# An Efficient Method for the Mass Unbalance Analysis of a Rotor System Using FFT and Lissajous Diagram

Hua Su*, and Kil To Chong**

* Department of Control and Instrumentation Engineering, Chonbuk National University, Jeonju, Korea
(E-mail: hua_su@chonbuk.ac.kr)

**Department of Control and Instrumentation Engineering, Chonbuk National University, Jeonju, Korea
(Tel : +82-63-270-2478; E-mail: kitchong@chonbuk.ac.kr)

**Abstract**: Unbalance analysis is essential in the rotor system. However, some problems still remain in the aspects of computational efficiency and accuracy. In the present paper a new method is proposed for estimating the mass unbalance of a rotating shaft by using the vibration signals. This is an advanced new method for the detection of a mass unbalance and its phase position. Based on the signal processing with FFT, an estimator is designed to detect the mass of unbalance. And an improved Lissajous diagram is also introduced with statistical analysis, which make it possible to compute the phase position of the mass unbalance efficiently and arranged at a certain location of the shaft. The proposed method is demonstrated and validated through several test examples.

**Keywords:** rotor system, mass unbalance, FFT, Lissajous diagram

## 1. INTRODUCTION

The rotor unbalance considered does not only cause vibration, it also transmits rotational force to the rotor system and to the supporting structure. The forces thus transmitted may damage the machine and shorten its working life [1]. Very often, a mass unbalance results very dangerous damage in rotor system, which can lead to a catastrophe. Actually, there have been many reports to these disasters [2-4]. Thus, it is necessary to estimate the mass unbalance and its phase position, as carefully as possible, to keep the stability of a system, to guarantee the safety for the men and to save the running cost.

Here, a suitable step should be taken. Over the years, several methods have commonly been employed. The modal balancing technique was developed and further investigated by several researchers [5-7], but in this method, the critical speed mode shapes of the rotor must be known in advance and mass distribution determined from the geometry of the rotor is not accurate. Also the finite element method (FEM) played an important role in the analysis of rotor system because of its usefulness in vibration diagnosis. However, there remain some difficulties in the computational aspect of unbalance analysis due to its inconvenient classical modal and complicated dynamics properties such as rotational speed dependency and anisotropy [8]. Furthermore, various field unbalance analysis methods have also been developed and discussed [9], but due to the deficiencies in the former methods for detecting the mass unbalance, the results are still not satisfactory.

Therefore, in the present paper, a new method for estimating the mass unbalance during the process of operation is proposed, which can indicate the unbalance weights and phase position of the rotor efficiently. The proposed method is based on the FFT and Lissajous diagram of analyzing the vibration signal with statistical method. The rotor system considered here is a motor and clutch arrangement, through which the rotor is brought up to a predetermined speed to obtain unbalance vibration signal and reference time signal of the rotor. By transforming the vibration signal into its spectrum with FFT, the mass unbalance can be distinguished by the designed estimator. Next, the phase position can then be obtained by computation of the Lissajous diagram with combined uses of the vibration signal and reference time signal after statistical signal processing. The proposed method causes no errors, even though the computational time is drastically reduced.

Finally, several test examples are also conducted to validate the efficiency and applicability of the proposed method. In these tests, the rotor is brought up to several different speeds with varying mass unbalances to analyze the results of the proposed method.

## 2. PROPOSED MASS UNBALANCE ESTIMATION SYSTEM

### 2.1 Design of the Detection System

In this section, the proposed new method is presented, in which the mass unbalance and its phase position are detected.

The criteria to detect a mass unbalance are based on the magnitude of forces. In order to measure the forces, two force transducers are used to measure the vertical vibration forces exhibited by the mass unbalance. An optical sensor is also used to produce the reference time signal of the rotor rotation. The rotor system with the sensors is illustrated in Fig. 1.

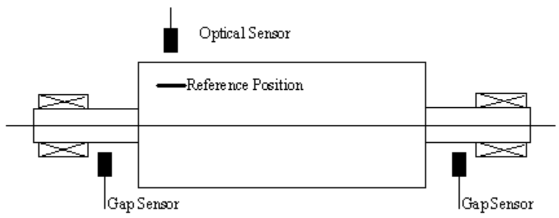


Fig. 1 The structure of rotor system

The proposed mass unbalance detection system is schematized in Fig. 2. The maximal magnitudes of the force from the two transducers are compared in order to select a vibration signal for analyzing. The chosen vibration signal $V(t)$ and time reference signal $T(t)$ are converted to $V_{DS}(t)$ and $T_{DS}(t)$ by the low pass filtering and down-sampling to filter out the noise of the motor and support bearings, so that the proper analysis result can be obtained. The vibration signal $V_{DS}(t)$ is used to design the unbalance detector $I_{rate}$. When the rotor system is determined to be unbalance, the spectrum $V_{DS}(f)$ is obtained by using FFT, which produces the mass unbalance estimator $M_f$. To detect the phase position of the mass unbalance, the vibration signal

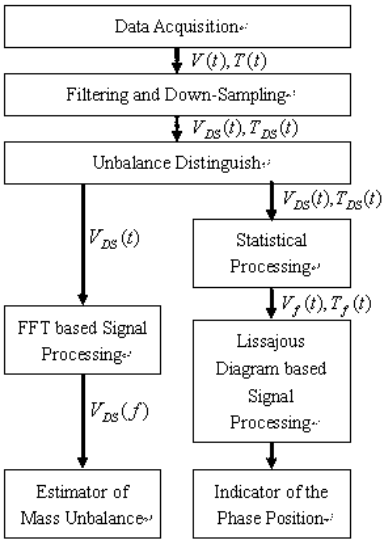


Fig. 2 Proposed mass unbalance analyze system

$V_{DS}(t)$ and time reference signal $T_{DS}(t)$ are processed statistically to produce the $V_f(t)$ and $T_f(t)$, which Lissajous diagram analysis can use. With the implementation of the algorithm introduced in this paper, the Lissajous diagram can be used to detect the phase position with the phase relation between $V_f(t)$ and $T_f(t)$.

**2.2 Unbalance Detector**

As it has been known, the forces of mass unbalance are a clear indicator for the existence of a mass unbalance on the rotor, so by analyzing the vibration signal obtained from the force transducers, it is possible to detect the existence of the mass unbalance efficiently.

The design of unbalance detector is to produce a model to distinguish the balance or unbalance vibration signals. This research focuses on the development of an unbalance detector that uses the statistical analysis of vibration signals.

Statistical analysis is mostly applied to random signals where other signal analysis methods based on the assumptions on deterministic signals are not applicable [10]. Here, the root mean square value (RMS) of vibration signal magnitude is used for the detector model.

The vibration signals of a rotor system contain both the transient signals and steady periodical signals. If the rotor system is balance, the vibration signals are mainly composed of transient signals, while the centrifugal force must also be detected if the rotor system is unbalance. From the work by Kent [11], the centrifugal force is,

$$F = \frac{mN^2 D}{7200} \tag{1}$$

where $m$ is the mass of unbalance, $N$ is the rotation speed in RPM, and $D$ is the diameter of the rotor. Thus, it is easy to know that the amplitude of the unbalance vibration signal varies with respect to the mass of unbalance and rotation speed.

Let us assume that the time domain vibration signal $V_{DS}(t)$ can be described as,

$$V_{DS}(t) = A\cos(2\pi f_r t + \phi) + V_N(t) \tag{2}$$

where $A$ is the amplitude of vibration, $f_r$ is the rotational speed of unbalance rotor, $\phi$ is the initial phase position, and $V_N(t)$ is the signal of random noise.

From Eqs. (1) ~ (2), we find that the amplitude of the vibration can reveal existence of mass unbalance. As it is difficult to distinguish the vibration from the noise in the time domain, the Fourier transform is used to convert the $V_{DS}(t)$ into its frequency domain form $V_{DS}(f)$.

The Fourier transform is based on Fourier series, which are expansions of periodic functions in terms of an infinite sum of sines and cosines. Because the data obtained are discrete data, the discrete Fourier transform (DFS) is used to handle the discrete sampled value. The DFT is given by,

$$X_k = \sum_{n=0}^{N-1} x(n)\exp(-j2\pi kn/N) \tag{3}$$

where $X_k$ is expressed as discrete Fourier coefficient, $N$ is the frame size and $x(n)$ is the input signal on time domain. To realize the computation of DFT, the FFT method is used here.

The FFT is a well-established technique, based on the split-radix algorithm [12], which minimizes the number of multiplications and additions. The amplitude of vibration signal on frequency domain $V_{DS}(f)$ is obtained by,

$$V_{DS}(f) = FFT\{V_{DS}(t)\} \tag{4}$$

According to this, in a balance system, the RMS value of the magnitude of vibration signal will be a very small value since the balance vibration signals do not contain the unbalance centrifugal force.

Therefore the unbalance detector can be designed to distinguish the existence of mass unbalance. If the value of the detector is less than the threshold value, it signifies a balance state. Otherwise, it means a mass unbalance exists. The RMS value in frequency domain is obtained as,

$$V_{RMS} = \sqrt{\frac{1}{N}\sum_{n=1}^{Nf}[V_{DS}(f)]^2} \tag{5}$$

And the detector $I_{rate}(t)$ is defined as,

$$I_{rate}(t) = V_{RMS}(i) \tag{6}$$

where i=1, 2, 3 …..N represents the ith sampling period. The threshold value is an arbitrary small number defined by the balance data model.

**2.3 Mass Unbalance Estimator**

From the above section, it is known that the periodical signal represents the vibration of the mass unbalance because of the centrifugal force. For the practical unbalance distribution of a rotor, convergence conditions [13] are always satisfied and would not cause any convergence problem. So the unbalance distribution should converge at some point of the rotor. In this paper, the work focuses on the detection of the equivalent mass unbalance and its phase position of such point. We find that the amplitude of the vibration can reveal the mass of unbalance, making it possible to estimate the mass by analyzing the amplitude of vibration signal.

The spectral have been obtained by Eq. (4). Each value displays the energy of harmonics. Let us use the $M_f$ as an estimate of the mass unbalance of the vibration signal, then,

$$M_f = \max\{V(f) \mid f \subset B_r\} \tag{7}$$

where $B_r$ is the frequency band of rotor rotation speed

$[f_r - \delta, f_r + \delta]$ and $\delta$ is the frequency correction factor which will be explained later. The maximum value reveals the mass of unbalance, in comparison with unit mass amplitude $M_{UNIT}$ which is defined in advance by experiment; the weight can be shown as,

$$W = M_f / M_{UNIT} \quad (8)$$

There is an inaccurate problem provided by the FFT method, which results in the spectral leakage. The spectral leakage causes the spreading of the energy distribution of each harmonic. To reduce this problem, windowing must be applied to sampled time data.

In this paper, the Hanning window is used, defined as,

$$W(n) = 0.5 - 0.5\cos(2\pi n / N + 1) \quad (9)$$

where $n$ is taken from 0 to $N$ in each case. So the new spectral of vibration signal $V_W(t)$ is expressed as,

$$V_W(f) = FFT\{V_W(t)\} = V_{DS}(f) * W(f) \quad (10)$$

Because of the spectral leakage, the interpolation for the frequency of amplitude is also needed. Letting the highest peak be at the $i$ th value with a magnitude $v_i$ and defining,

$$r = \frac{v_i}{\max(v_{i+1}, v_{i-1})} \quad (11)$$

then interpolation involves calculating a frequency correction factor $\delta$ so that the improved peak position is $f(v_i) + \delta$. The correction factor is a function of r which depends on the data window being used. The general relationship between $\delta$ and $r$ is obtained from the solution of,

$$0 = \delta \sum_{m=0}^{N/2} \frac{(-1)^m a_m}{\delta^2 - m^2} - r(\delta - 1) \sum_{m=0}^{N/2} \frac{(-1)^m a_m}{(\delta - 1)^2 - m^2} \quad (12)$$

where $a_m$ are the weighting coefficients of the data window. As the Hanning window is used in this research, Eq. (9) yields,

$$\delta = \frac{2 - r}{1 + r} \quad (13)$$

The algorithm assumes that the spreading of two close frequency harmonics will not overlap since then the value of $\max(v_{i+1}, v_{i-1})$ will be subject to error. This error can be reduced by decreasing $f_{res}$ which decreases the spectral leakage.

### 2.4 Phase Position Estimator

It is feasible to balance the mass unbalance rotor system by adding weights or digging holes at a given point. Therefore the phase position is the most important and difficult in unbalance analysis.

Only with the vertical vibration signal, it is impossible to find the position of the mass unbalance. So the time reference signal is also used in this proposed method. The time reference signal is produced by the optical sensor, whose frequency is the same as the vibration signal. After low pass filtering and down-sampling, the two signals are sine-wave signals with the same frequency, which is equal to the rotor rotation speed. Assuming the time signal is defined as,

$$T_{DS}(t) = b_0 + b_1 \cos(2\pi f_r t + \theta) \quad (14)$$

where $b_0$ is the base amplitude because of the optical sensor, $b_1$ is the amplitude of time signal, $f_r$ is the frequency of rotational speed of unbalance rotor, and $\theta$ is the initial phase position.

There is a phase lag between the vibration signal and time signal according to the position of the mass unbalance on the rotor. By calculating the phase lag, it is possible to find the phase position of the mass unbalance with the reference point. A Lissajous diagram is used in this paper to calculate the phase lag.

The Lissajous diagram is a basic approach to determine the relative characteristics of two sources, primarily their frequency and phase relations. By applying two signals as vertical axis and horizontal axis inputs, an ellipse trace can be obtained, except for the phase lag is the multiple of $\pi/2$. From the ellipse curve, the phase lag between two inputs can be obtained as,

$$\sin\phi = \frac{y - \text{intercept}}{y - \text{amplitude}} = \frac{y_1}{y_2} \quad (15)$$

where $\phi$ is the phase lag between two signal ($0 < \phi < \pi/2$), $y_1$ is the value where $x$ is 0 and $y_2$ is the max value of magnitude.

The time signal is set as x input while vibration signal is y input. As the two signals are periodical signals, for the calculation of Lissajous diagram, the statistical method is used to process the signals. There is an offset $b_0$ in the time signal from the x axis, which is the mean value of the time signal, and is removed to make time signal a sine wave signal on x axis. Here the mean value is used to get the intercept value and amplitude value of the vibration periodical signal. The y axis intercept value is obtained as,

$$y_1 = \frac{1}{N} \sum_{n=1}^{N} V_{\text{intercept}} \quad (16)$$

where $N$ is the number of periods of the vibration signal, $V_{\text{intercept}}$ is the magnitude value of each period where time signal magnitude is 0. And the y axis amplitude value is obtained as,

$$y_2 = \frac{1}{N} \sum_{n=1}^{N} V_{amplitude} \quad (17)$$

where $N$ is the number of period of vibration signal, $V_{amplitude}$ is the max value of the magnitude in each period. So the phase lag $\phi$ can be obtained from $y_1$ and $y_2$.

As the Lissajous diagram only can indicate the phase lag less than $\pi$, a modified Lissajous diagram is introduced in this paper. The new algorithm divides the Lissajous diagram into four forms according to clockwise rotation or anti-clockwise rotation of the diagram. By distinguishing the diagram form first, the phase lag can be calculated in $2\pi$ range. The algorithm is illustrated in Fig. 3.

So the phase lag $\alpha$ of the mass unbalance on the rotor is defined as,

$$\alpha = \begin{cases} \phi & 0 \le \alpha < \pi/2 \\ \pi - \phi & \pi/2 \le \alpha < \pi \\ \pi + \phi & \pi \le \alpha < 3\pi/2 \\ 2\pi - \phi & 3\pi/2 \le \alpha < 2\pi \end{cases} \quad (18)$$

where $\alpha$ is the phase lag on the rotor, $\phi$ is obtained in Eq. (15).

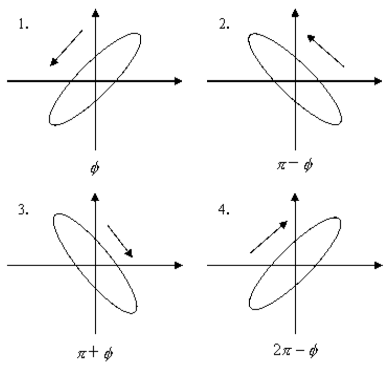


Fig. 3 Phase lag algorithm based on Lissajous Diagram

Therefore the phase position of the mass unbalance on the rotor can be calculated according to the phase lag. The geometries of a rotor with unbalance mass are shown in Fig. 4. First, a zero reference point of the mass unbalance is defined on the rotor. The $\alpha_0$ is the phase lag between the zero reference point and time reference point, which is obtained in advance by experiment. Then the calculated position of mass unbalance would be,

$$\alpha_{MASS} = \alpha - \alpha_0 \tag{19}$$

With the reference of zero reference point, the position of mass unbalance can be located efficiently and accurately through this method.

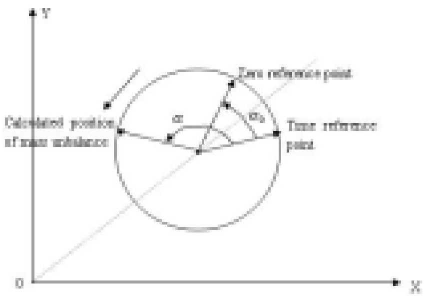


Fig. 4 geometries of a rotor with unbalance mass

## 3. EXPERIMENTS AND ANALYSIS

The performance of the mass unbalance detection system is investigated by applying the proposed method to a real plant. Several examples are presented to illustrate the feasibility and applicability of the proposed method.

### 3.1 Setup of Experimental System

As shown in Fig. 5, the structure of an experimental rotor system is consists of a test rig, the measuring transducer, an ADC model and an analysis computer. The experiment equipment is shown in Fig. 6.

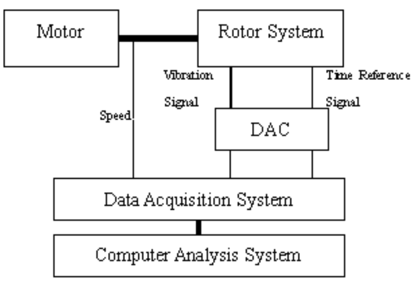


Fig. 5 Structure of Experiment System

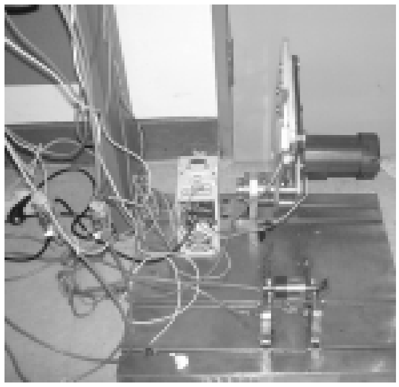

Fig. 6 Experimental Rotor System

The rotor is driven by a motor through a belt and pulley arrangement and is brought up to a predetermined RPM value for test. A pair of piezoelectric sensors are mounted in the pedestal adjacent to and spaced axially along the rotor and an optical sensor above the rotor. The positions of sensors are shown in Figure 1. The force transducers are coupled mechanically to the shaft and provide periodic electrical output signals indicative of unbalance forces transmitted through the shaft when the rotor is driven rotationally. The reference position of the rotor is monitored with respect to the optical sensor during every rotation. The vertical force exhibited by the mass unbalance and rotation time data are converted to digital form through the use of NI 6070E devices. The data is stored by the data acquisition system from which the mass unbalance and phase position can be calculated. A computer program is developed to analyze the data. The sampling frequency is 10800Hz and the rotation speed of the rotor is varied from 360 to 2160 RPM.

### 3.2 Result and Analysis of Experiments

First the results of unbalance detection are shown in Fig. 7 to 9.

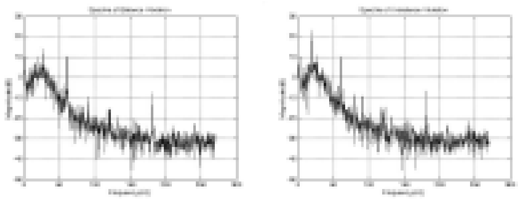

Fig. 7 Vibration Spectra of Balance and Unbalance Rotor

The vibration spectra on balance rotor and unbalance rotor are compared in Fig. 7. It is hard to tell the existence of mass unbalance with the naked eye. However, the indicator of mass unbalance is shown in Fig. 8 and 9.

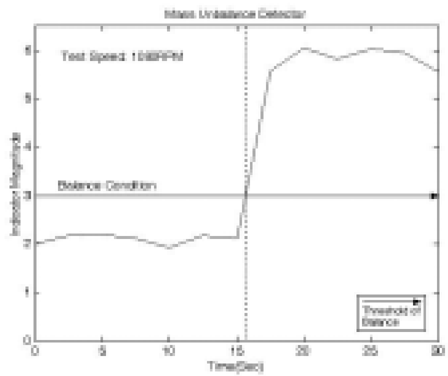


Fig. 8 Unbalance Detection with 2.1g at 1080RPM Test Speed

The two figures show that the indicator can tell the existence of mass unbalance easily according to the threshold value. The test speed is 1080RPM with 2.1g and 3.2g mass unbalance respectively. In the first 15 seconds, the rotor is in balance status and then a mass unbalance is added to the rotor. The threshold value is set to 3 according to the experimental data.

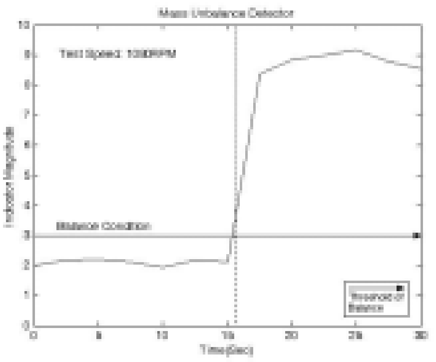


Fig. 9 Unbalance Detection with 3.2g at 1080RPM Test Speed

Fig. 12 through 14 show the three experimental results for the weight estimator of mass unbalance. The four test masses of unbalance on the rotor are set to 1g, 2.1g and 3.2g in one position and two separate 1g in adjacent positions, respectively. The phase between the separate 1g mass unbalance is 45 degrees. The spectra for the mass unbalance vibration signal is shown in Figure 12 and enlarged in Figure 13 to focus on the amplitude. The test speed is 1080 RPM and it is easy to find that the mass of unbalance is different from the spectra. From the above section, we know that the higher amplitude signifies a state of more mass unbalance. But it is still difficult to tell the exact weight of the unbalance with the spectra.

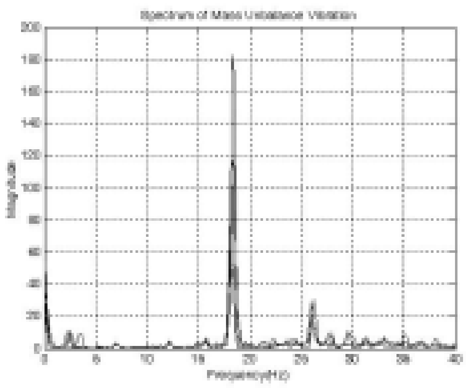


Fig. 12 Vibration Spectra of Mass Unbalance
at 1080 RPM test speed

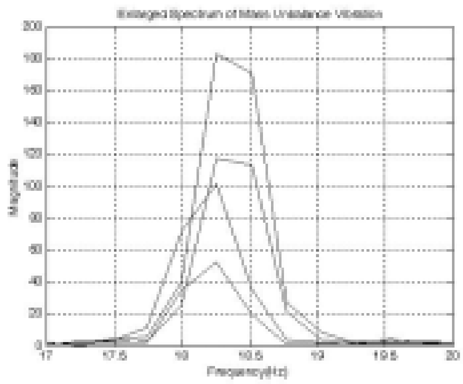


Fig. 13. Enlargement of amplitude of Vibration Spectra

The weight is calculated by the estimator and shown in Fig. 14. Checked with theoretical values, we can find that the estimator has a reliable performance for detecting the weight of mass unbalance. The convergence weight of the two separate 1g mass unbalance is also estimated accurately, which proves that the convergence conditions are always satisfied and this method can be used to balance the unbalance rotor practically.

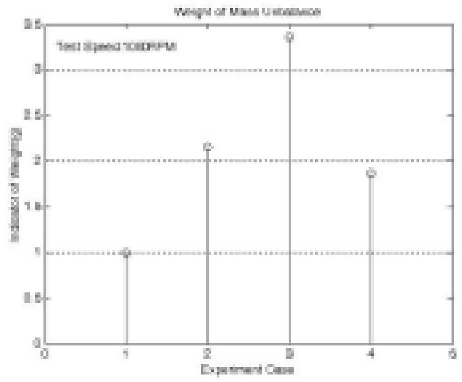


Fig. 14. Indicator of the Weight of Mass Unbalance
at 1080 RPM test speed

Finally the phase position of the mass unbalance is obtained and analyzed with three examples in the proposed method. The numerical results of the examples are shown in Table 1 and 2. The difference of the first two cases is the unbalance distribution on the rotor. Also, the two separate 1g apart from 45 degrees is tested in the last case, which the convergence position is calculated.

Table 1 The Numerical results of Case 1

| Test Speed: 1800 RPM | Test Position | Calculated Position |
|---|---|---|
| | 0° | 0° |
| | 45° | 44.1204° |
| | 90° | 89.7250° |
| Mass Unbalance: 3.2g | 135° | 131.8349° |
| | 180° | 180.8455° |
| | 225° | 225.0710° |
| | 270° | 272.5329° |
| | 315° | 319.3762° |

Compared between the test position and the calculated result, the tests show that the proposed method can give a very accurate result of the phase position of the mass unbalance. The calculation speed is very fast as the statistical method is combined used with Lissajous diagram. The proposed method is more effective than other methods as this method tells the exact unbalance position where others only indicate the subsystem or bound. The present examples confirm that the proposed method can significantly reduce the computation time without resulting in any inaccuracy. It is very useful in the design and balancing of rotor system.

Table 2 The Numerical results of Case 2

| Test Speed: 1800 RPM | Test Position | Calculated Position |
|---|---|---|
| | 0° | 0° |
| | 45° | 45.5976° |
| | 90° | 94.1405° |
| Mass Unbalance: 2.1g | 135° | 131.0347° |
| | 180° | 178.8445° |
| | 225° | 221.3901° |
| | 270° | 275.0861° |
| | 315° | 319.2706° |

Table 3 The Numerical results of Case 3

| Test Speed: 1800 RPM | Theoretic Convergence Position | Calculated Position |
|---|---|---|
| | 22.5° | 24.2435° |
| | 67.5° | 61.7706° |
| | 112.5° | 110.3993° |
| Mass Unbalance: 1g×2 | 157.5° | 155.0006° |
| | 202.5° | 197.0303° |
| | 247.5° | 247.3254° |
| | 292.5° | 296.9375° |
| | 337.5° | 339.5712° |

## 4. CONCLUTION

In the present paper a new method is proposed for the mass unbalance analysis of rotor systems. Based on FFT and Lissajous diagram, the estimator of weight and phase position of mass unbalance is developed. With this estimator the detection of a mass unbalance and localization have been done. The above method gives an efficient way to solve the unbalance problem. The use of the statistical method reduces the computation time and increases the accuracy. Theoretical results have been given. Several examples are illustrated and compared to verify the proposed method. The applicability of the proposed method is shown through the examples.

The proposed method is very effective in the facts that it ca n be applied to most mass unbalance rotor systems and can be extended to a complicated rotor system easily. The proposed m ethod shows the possibility of unmanned automated system fo r rotor balancing and real-time analysis, as it utilizes a mass un balance detection method with vibration signals. It may be a n ew start for achieving the better balance in actual systems in th e future.

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