I-PDA Controller Designed by CDM Incorporating FFC for Two-Inertia System

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Abstract: The two-inertia system, which has the torsion vibration, is typically found in several industrial applications. This torsion vibration will effect the quality of the rolled material as well as the stability of the drive system. Thus the speed and torsion vibration of the system have to be properly controlled. This paper, I-PDA controller designed by Coefficient Diagram Method to control a two-inertia system is proposed. The experimental result shows that both of transient and steady state specification can be fulfilled but the transient response still has long rise time. In order to improve the speed of the system response, a phase lag structure of feedforward controller is introduced to I-PDA control system. It is shown that the performance of the I-PDA control system with suitable FFC has shorter rise and settling times, no overshoot and the torsion vibration can be suppressed.

Keywords: I-PDA controller, feedforward controller, CDM, two-inertia system

1. INTRODUCTION

The servo control systems driving a load through a shaft or a transmission system are widely used in the industrial applications. Several industrial applications, such as an industrial rolling mill drive system has the phenomenon as the two-inertia system that a flexible shaft has very low natural resonant frequency because long shaft and low stiffness between motor and load. Hence, it is difficult to achieve the precise speed control due to torsion vibration [1]. It is known that the speed and torsion vibration of the two-inertia system must be properly controlled, otherwise the vibration may occur and the stress on the shaft may result in damage to the shaft.

The design and adjustment of two-inertia system are difficult because most servo systems only provide direct measurement of motor variables, not load variables. In recent years, many control schemes have been proposed to control the speed of the two-inertia system in order to obtain a good response. The LQG-based speed controller developed to achieve torsion vibration suppression has been reported [1]. The PI and PID control designed by three kinds of typical pole assignments with identical radius, damping coefficient and real part have been investigated [2]. On the other hands, the parameter design method for PIDA (Proportional - Integral - Derivative - Acceleration) controller proposed by S. Jung and R. C. Dorf [3] to be utilized especially for a third-order plant has been reported in [4]. The I-PDA controller scheme has also been reported in [5]. Both of controllers which give the satisfied transient and steady state performances were designed based on coefficient diagram method (CDM) [6]. However, the transient response of the control systems generally still has long rise time. In order to improve the speed of the transient response of the I-PDA control system, a feedforward controller (FFC) has been introduced [7].

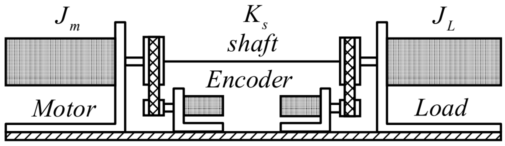
In this paper, I-PDA controller designed by CDM incorporating FFC for two-inertia system is presented. The model of the two-inertia system is the third order system. Using CDM concept, I-PDA controller parameters can be designed from the stability index and the equivalent time constant respectively. The stability index and the equivalent time constant are defined based on the coefficient of the characteristic polynomial of the closed-loop transfer function. Normally, the settling time t_s of the control system is first selected and then the equivalent time constant τ can be obtained. However, in case of the I-PDA controller, only stability index γ_i can be specified due to the equivalent time constant τ has been assigned implicitly from the design procedure.

Since the response of I-PDA control system is slow, the FFC whose structure is a phase lag structure with two parameters and one integral time is added. The integral time T_i of the FFC is chosen base on Zeigler-Nichols method. The two parameters α and β, on the other hand, must be designed properly by utilizing the advantage of CDM. A new parameter known as σ_f which is a percentage of equivalent time constant τ is then introduced. By varying σ_f and standard stability index γ_i, the desired result can be obtained.

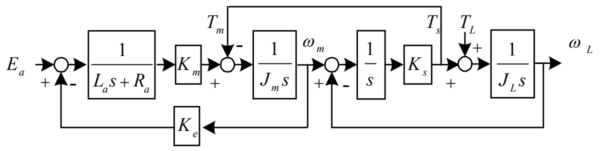
The experimental results of two-inertia system in laboratory employing the I-PDA controller and FFC are shown. The values of α and β are assigned from σ_f equals to 60% of the equivalent time constant. The response of I-PDA control system with FFC is then compared to the system without FFC when σ_f equals to 60% of the equivalent time constant. The disturbance effect rejection capability and the effect due to the variation of σ_f are also shown in this paper.

2. MODEL OF TWO-INERTIA SYSTEM

The mathematical model of the two-inertia system in the laboratory is shown in Fig.1 (a). The precise mathematical model of the actual system is complex but it can be approximately reduced to the simplified model shown in the block diagram of Fig. 1(b).



(a) Model of the two-inertia system.



(b) Block diagram of the two-inertia model.

Fig.1 Two-inertia system.

The transfer function expressed as a third-order system by neglecting the electrical parts (L_a / R_a) is given by Eq. (1)

$$G_p(s) = \frac{\omega_m(s)}{E_a(s)} = \frac{n_1 s^2 + n_0}{d_3 s^3 + d_2 s^2 + d_1 s + d_0}\,, \tag{1}$$

where $n_1 = K_m J_L$, $n_0 = K_m K_s$, $d_3 = J_m J_L$, $d_2 = K_s K_e J_L$, $d_1 = K_s J_m + K_s J_L$, $d_0 = K_s^2 K_e$ and where

E_a : Input voltages apply to the armature
K_m : Motor torque constant
K_e : Electromotive force constant
T_m : Motor torque
T_s : Shaft torque
ω_m : Motor speed
K_s : Torsion stiffness of drive shaft
J_m : Moment of inertia of motor
J_L : Moment of inertia of load
ω_L : Load speed
T_L : Torque disturbance.

3. CONTROL SYSTEM STRUCTURE

3.1 Overview of I-PDA incorporating FFC

The I-PDA control system with FFC shown in Fig. 2 consists of a feedforward controller, a feedback controller (FBC), an integral controller and a third-order plant. α , β and T_i are the parameters of FFC. K_p , K_d and K_a are the proportional gain, derivative gain and acceleration gain of the FBC respectively, K_i is the integral gain of the integral controller. $D(s)$ is the process step disturbance to be applied to the system. The transfer function from $R(s)$ to $C(s)$ and from $D(s)$ to $C(s)$ are then respectively given by

$$\frac{C(s)}{R(s)} = \frac{G_p(s)\left[(\alpha T_i)s^2 + (\beta + K_i T_i)s + K_i\right]}{(T_i s+1)\left(s + G_p(s)\left[s\left(K_p + K_d s + K_a s^2\right) + K_i\right]\right)} \tag{2}$$

and

$$\frac{C(s)}{D(s)} = \frac{G_p(s)}{1 + G_p(s)\left(\frac{K_i}{s} + K_p + K_d s + K_a s^2\right)}\,. \tag{3}$$

From Eq. (3), it is shown that the disturbance response is irrelevant to FFC. Its means that we have a freedom in selecting α and β to improve transient specification while disturbance rejection and tracking capability designed by I-PDA are not influenced.

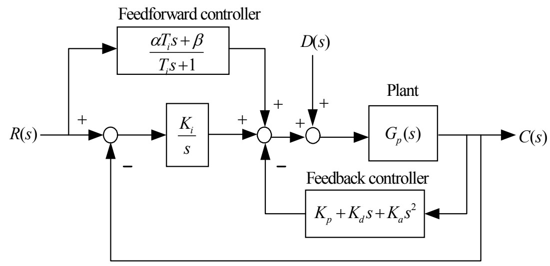


Fig. 2 Structure of I-PDA control system with FFC.

In order to employ CDM concept to design the controller gains properly, the controlled system consists of the CDM standard block diagram of SISO system with the FFC and FBC is shown in Fig. 3. $A_p(s)$ and $B_p(s)$ are the polynomials of the plant $G_p(s)$. $A_c(s)$, $B_c(s)$ and $B_a(s)$ are the polynomials of the CDM controller. $B_{fb}(s)$ is the polynomial of the FBC. $A_{ff}(s)$ and $B_{ff}(s)$ are the numerator and denominator of the FFC.

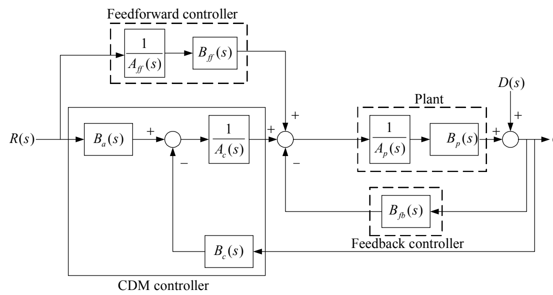


Fig. 3 SISO system with FBC and FFC.

By rearranging the polynomials of the plant and FBC, the modified plant can be obtained. The modified FFC can also be obtained by rearranging the pre-filter $B_a(s)$ of the CDM controller and the polynomial of the FFC. The rearranged CDM block diagram is then shown in Fig. 4. Therefore, the transfer function from $R(s)$ to $C(s)$ is

$$\frac{C(s)}{R(s)} = \frac{B_{ff}^*(s)B_p^*(s)}{A_{ff}^*(s)\left(A_c(s)A_p^*(s) + B_c(s)B_p^*(s)\right)} \tag{4}$$

and the transfer function from $D(s)$ to $C(s)$ is

$$\frac{C(s)}{D(s)} = \frac{A_c(s)B_p^*(s)}{A_c(s)A_p^*(s) + B_c(s)B_p^*(s)}\,. \tag{5}$$

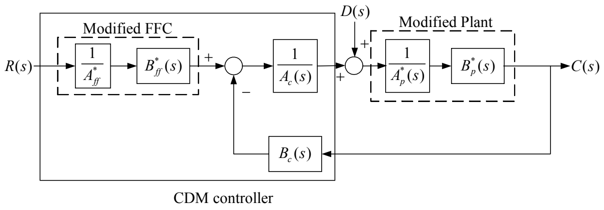


Fig. 4 Rearranged CDM block diagram.

3.2 Concept of CDM

In this sub-section, the concept of CDM used to design the parameters of a controller so that the step response of the control system satisfies stability, fast response and robustness requirements [7] is described. Generally, the order of the controller designed by CDM is less than the order of the plant. From Fig. 4, the polynomial forms of the modified plant form are

$$A^*_p(s) = p_k s^k + p_{k-1}s^{k-1} + ... + p_0 \tag{6a}$$

$$B^*_p(s) = q_m s^m + q_{m-1}s^{m-1} + ... + q_0 \tag{6b}$$

and the CDM controller polynomial forms are

$$A_c(s) = l_\lambda s^\lambda + l_{\lambda-1}s^{\lambda-1} + ... + l_0 \tag{7a}$$

$$B_c(s) = k_\lambda s^\lambda + k_{\lambda-1}s^{\lambda-1} + ... + k_0 \tag{7b}$$

$$B_a(s) = k_0 , \tag{7c}$$

where $\lambda < k$ and $m < k$. $B_s(s)$ is called as a pre-filter and has to be set to k_0 so that the step response with zero steady-state error is obtained. The characteristic polynomial of the closed-loop system without FFC structure as shown in Fig. 4 is given in the following form

$$\begin{aligned} P(s) &= A_c(s)A^*_p(s) + B_c(s)B^*_p(s) \\ &= \sum_{i=0}^{n} a_i s^i \end{aligned} \tag{8}$$

where $a_0, a_1,..., a_n$ are the coefficients of the characteristic polynomial. The stability index γ_i , the equivalent time constant τ and stability limit γ^*_i are defined as

$$\gamma_i = \frac{a_i^2}{a_{i+1}a_{i-1}} \tag{9}$$

$$\tau = \frac{a_1}{a_0} \tag{10}$$

$$\gamma^*_i = \frac{1}{\gamma_{i+1}} + \frac{1}{\gamma_{i-1}} \quad ; \quad \gamma_0 , \gamma_n = \infty \tag{11}$$

where $i = 1,..., n-1$. In order to meet the specifications, the equivalent time constant τ and stability index γ_i are chosen as

$$t_s = 2.5 - 3\tau \tag{12}$$

$$\gamma_i > 1.5\gamma^*_i . \tag{13}$$

In general, settling time is selected to be $t_s = 2.5\tau$, and the standard stability index

$$\gamma_{n-1} = ... = \gamma_3 = \gamma_2 = 2, \gamma_1 = 2.5 \tag{14}$$

is recommended.

The standard values in Eq. (14) can be used to design the controller if the following condition is satisfied

$$p_k / p_{k-1} > \tau / (\gamma_{n-1}\gamma_{n-2} \cdots \gamma_1) \tag{15}$$

where p_k and p_{k-1} are the coefficients of the plant at kth and (k-1)th. If the above condition is not satisfied, we can first increase γ_{n-1} then γ_{n-2} and so on, until Eq. (15) is satisfied. From Eq. (9)-Eq. (11), the coefficient a_i can be written by

$$a_i = a_0\tau^i \frac{1}{\gamma_{i-1} \cdots \gamma_2^{i-2}\gamma_1^{i-1}} = a_0\tau^i \prod_{j=1}^{i-1} \frac{1}{\left(\gamma_{i-j}\right)^j} . \tag{16}$$

Then the characteristic polynomial used for designing the parameters of a controller is expressed as

$$P(s) = a_0 \left[\left\{ \sum_{i=2}^{n} \left(\prod_{j=1}^{i-1} 1 \Big/ \gamma_{i-j}^{j} \right) (\tau s)^i \right\} + \tau s + 1 \right] . \tag{17}$$

By equating the characteristic polynomial in Eq. (8) including a controller to the characteristic polynomial in Eq. (17) resulting from the known equivalent time constant τ and stability index γ_i , then the parameters of a controller can be obtained.

4. CONTROLLER DESIGN

This section describes the procedure for assigning the parameters of I-PDA controller and the parameters of FFC will be designed respectively.

4.1 I-PDA controller design

By using CDM concept previously described, parameters of I-PDA controller including K_p , K_d , K_a and K_i can be designed by the following procedures:

1) Determine the proper stability index.
2) Since the settling time t_s can not be specified, set the equivalent time constant τ as a variable that will be found later.
3) Derive the characteristic polynomial of Eq. (8) and then equates to the characteristic polynomial obtained by Eq. (17). Thus the parameters of I-PDA controller are obtained.
4) Set the pre-filter $B_a(s) = k_0$, where $k_0 = K_i$.

4.2 FFC design

The structure of the proposed FFC as shown in Fig. 2 is a phase lag structure with the following transfer function

$$\frac{B_{ff}(s)}{A_{ff}(s)} = \frac{\alpha T_i s + \beta}{T_i s + 1} . \tag{18}$$

The value of α and β must be properly selected and T_i is the integral time obtained from reaction curve of the two-inertia system. From Fig. 4, the polynomial forms of the modified FFC are

$$B_{ff}^{*}(s) = B_a(s)A_{ff}(s) + A_c(s)B_{ff}(s) = m_2 s^2 + m_1 s + m_0 \tag{19}$$

and

$$A_{ff}^{*}(s) = A_{ff}(s) = T_i s + 1, \tag{20}$$

where $m_2 = \alpha T_i$, $m_1 = (\beta + k_0 T_i)$ and $m_0 = k_0$. The proper values of α and β can be designed from the following procedures:

1) Let $m_1 / m_0 = \sigma_f$ be 60% of the equivalent time constant τ
2) Find m_1 from the known values of m_0 and σ_f
3) Find m_2 from $m_2 / m_1 = \sigma_f / \gamma_1$
4) Find α and β from $m_2 = \alpha T_i$ and $m_1 = (\beta + k_0 T_i)$

5. EXPERIMENTAL RESULTS

In this section, the proposed controllers are implemented to control the two-inertia system shown in Fig. 5. Therefore, the transfer function of two-inertia system has to be known. Two 80 W motors are connected by a shaft which has a diameter of 0.16 cm and a length of 35 cm. One motor is operated as the drive power and the other is used as the load that can impose the disturbance torque. Two encoders are equipped to measure the motor and load speeds. Flywheels are fixed at the axes of motor and load via clamp locks. Chucks fix the shaft and they are easily alterable. The input of driver is the control-input voltage, which represents the desired speed of the motor.

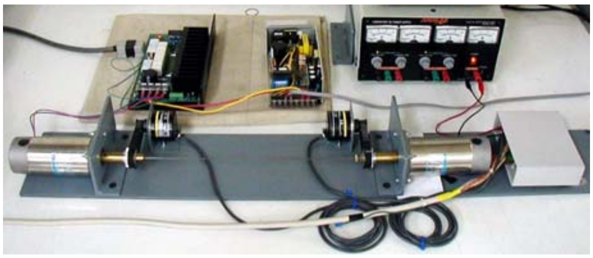


Fig. 5 Structure of two-inertia system.

Table 1 Parameters of the two-inertia system.

K_e	K_m	J_m	J_L	K_s
V/rpm	$N \cdot m / A$	$kg \cdot m^2$	$kg \cdot m^2$	$N \cdot m / rad$
3.75	0.0775	$0.45 \cdot 10^{-4}$	$0.55 \cdot 10^{-4}$	9

The transfer function of two-inertia system can be found from Eq. (1) by using the data shown in Table 1 and then the transfer function from armature input voltage to the motor speed is given by

$$\frac{\omega_m(s)}{E_a(s)} = \frac{17.22 s^2 + 28180}{s^3 + 64.58 s^2 + 3636 s + 105700}. \tag{21}$$

The experimental open-loop response that has the torsion vibration of the two-inertia system is shown in Fig. 6 when the input voltage 5 volts is applied. In order to suppress this undesired torsion vibration, the proposed controllers will be implemented. The results of the proposed control system will be shown in three aspects. First is the system performance of the two-inertia system, second is the system response when take the disturbance and varies the control speed and third is the effect of the σ_f variation.

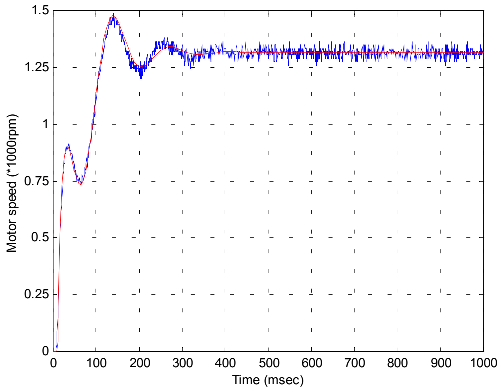


Fig. 6 Open-loop response of two-inertia system.

5.1 System performance

According to design procedure stated in sub-section 4.1, the standard stability index can not be used because the negative gains will be obtained. So the stability index has to be adjusted for obtaining the positive gains. By specifying the stability index $\gamma_1 = 5.0, \gamma_2 = 1.5, \gamma_3 = 2.0$ and $\gamma_4 = 2.0$, the equivalent time constant $\tau = 0.1659$ seconds can be obtained. The I-PDA controller gains K_p, K_d, K_a and K_i are then obtained as 0.6155, 0.0037, 0.00029 and 26.614 respectively. In the case of addition of FFC, the FFC design procedure is used. The system responses are depicted in Fig. 7.

It is seen that by choosing the suitable σ_f, the I-PDA control system with FFC will have faster response with zero overshoot. The torsion vibration of the controlled system with and without FFC can be suppressed.

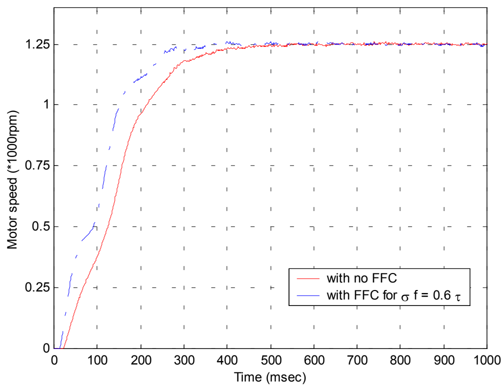


Fig. 7 System responses.

The performance of these system responses can be summarized in Table 2. It is found that the speed of the system response for the two-inertia system using the I-PDA controller designed by CDM with FFC is faster than the system without FFC. Both control systems have no overshoot P_0 and no steady-state error E_{ss}, while the rise time t_r and settling time t_s of the I-PDA control system with FFC are shorter than the system without FFC.

Table 2 Systems performance comparison.

Controller	t_r (sec)	t_s (sec)	P_o (%)	E_{ss} (%)
I-PDA with FFC	182.75	275.00	0.0	0.0
I-PDA	224.50	375.00	0.0	0.0

5.2 System responses with torque disturbance effect

In order to demonstrate the effectiveness of the I-PDA controller with FFC, the controller is implemented to control the speed at 1500 rpm, 1250 rpm and 1000 rpm without redesign.

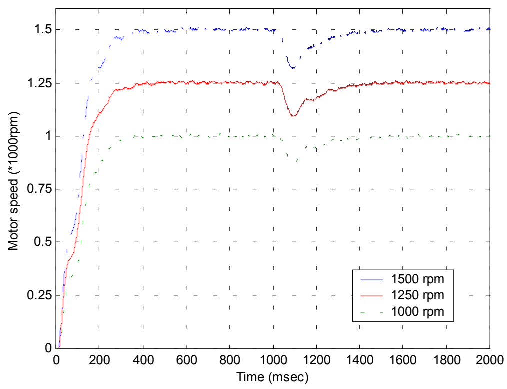


Fig.8 System responses with disturbance.

It is seen from Fig. 8 that the I-PDA controller with FFC can control each speed of the two-inertia system without overshoot P_0 and steady-state error E_{ss}. It can be noticed that the rise time and settling time of each system response almost unchanged. It means that the I-PDA controller with FFC can keep the speed of the system response of each speed control system similarly. The torque disturbance entering to the system at 1000 milliseconds is also investigated in these experiments. It is also shown that the effects of torque disturbances can be rejected.

5.3 System responses with the variation of σ_f

In this sub-section, σ_f will be varied 60%, 70% and 80% of τ in order to observe its effect. The performance of the system responses due to the variation of σ_f can be shown in Fig. 9.

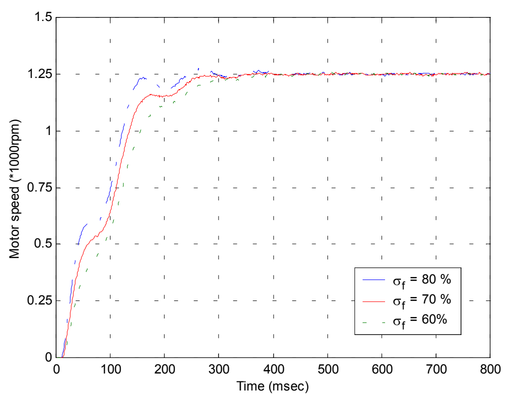


Fig. 9 System responses due to the variation of σ_f.

It is shown that when the percentage of the equivalent time constant τ greater than 60% and closed to 80% is used for designing FFC, faster rise time but higher overshoot will be obtained. Rise time and settling time can be improved. However, in case of high σ_f the worse torsion vibration will be resulted.

6. CONCLUSIONS

The speed control system using the I-PDA controller designed by CDM technique with the proposed FFC for controlling the two-inertia system has been proposed in this paper. It has been shown that the gains for the I-PDA controller can be properly designed by employing the concept of CDM. After applying FFC designed based on the advantage of CDM, the faster response of the control system was improved. Also with the proper selection of σ_f zero overshoot P_0, torsion vibration suppression and disturbance rejection can be achieved.

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