency domain based control design. Instead of above-mentioned analytical relations, graphical relations are provided in this paper that has been obtained based on extensive computer simulations.

Among five adjustable parameters in generalized predictive controller, three of them i.e. the control and prediction horizons and the sampling interval are determined by the process step response characteristics [2]. The control move suppression parameter, λ, and the smoothing filter coefficient, α are the ones that can be adjusted independently to achieve an acceptable closed loop performance in time or frequency domain [3]. In the present work, first, we have obtained gain and phase margins of a first order plus delayed time process with different normalized delay and controlled by a generalized predictive controller with different λ and α. In this stage, approximated ranges of the achievable gain and phase margins are determined. Then based on the gathering data, different curves are presented from which the required values of λ and α to get a pair of specific gain and phase margins can be determined. Finally numerical formulas are given that relate the desired gain and phase margins and the normalized delay of the process to the control parameters λ and α.

The paper is organized as follows. In Section 2, RST structure of generalized predictive controller is derived. Relations among the control parameters and gain and phase margins are discussed in Section 3. Numerical formulas that are obtained using curve fitting techniques are given in Section 4. Section 5 discusses the results and concludes the paper.

2. RST STRUCTURE OF GENERALIZED PREDICTIVE CONTROLLER

Discrete form of a FOPDT model is given as:

$$y_m(z) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1}} z^{-d} u(z) \tag{1}$$

Using the following definitions and also Bezout identity.

$$\widetilde{A}(z) = (1 - z^{-1})(1 + a_1 z^{-1}), \quad \Delta u(z) = (1 - z^{-1}) u(z),$$
$$1 = E_j(z)\widetilde{A}(z) + z^{-j} F_j(z)$$

one can derive j-step ahead prediction of the model output as:

$$y_m(k+j) = F_j(q) y_m(k) + \breve{G}_j(q) \Delta u(k+j-d) \tag{2}$$

In which the following definitions are employed.

$$F_j(z) = f_{j,0} + f_{j,1} z^{-1}$$
$$\breve{G}_j(z) = (b_1 z^{-1} + b_2 z^{-2}) E_j(z)$$
$$E_j(z) = e_{j,0} + e_{j,1} z^{-1} + \cdots + e_{j,j-1} z^{j-1}$$

Dividing the second term in the right hand side of Eq. (2) into known and unknown parts at sample time k

$$\breve{G}_j(q)\Delta u(k+j-d) = G_j(q)\Delta u(k) + \overline{G}_j(q)\Delta u(k+j-d),$$

the j-step ahead prediction of the output is written as:

$$y_m(k+j) = G_j(q)\Delta u(k) + y_{past}(k+j) \tag{3}$$

$y_{past}(k+j)$ represents those parts of the output that depend on the measurements available at sample time k.

$$y_{past}(k+j) = F_j(q) y_m(k) + \overline{G}_j(q)\Delta u(k+j-d) \tag{4}$$

The control moves in GPC controller is determined as follows [4].

$$\Delta U(k) = K_{GPC} E(k+1) = (G^T G + \gamma I)^{-1} G^T E(k+1) \tag{5}$$

Where G is a Toeplitz matrix constructed from the step response coefficients of the process. $E(k+1)$ is the prediction error and is defined as:

$$E(k+1) = Y_d(k+1) - Y_{past}(k+1) - D(k+1) \tag{6}$$

$Y_d(k+1)$ indicates the desired output and is defined as:

$$Y_d(k+1) = [y_d(k+d+1)\; y_d(k+d+2) \cdots y_d(k+d+P)]^T$$

$$y_d(k) = y_p(k)$$
$$y_d(k+i) = \alpha y_d(k+i-1) + (1-\alpha) r(k), \text{ for } i = 1, \cdots, d+P$$

This relation can be summarized as follows.

$$\begin{aligned} Y_d(k+1) &= \overline{\alpha} y_p(k) + (\mathbf{1} - \overline{\alpha}) r(k) \\ \overline{\alpha} &= \left[\alpha^{d+1}\; \alpha^{d+2} \cdots \alpha^{d+P}\right]^T \end{aligned} \tag{7}$$

$Y_{past}(k+1)$ is obtained from Eq. (4) and is given as follows:

$$Y_{past}(k+1) = \begin{bmatrix} F_{d+1}(q) y_m(k) + \overline{G}_{d+1}(q) \Delta u(k+1) \\ F_{d+2}(q) y_m(k) + \overline{G}_{d+2}(q) \Delta u(k+2) \\ \vdots \\ F_{d+P}(q) y_m(k) + \overline{G}_{d+P}(q) \Delta u(k+P) \end{bmatrix}$$

It can be further simplified as:

$$\begin{aligned} Y_{past}(k+1) = F\left[y_m(k)\; y_m(k-1)\right]^T + \\ \breve{G}\left[u(k-1)\; u(k-2) \cdots u(k-d)\right]^T \end{aligned} \tag{8}$$

$D(k+1)$ represents model/process mismatch and external disturbances. It is approximated as:

$$D(k+1) = d(t)[1\,1 \cdots 1]^T = \left(y_p(k) - y_m(k)\right)[1\,1 \cdots 1]^T \tag{9}$$

Usually the first component of $\Delta U(k)$ in Eq. (5) is used to determine the control signal i.e.

$$\Delta u(k) = K\left(Y_d(k+1) - Y_{past}(k+1) - D(k+1)\right) \tag{10}$$

Where K is the first row of matrix K_{GPC}.

To obtain the transfer function representation of GPC controller, we substitute for the right hand side variables of Eq. (10) from what are given in Eqs. (7-9).

$$\begin{aligned} \left[1\; K\breve{G}\right] \begin{bmatrix} \Delta u(k) \\ \Delta u(k-1) \\ \vdots \\ \Delta u(k-d-1) \end{bmatrix} &= K(\mathbf{1} - \overline{\alpha}) e(k) + \\ & K\left(\mathbf{1} y_m(k) - F \begin{bmatrix} y_m(k) \\ y_m(k-1) \end{bmatrix}\right) \end{aligned} \tag{11}$$

Using the following definitions, this equation is written as RST structure.

$$R(q)(1 - q^{-1}) u(k) = T(q) e(k) - S(q) y_m(k) \tag{12}$$

$$T(q) e(k) = T(q)(r(k) - y_p(k)) = K(\mathbf{1} - \overline{\alpha}) e(k)$$

$$\begin{aligned} S(q) y_m(k) &= K(F \begin{bmatrix} y_m(k) \\ y_m(k-1) \end{bmatrix} - \mathbf{1} y_m(k)) \\ &= s_0 y_m(k) + s_1 y_m(k-1) \end{aligned}$$

$$R(q) \Delta u(k) = \left[1\; K\breve{G}\right] \begin{bmatrix} \Delta u(k) \\ \Delta u(k-1) \\ \vdots \\ \Delta u(k-d-1) \end{bmatrix}$$

Block diagram of the controller is shown in Fig. 1.

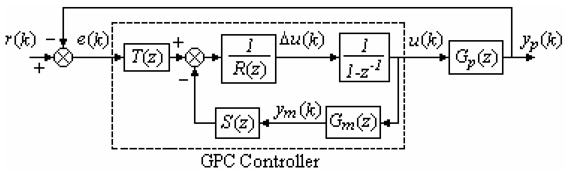

Fig. 1. Block diagram of the GPC controller.

As it seen from Fig. 1 and Eq. (12), only α has explicit effect on the RST control parameters. It can be adjusted to get a specific loop gain and therefore change the gain and phase margins. Other parameters of the GPC controller affect both zeros and poles of the loop gain but not in explicit and clear way. Therefore, it is difficult (if not impossible) to determine mathematical relations among GPC parameters and the margins analytically. We try to find these mathematical relations using the extensive computer simulation results and numerical methods.

3. EFFECT OF THE CONTROL PARAMETERS

Although all five parameters ($P, M, \alpha, \gamma (or\ \lambda), T_s$) of GPC controller affect the closed loop characteristics and performance, only few of them indicate wide range influences. On the other hand, considering all these parameters complicates analysis and increases number of the simulations that should be done to get an appropriate result. Among the control parameters, P, M and T_s are selected based on the process dynamics [2]. α and λ (in its scaled form) are somehow independent from the process dynamics and can be selected to improve the closed loop response. α is the smoothing filter pole and therefore confine in $[0,1)$ for stability reason. Its variation rate depends on the process normalized delay. Table 1 indicates effective range of α for some normalized delays. The normal value of λ is 1. In all the simulations λ was set to change from 0 to 5 by steps of 0.01. The sampling time was selected 0.1τ in which τ is the time constant of the process. Decreasing of the sampling time improves the closed loop performance. However this should be accompanied by the increase in P and M which in turns increases computational requirement.

Results given in this paper have been obtained based on 350700 simulation runs for GPC with different control parameters and the normalized FOPDT process model.

$M = 2, P = 25, T_s = 0.1$, $\lambda \subset \{0:0.01:5\}$

$G(s) = \frac{e^{-Ls}}{s+1}$, $L \subset \{0:0.117:3.978\}$

For each selection of L (normalized delay) and λ, 20 different α was implemented. Table 1 indicates sets of the selected α for the shown normalized delay. 501 λ and 35 normalized delays were considered.

Table 1 Values of α used in some of the simulations.

No. of simulation	α	**Normalized delay**
20	0.3985:0.0285:0.94	0
20	0.5325:0.0225:0.96	0.468
20	0.6850:0.0150:0.97	1.053
20	0.7425:0.0125:0.98	1.638
20	0.8000:0.0100:0.99	2.223

4. DETERMINATION OF THE CONTROL PARAMETERS

To determine the control parameters α and λ for specific Gain and Phase margins, results obtained in the simulations were drawn in contour form. Each contour indicates what Gain and Phase margins can be obtained by a specific value of α or λ. Some of these contours are shown in Figs. 2-6 for the normalized delay given in Table 1.

Based on these figures, one is able to specify required values of α and λ for a pair of desired Gain and Phase margins.

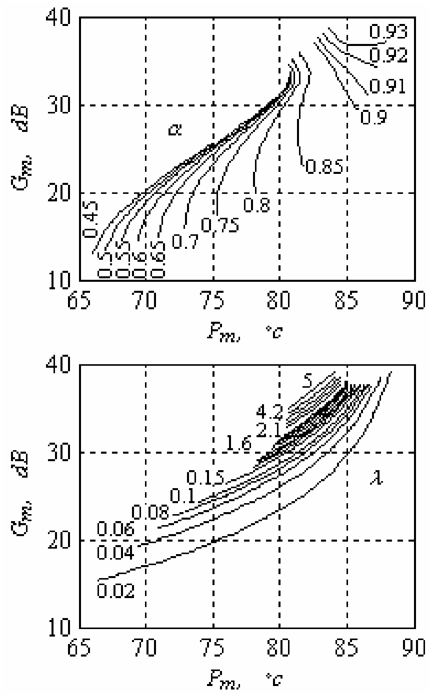

Fig. 2. Gain and Phase margins contour for $L = 0$.

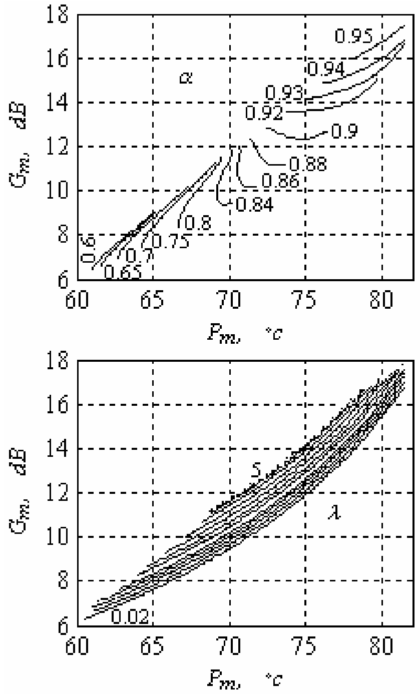

Fig. 3. Gain and Phase margins contour for $L = 0.468$.

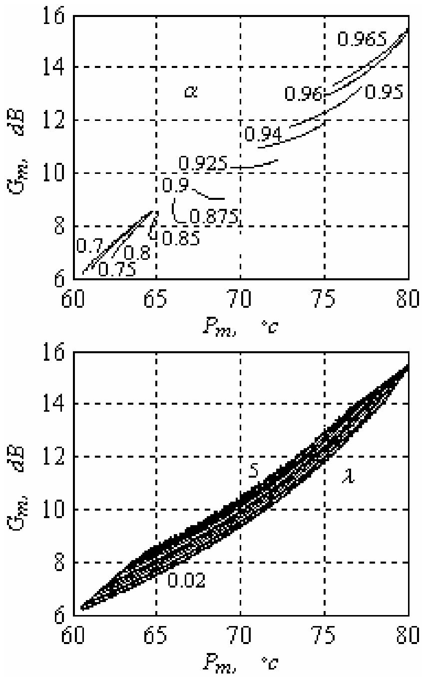

Fig. 4. Gain and Phase margins contour for $L = 1.053$.

Some general comments are deduced from the figures obtained for each normalized delay.

1) For a specific α, the Phase margin is almost constant and Gain margin increases by increase in λ.
2) For a specific α, the Gain margin is almost constant and Phase margin decreases by increase in λ.
3) Increase in the normalized delay reduces the effective

range of λ.

4) Increase in λ always increases the Gain margin. However, the magnitude of the change decreases by increase in the normalized delay.
5) Decrease in λ always increases the Phase margin. However, the magnitude of the change decreases by increase in the normalized delay.

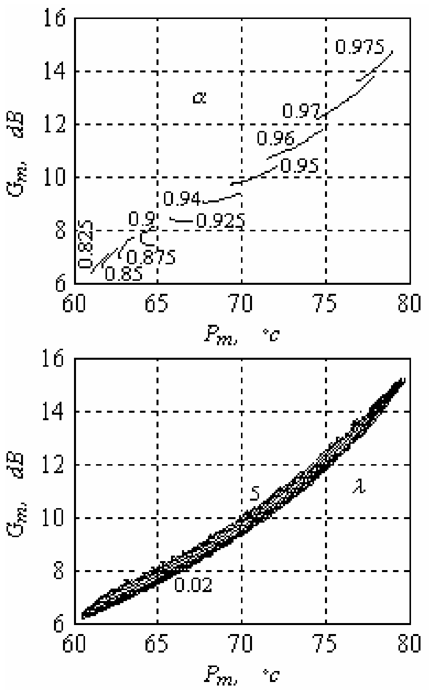

Fig. 5. Gain and Phase margins contour for $L = 1.638$.

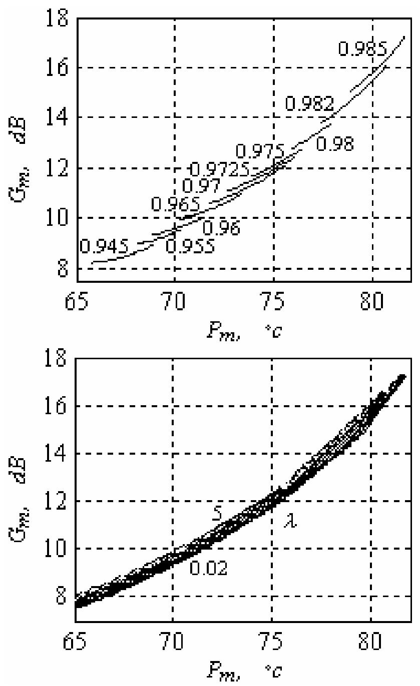

Fig. 6. Gain and Phase margins contour for $L = 2.223$.

6) Minimum and maximum values of the Gain margin are approximately 6 and 55 dB.
7) Minimum and maximum values of the Phase margin are approximately 6 0 and 90 ^{o}c.

An important concluding result that can be stated is that for each λ, one is able to get an appropriate pair of Gain and Phase margins only by adjusting α. This property could be used to simplify the process of getting mathematical relation among the control parameters and desired values of the margins. It should be noted however, that the Gain and Phase margins are not completely independent. In other words, assigning a value for one of them imposes a constrained range in the selection of the other.

In the following example we try to design a GPC controller to obtain Gain margin of $10.2\,dB$ and Phase margin of $70\ ^{o}c$.

$$G(s) = \frac{3e^{-15.795s}}{15s+1}$$

The normalized delay of the process is $15.795/15 = 1.053$ and therefore Fig. 4 is employed to assign the control parameters α and λ. Since the steady state gain of the process is 3, the input variation weight in GPC controller is determined as $3^2\lambda$. Results presented in Figs. 7 and are obtained for the following parameters of the controller.

$$M = 2, P = 25, \gamma = 3^2\lambda = 9 \times 0.45, \alpha = 0.928, T_s = 1.5\,\text{sec}$$

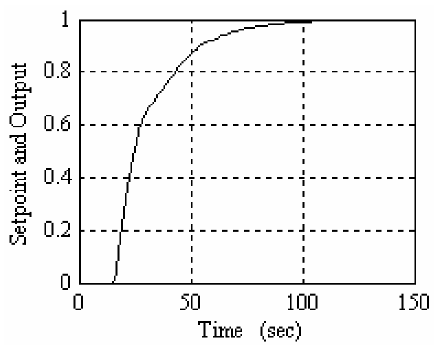

Fig. 7. Unit step response of the process.

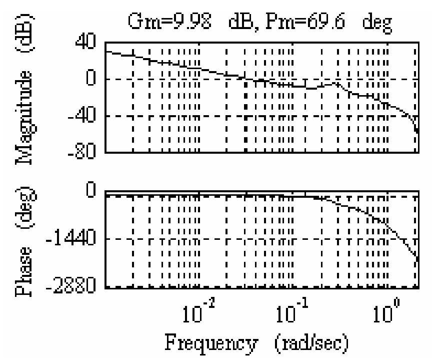

Fig. 8. Bode diagram of the system.

As it is seen results are almost compatible with the desired values.

5. CONCLUSIONS

We investigate design of GPC controller based on the frequency domain characteristics. Specifically, two of the control parameters were determined to obtain desired Gain and Phase margins. It is expressed and illustrated that these to parameters have wide range of influence on the Gain and Phase margins. Results obtained from extensive simulations were presented in graphics form to use in the parameters adjustment. These graphics would be used in the next step to determine mathematical relations among the control parameters and the desired Gain and Phase margins.

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