Traction Control of Automobiles using a Disturbance Observer with the Approach of Sliding Mode Control

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Abstract: This paper presents an automobile traction control system by using a sliding mode controller with disturbance observer for estimating the car-body speed. First, we show that the control system, which combines an automobile system and a disturbance observer, can be divided into a controllable system and an estimated one. And, we found out that the effect of the traction control and ABS depends on the air resistance of the car. Then, the sliding mode control system is designed using the obtained combined system. And finally, the stability of this control system is verified by simulation and it shows a very satisfactory results.

Keywords: Automobiles, traction control, antilock braking system, sliding mode control, disturbance observer.

1. INTRODUCTION

An automobile loses running stability when it slips due to the rapid acceleration or braking. The driving force of automobiles is transmitted by the frictional force between the tires and the road surface. This frictional force is a function of the car-body s weight and the friction coefficient between the tires and the road surface. The friction coefficient is also a function of the following parameters: the slip ratio determined by car-body speed, wheel speed and the condition of the road surface. Due to variations in this friction coefficient, the controlled object has uncertainty. As mentioned above, the traction control and antilock braking system (ABS) problems treated in this paper are non-linear and subject to disturbances and uncertainties. In the case of a control system designed for a controlled object with uncertainty, it is important to design a control system, which allow for uncertainties and the sliding mode control is known as one such approach [2].

Our objective is to develop a control system design to enhance the stability of the automobiles, especially during acceleration and braking. Generally, car-body speed is indispensable information for traction control and ABS. So, a disturbance observer is used to estimate the car-body speed since it is difficult to measure directly. That is: we used a simple method of calculating the car-body speed from a friction coefficient μ estimated by the disturbance observer [1].

A non-linear observer to estimate the car-body speed from the output of the system (i.e. the wheel speed) has been discussed by Unsal and Kachroo. This work gave a comparison between extended Kalman filter and sliding observer. They showed that the usage of extended Kalman filter gave an unsatisfactory result. So, they replace the estimator by the sliding observer. However, they face the problem in determining the gain coefficient for the sliding observer even though the result is satisfactory. They use the trial-and-error method, which is difficult to set. The stability of the system is not discussed, but they only state the necessary condition in order to control the wheel slip [3].

For the implementation of the system, Hori et al. proposed two types of traction control technique of electric vehicle, which are the model following control (MFC) and the optimal slip ratio control. They demonstrate the effectiveness of the system by using the test vehicle. They confirmed that MFC could reduce its torque quickly when the motor speed is suddenly increased by the tire slip. However, the stability of their control system is not discussed [4].

As to date, the stability of the system that combines the vehicle system and the disturbance observer has not been proved yet. In this paper, we first prove the stability of the combined system divided into a controllable system and an estimated one. Furthermore, we propose a new control system, which uses a disturbance observer with the approach of sliding mode control. Finally, we show that the stability of the system is achieved from the satisfactory simulation results. The verification is done via simulation using MATLAB.

2. CONTROLLED OB ECT

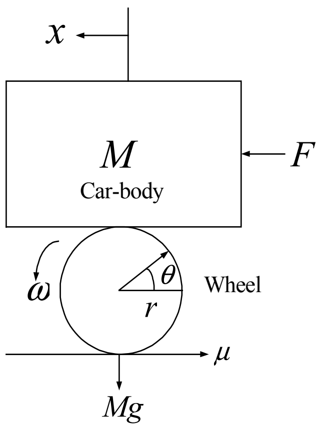


Fig. 1 Vehicle model.

Consider a vehicle model depicted in Fig. 1. The equation of motion for this controlled model is expressed by

$$\begin{cases} M\ddot{x} + C\dot{x} = \mu \\ J\ddot{\theta} = (F - \mu \quad)r \\ \mu := f(\lambda). \end{cases} \tag{1}$$

Friction coefficient μ is a non-linear function which has a slip ratio λ

$$\lambda := \frac{\omega - \quad / r}{\omega}. \tag{2}$$

For example, the μ-λ characteristic on a dry road can be expressed as shown in Fig. 2. And from Fig. 2, we define μ as

$$\mu = f(\lambda) = k_m g(\lambda) \cdot \lambda . \tag{3}$$

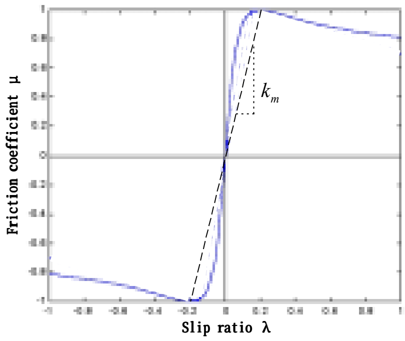


Fig. 2 μ − λ characteristics.

Substituting Eq. (2) into Eq. (3), friction coefficient μ can be expressed as

$$\mu = \frac{k_m g(\lambda)}{\omega}(\omega - \frac{}{r}) \qquad (4)$$
$$= (c_d + \delta)(\omega - \frac{}{r}),$$

where

$$c_d := \frac{k_m}{\omega_0},\ \delta := k_m \left(\frac{g\ \lambda}{\omega} - \frac{1}{\omega_0}\right).$$

Here, using $:= \dot{x},\ \omega := \dot{\theta},\ \tau := Fr$, Eq. (1) is rewritten as follows:

$$\begin{cases} M\dot{} + C\ = \mu \\ J\dot{\omega} = \tau - \mu\ \ r. \end{cases} \qquad (5)$$

Furthermore, we make the following equations from Eqs. (4), (5).

$$\dot{} = a_{11}\ + b_{m1}\mu \quad (a_{11} := -C/M,\ b_{m1} :=\ \ /M)$$
$$= (a_{11} - b_{m1}c_d/r)\ + b_{m1}c_d\omega + b_{m1}\delta(\omega -\ /r), \qquad (6)$$
$$\dot{\omega} = b_2\tau + b_{m2}\mu \quad (b_2 := 1/J,\ b_{m2} := -\ \ r/J)$$
$$= -(b_{m2}c_d/r)\ + b_{m2}c_d\omega + b_2\tau + b_{m2}\delta(\omega -\ /r).$$

Here, the following torque system is added to the controlled model in Eq. (5):

$$T_f\dot{\tau} + \tau = u. \qquad (7)$$

And we rewrite Eq. (7) as follows:

$$\dot{\tau} = a_\tau\tau + b_\tau u \quad (a_\tau := -1/T_f, b_\tau := 1/T_f).$$

From the previous-shown equations, the block diagram of the controlled object can be drawn as below.

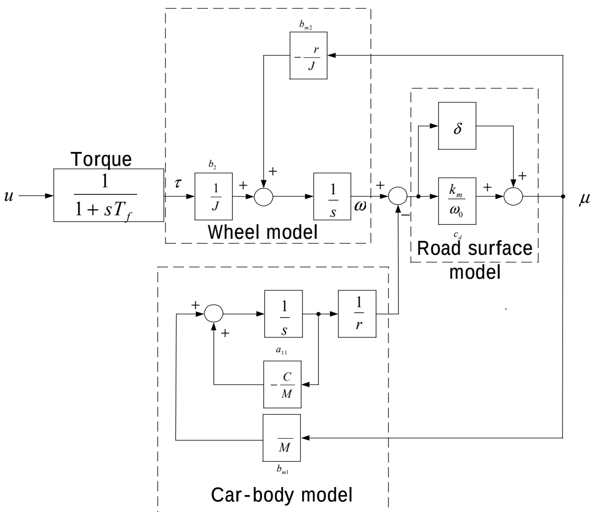


Fig. 3 Block diagram of the controlled object.

3. ESTIMATION OF THE CAR BODY SPEED BY A DISTURBANCE OBSERVER

Since the car-body speed v can not be measured directly, we estimate v by a disturbance observer.

Assuming $\dot{\mu} = 0$, the following equation is obtained

$$\begin{bmatrix}\dot{\omega}\\ \dot{\mu}\end{bmatrix} = \begin{bmatrix}0 & b_{m2}\\ 0 & 0\end{bmatrix}\begin{bmatrix}\omega\\ \mu\end{bmatrix} + \begin{bmatrix}b_2\\ 0\end{bmatrix}\tau \Rightarrow \dot{x} = Ax +\ \ u,$$
$$y = [1\ \ 0]\begin{bmatrix}\omega\\ \mu\end{bmatrix} \Rightarrow y = Cx. \qquad (8)$$

Following the Gopinath s method, we choose a transfer function as $S := \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$. Then, we obtain a minimal order observer as

$$\begin{cases} \dot{z} = Az +\ \ y + Ju \\ x = Dy + Cz \end{cases}, \qquad (9)$$

where

$$\begin{cases} A = \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix} = \begin{bmatrix}0 & b_{m2}\\ 0 & 0\end{bmatrix} \\ \ = \begin{bmatrix}b_2\\ 0\end{bmatrix} \\ A = -LA_{12} + A_{22} = -b_{m2}L \\ \ = -LA_{11} + A_{21} + AL = -b_{m2}L^2 \\ C = S^{-1}\begin{bmatrix}0\\ I\end{bmatrix} = \begin{bmatrix}0\\ 1\end{bmatrix},\ D = S^{-1}\begin{bmatrix}I\\ L\end{bmatrix} = \begin{bmatrix}1\\ L\end{bmatrix} \\ J = -L\ _1 +\ _2 = -b_2L. \end{cases} \qquad (10)$$

From Eqs. (9), (10), the minimal observer which estimate μ can be expressed as

$$\begin{cases} \dot{z} = -b_{m2}Lz - b_{m2}L^2\omega - b_2L\tau \\ \mu = L\omega + z \end{cases} \qquad (11)$$

Then, from Eq. (11), the differential of μ can be written as

$$\dot{\mu} = L\dot{\omega} + \dot{z}$$
$$= b_{m2}L(\mu - \mu). \qquad (12)$$

By substituting Eq. (4) into Eq. (12), we obtain the following equation to express $\dot{\mu}$.

$$\dot{\mu} = -(b_{m2}c_dL/r)\ + b_{m2}c_dL\omega - b_{m2}L\mu$$
$$+ b_{m2}L\delta(\omega -\ /r). \qquad (13)$$

Putting $\ \to\ $ and $\mu \to \mu$ as the estimated values, Eq. (6) can be rewritten as

$$\dot{} = a_{11}\ + b_{m1}\mu\ , \qquad (14)$$

where a_{11} is the value with a perturbation.

4. COMBINED SYSTEM

From Eqs. (6), (7), (13) and (14), the state-space equation can be expressed as

$$\begin{bmatrix} \dot{}_{0} \\ \dot{\omega} \\ \dot{\tau} \\ \dot{\mu} \\ \dot{}_{0} \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 & b_{m1}/r & 0 \\ 0 & b_{m2}c_d & b_2 & 0 & -b_{m2}c_d \\ 0 & 0 & a_\tau & 0 & 0 \\ 0 & b_{m2}c_d L & 0 & -b_{m2}L & -b_{m2}c_d L \\ 0 & b_{m1}c_d/r & 0 & 0 & a_{11}-b_{m1}c_d/r \end{bmatrix} \begin{bmatrix} {}_{0} \\ \omega \\ \tau \\ \mu \\ {}_{0} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 0 \\ b_\tau \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ b_{m2} \\ 0 \\ b_{m2}L \\ b_{m1}/r \end{bmatrix} \delta\,(\omega - {}_{0}) \qquad (15)$$

where ${}_0 := \ /r$, ${}_0 := \ /r$ is defined.

We choose a transform matrix T as follows:

$$\begin{bmatrix} {}_{0} \\ \omega \\ \tau \\ \mu \\ \eta \end{bmatrix} = T \begin{bmatrix} {}_{0} \\ \omega \\ \tau \\ \mu \\ {}_{0} \end{bmatrix}, T := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -b_{m1}/(b_{m2}Lr) & 1 \end{bmatrix}.$$

Then, Eq. (15) can be written as

$$\begin{bmatrix} \dot{x} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} A & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x \\ \eta \end{bmatrix} + \begin{bmatrix} \\ 0 \end{bmatrix} u + \begin{bmatrix} {}_{m} \\ 0 \end{bmatrix} \mu_d , \qquad (16)$$

where

$$x := \begin{bmatrix} {}_{o} \\ \omega \\ \tau \\ \mu \end{bmatrix}, A := \begin{bmatrix} a_{11} & 0 & 0 & b_{m1}\ \ r \\ -b_{m2}c_d & b_{m2}c_d & b_2 & -b_{m1}c_d\ \ Lr \\ 0 & 0 & a_\tau & 0 \\ -b_{m2}c_d L & b_{m2}c_d L & 0 & -\ b_{m2}L + b_{m1}c_d\ \ r \end{bmatrix}$$

$$A_{12} := \begin{bmatrix} 0 \\ -b_{m2}c_d \\ 0 \\ -b_{m2}c_d L \end{bmatrix}, \ := \begin{bmatrix} 0 \\ 0 \\ b_\tau \\ 0 \end{bmatrix}, {}_{m} := \begin{bmatrix} 0 \\ b_{m2} \\ 0 \\ b_{m2}L \end{bmatrix} \ \mu_d := \delta\ \ \omega - {}_{0}$$

$$A_{21} := \begin{bmatrix} a_{11} - a_{11} & 0 & 0 & a_{11}b_{m1}\ \ b_{m2}Lr \end{bmatrix}, A_{22} := a_{11}.$$

From Eq. (16), Fig. 4 is obtained.

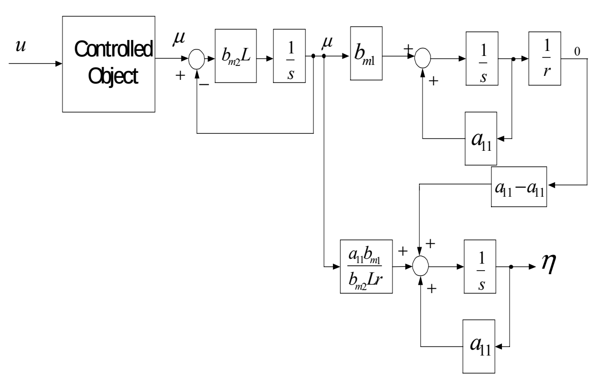


Fig. 4 Combined systems.

If $a_{11} = a_{11}$ and the observer gain $|L| >> 0$, then $a_{11}b_{m1}/(b_{m2}Lr) \to 0$ and Eq. (16) can be separated into a controllable and uncontrollable state value as expressed as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A & A_{12} \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} \\ 0 \end{bmatrix} u + \begin{bmatrix} {}_{m} \\ 0 \end{bmatrix} \mu_d . \qquad (17)$$

Here, η is defined as $e := {}_{o} - {}_{o}$.

If the following control law is applied

$$u = \begin{bmatrix} -F & 0 \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} ,$$

Eq. (17) can be rewritten as

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - \ F & A_{12} \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} {}_{m} \\ 0 \end{bmatrix} \mu_d . \qquad (18)$$

If A, is controllable (see Appendix), and A_{22} takes a negative value (that is: if the combined system is stabilizable), we can stabilize the system expressed in Eq. (18). Here, A_{22} takes a negative value, since it is considered as the air resistance of the car, etc.

From the result above, we found out that the effect of the traction control and ABS depends on the air resistance of the car.

5. SLIDING MODE CONTROL

When we have enough time to make e in Eq. (17) becomes zero. Then, Eq. (17) can be expressed as

$$\dot{x} = Ax + \ u + \ {}_{m}\mu_d . \qquad (19)$$

By considering only a linear term in Eq. (19), the equation for a reference model is obtained as

$$\dot{x}_r = Ax_r + \ u_r . \qquad (20)$$

Then, by subtracting Eq. (20) from Eq. (19), an error system can be expressed by

$$\dot{x}_e = Ax_e + \ u_e + \ {}_{m}\mu_d$$
$$x_e := x - x_r \ , \ u_e := u - u_r \ . \qquad (21)$$

Defining a sliding surface as $S = \ x_e$, the control law u_e as $S \to 0\ (t \to \infty)$ is separated into a linear term u_1 and a non-linear term u_2 , which is $u_e = u_1 + u_2$.

5.1 Linear Controller

From Eq. (21), we assume a non-linear term $\mu_d = 0$. Then, we obtain

$$\dot{x}_e = Ax_e + \ u_e , \qquad (22)$$

where

$$u_e = -Fx_e \quad (u_1 := -Fx_e \ , \ u_2 := 0) .$$

Next, we consider $\dot{S} = 0$:

$$\dot{S} = \ \dot{x}_e = \ \ A - \ F\ \ x_e = 0 . \qquad (23)$$

Then, a linear feedback gain F can be expressed as

$$F = \ \ {}^{-1}\ \ A \qquad (24)$$

By substituting Eq. (24) into Eq. (22), the following equation is obtained

$$\dot{x}_e = Ax_e , \quad A := A - \ (\ \ )^{-1}\ \ A . \qquad (25)$$

The eigen values of A have the same number of zeroes as the input variables and the zeroes of $\ (sI - A)^{-1}$. And, it is known that they are stable when

$$= \ {}^{T}P , \qquad (26)$$

where P is a positive definite solution which satisfy the Riccati equation:

$$PA + A^T P - P\ \ {}^{T}P + \ = 0 \quad \ > 0 . \qquad (27)$$

5.2 Non-linear Controller

In order to stabilize a non-linear control system, we consider a Lyapunov function as

$$V(S) = S^T S . \qquad (28)$$

From Eq. (28), the differential of $V(S)$ can be expressed as

$$\dot{V}(S) = (\quad u_2 + \quad {}_m\mu_d)^T S + S^T (\quad u_2 + \quad {}_m\mu_d). \quad (29)$$

Substituting

$$u_2 = R\frac{S}{\|S\|},\ R = -\rho(\quad)^T \ (\rho : const),\ \| \quad {}_m\mu_d \| \le \gamma,$$

we obtain the following equation to express

$$\dot{V}(S) \le \{ -\rho \cdot \lambda_{\min}(\quad)(\quad)^T + \gamma \} \times 2\|S\|.$$

Here, when satisfying

$$\rho > \frac{\gamma}{\lambda_{\min}(\quad)(\quad)^T},$$

we obtain $V(S) > 0$, $\dot{V}(S) < 0 \ (S \ne 0)$.

Therefore $S \to 0$ as $t \to \infty$ is obtained. This clearly indicates that this control system is stable.

Fig. 5 shows the sliding mode control system, which is obtained from the above results.

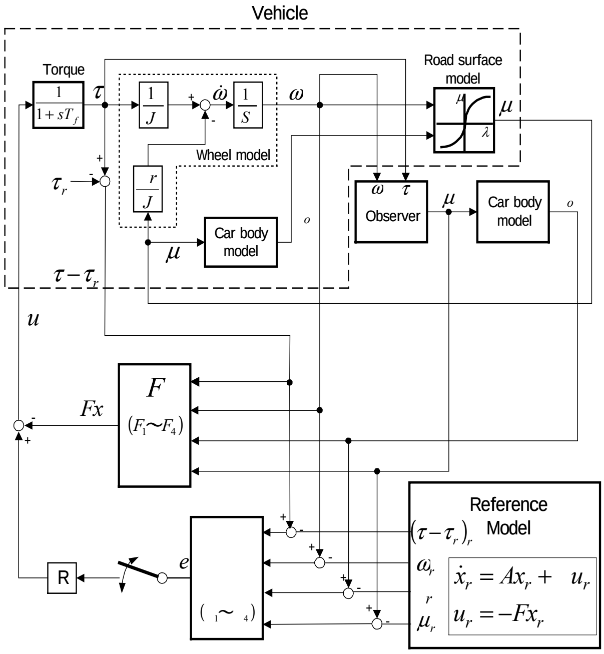


Fig. 5 Sliding mode control system.

. SIMULATION RESULTS

Simulations were carried out with all the parameters were set as follow:

$T_f = 0.2\,[\mathrm{sec}]$	: Torque s time constant
$M = 1500\,[kg]$	: Vehicle mass
$C = 0.01\,[kgs/m]$	: Friction of vehicle
$= 0.6 * M * g\,[N]$	: Vehicle weight
$J = 0.28\,[kgm^2]$	: Moment of inertia for wheel
$r = 0.3\,[m]$	: Wheel radius
$g = 9.8\,[m/s^2]$	: Acceleration of gravity
$\omega_0 = 10[m/\mathrm{sec}]$	: Rated wheel speed
$L = -100$	: Observer gain

For k_m in Eq. (3), the following value is derived from Fig. 2.

$$k_m = 1.0/0.2 = 5$$

Weighting matrix in the Riccati equation is chosen as

$$= \begin{bmatrix} q1 & 0 & 0 & 0 \\ 0 & q2 & 0 & 0 \\ 0 & 0 & q3 & 0 \\ 0 & 0 & 0 & q4 \end{bmatrix} \begin{cases} q1 = 10^9 \\ q2 = 0 \\ q3 = 0 \\ q4 = 0 \end{cases}.$$

Then, the matrices for a linear feedback gain F and a non-linear feedback gain is obtained

$$F = [-15.5 \quad 15.5 \quad 4.3 \quad -0.0031]$$
$$= [30981 \quad 628 \quad 8.6 \quad 6.3]$$

Setting $\gamma = 1500$ that satisfy

$$\| \quad {}_m\mu_d \| \le \gamma \ : \ |\mu_d| < 0.65 \times 10^{-3},$$

we obtain the switching gain $R = -34.8$.

The simulation results for a frozen road (µ 0.1) are shown in Figs. 6-8. An Input torque τ is assumed as follows: an accelerator pedal is stepped on so as to give a torque of 500 Nm at 0 sec, the input torque is reduced to 0 Nm after 15 sec, and the reverse torque of 400 Nm is given after 25 sec.

Fig. 6 shows the speed response for the non-control case. This figure indicates that the wheel speed is extremely higher or lower than the car-body speed.

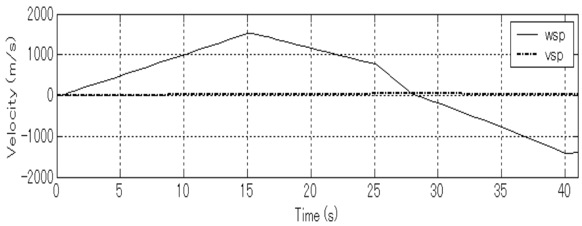
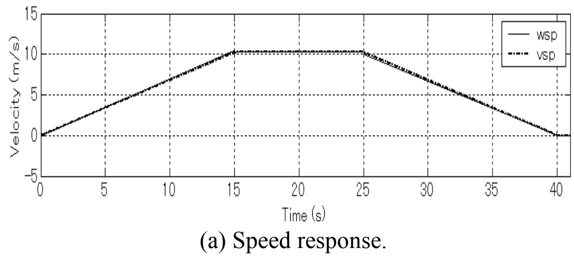
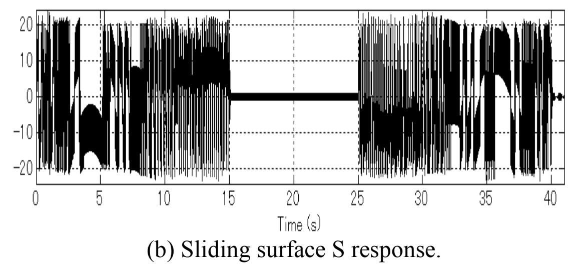


Fig. 6 Speed response (Non-control).

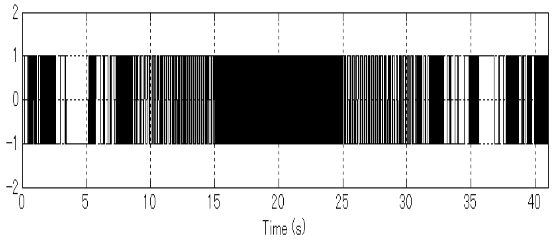
The simulation results for sliding mode control designed at $\gamma = 1500$ are plotted in Figs 7. The response of the wheel speed and the car-body speed is shown in Fig. 7(a). While, the response of the sliding surface S and the switching function are plotted in Fig. 7(b) and 7(c) respectively. From Fig. 7(a), it is clearly shown that the wheel speed is exactly follows the car-body speed. It verifies a very satisfactory performance as compare to the non-control case depicted in Fig. 6.



(a) Speed response.

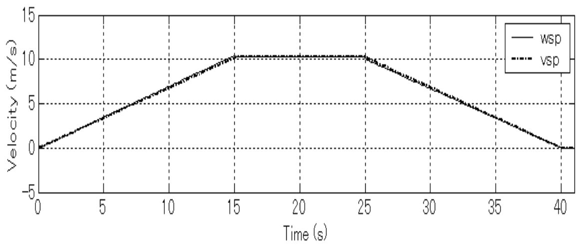


(b) Sliding surface S response.

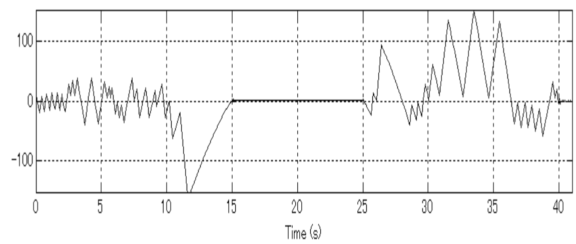


(c) Switching function S/||S||.
Fig. 7 Sliding mode control ($\gamma = 1500$).

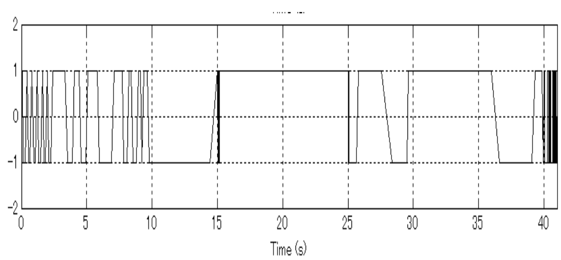
The simulation results for sliding mode control, which is designed at $\gamma = 150$ (that is: the estimate of uncertainty is smaller than the actual value) are shown in Fig. 8. From Fig. 8(b), it is shown that the sliding surface S is disturbed during acceleration and deceleration. Then, we can see that the switching in Fig. 8(c) hardly occurs.



(a) Speed response.



(b) Sliding surface S response.



(c) Switching function S/||S||.
Fig. 8 Sliding mode control ($\gamma = 150$).

Next, the verification of an automobile performance when the road condition suddenly changes from the dry road (µ 0.8) to the wet road (µ 0.1) is done. We assume that the transition happens at 20 sec after the start. The speed response for the non-control case is plotted in Fig. 9. It is shown that the wheel speed follows the car-body speed on the dry road. However, the tires slip just after the transition.

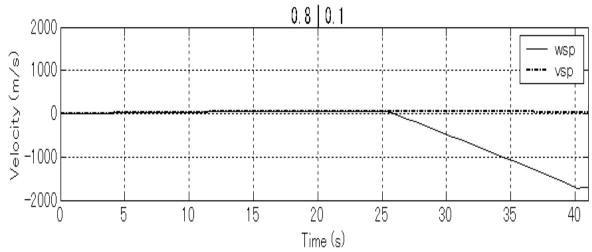
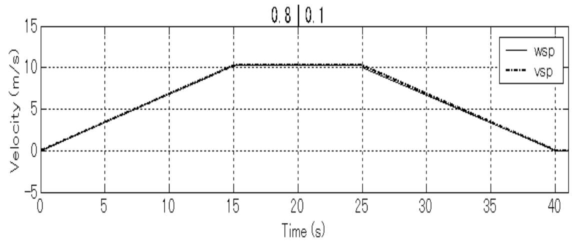
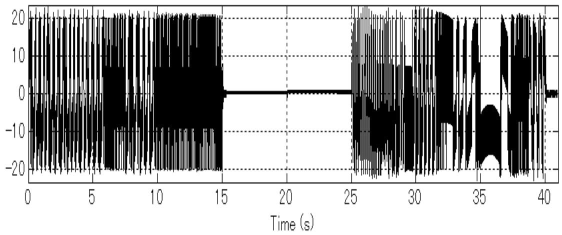


Fig. 9 Speed response (Non-control, µ 0.8 - 0.1).

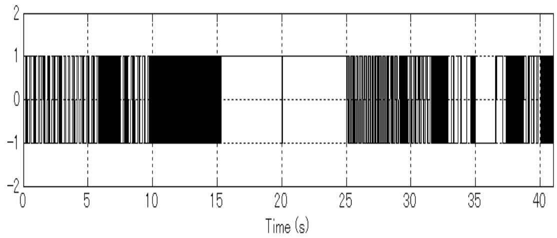
Then, the performance verification is done using the proposed controller. As indicated in Fig. 10, the performance of automobile is good even though the transition on the road condition happened.



(a) Speed response.

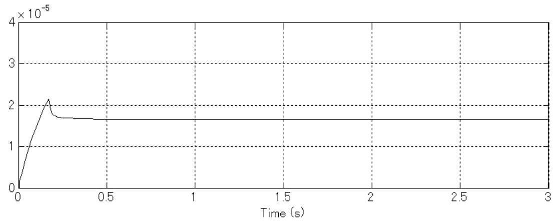


(b) Sliding surface S response.

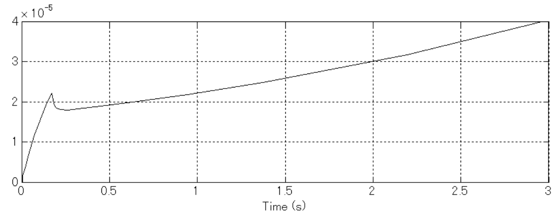


(c) Switching function S/||S||.
Fig. 10 Sliding mode control ($\gamma = 1500$, µ 0.8-0.1).

Fig. 11 shows the deviation e between the estimated value $_0$ and the car-body speed $_0$. From Fig. 11, we can confirm that if $a_{11} < 0$ in Eq. (16), e is almost 0, while if $a_{11} > 0$, e takes a large value. This shows that $_0$ and $_0$ are equal when $a_{11} < 0$, but $_0$ and $_0$ are not equal when $a_{11} > 0$. Here, even though $a_{11} < 0$, the deviation e is not 0 because the car is accelerating.



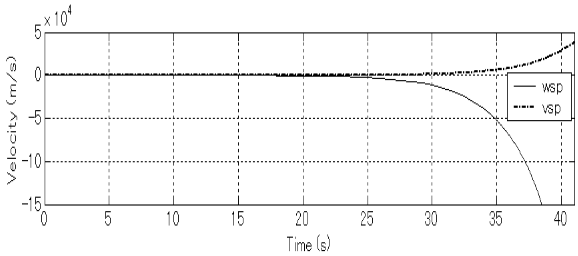
(a) a_{11} :   -6.6667e^{-006} (negative).



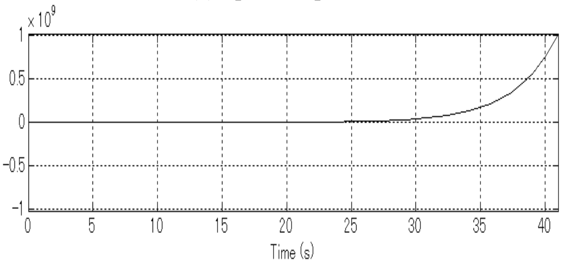
(b) a_{11} :   0.3 (positive).

Fig. 11 Relationship of $_0$ and $_0$.

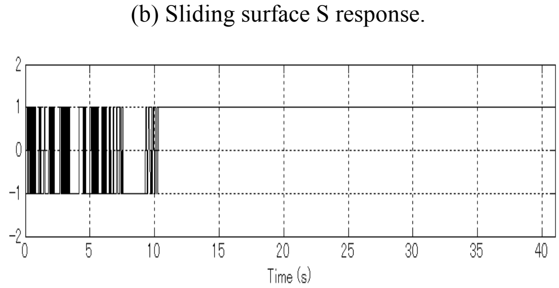
This unstable condition (when $a_{11} = 0.3$ is taken) is also shown in the following figure. From Fig. 12(a), it is clearly shown that the speed response is unstable even though it is under sliding mode control. Switching also hardly occurs as can be seen in Fig. 12(c).



(a) Speed response.



(b) Sliding surface S response.



(c) Switching function S/||S||.

Fig. 12 Sliding mode control ($a_{11} = 0.3$).

7. CONCLUSION

The proposed sliding mode controller showed the very satisfactory simulation results. And, we found out that the effect of the traction control and ABS depends on the air resistance of the car. A further study based on the experiment will be done to check the feasibility of the proposed controller.

However, the implementation of sliding mode control requires a certain sampling interval in which the control is constant. Thereby the switching frequency is limited by the sampling frequency, and the chattering or oscillation problems may arise [5]. Therefore, the implementation of discrete sliding mode control in our system will be considered in future study.

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APPENDI

1. The controllability of $(A, \ )$ in Eq. (18):

$\mathrm{rank(Uc)} = 4$ is obtained from

$$Uc := [\ , A\ , A^2\ , A^3 b]$$

$$= \begin{bmatrix} 0 & 0 & 0 & b_{m1}/r \cdot b_{m2} c_d L b_2 b_\tau \\ 0 & b_2 b_\tau & (b_{m2} c_d + a_\tau) b_2 b_\tau & \left\{ b_{m2} c_d (b_{m2} c_d + a_\tau - b_{m1} c_d / r) + a_\tau^{\ 2} \right\} b_2 b_\tau \\ b_\tau & a_\tau b_\tau & a_\tau^{\ 2} b_\tau & a_\tau^{\ 3} b_\tau \\ 0 & 0 & b_{m2} c_d L b_2 b_\tau & b_{m2} c_d L b_2 b_\tau \left\{ b_{m2}(c_d - L) + a_\tau - b_{m1} c_d / r \right\} \end{bmatrix}$$

The result above shows that $(A, \ )$ is controllable.