

Optimal Sliding Mode Control of Anti-Lock Braking System

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Abstract: Anti-lock brake systems (ABS) are being increasingly used in a wide range of applications due to safety. This paper deals with a high performance optimal sliding mode controller for slip-ratio control in the ABS. In this approach a sliding surface square is considered as an appropriate cost function. The optimum brake torque as a system input is determined by minimizing the cost function and used in the controller. Simulation results reveal the effectiveness of the proposed sliding mode controller.

Keywords: Optimal sliding mode control, cost function, slip control, ABS.

1. INTRODUCTION

The objective of an Antilock Brake System (ABS) is to prevent wheels from lockup. Wheel lockup often happens when braking on a wet and slippery road or during a severe braking. During wheel lockup, vehicle loses steering control and the friction force, which stops the vehicle, is greatly reduced.

There are many different variations and control algorithms for ABS systems. In this work a new sliding mode control is applied to an ABS. Sliding mode control Approach have been started in Russia by many researches [1-2]. The most distinguishing property of sliding mode control is that the closed loop system is completely insensitive to system uncertainties and external disturbances. Some of the concepts and theoretical advances of sliding mode control are covered in [3,4,5]. Due to its excellent invariance and robustness properties, variable structure control has been developed into a general design method and extended to a wide spectrum of system types including multivariable, large-scale, infinite-dimensional and stochastic systems. The ideas have successfully been applied to problems as divers as automatic flight control, control of electric motors, chemical process, helicopter stability augmentation systems, space systems and robots [6-12]. In sliding mode control, controller is designed to drive and then constrain the system to lie within a neighborhood of the switching function [13-14]. In this paper an optimal sliding mode controller is designed to minimize a cost function including sliding surface. The organization of the paper is as follows: In Section 2 the mathematical model of ABS is presented. In Section 3 the design procedure of sliding mode control is considered. The simulation results on the slip-ratio control of an ABS are presented in section 4. Finally, Section 5 concludes the paper.

2. MATHEMATICAL MODEL OF ABS

In this section to design a controller, a good model of the system is described [11]. This model will be used for design of control laws and computer simulations, see also [15-16]. Although the model considered here is relatively simple, it keeps the essential dynamics of the system. The model

identifies the vehicle speed and slip as state variables, and it identifies the torque applied to the wheel as the input variable. The two state variables in this model are associated with one-wheel rotational dynamics and linear vehicle dynamics.

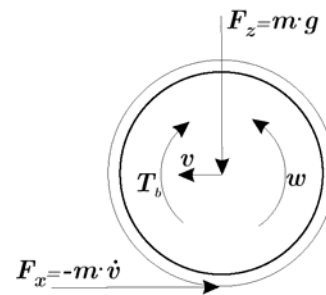


Fig.1 Wheel dynamics

The dynamic equations for the motion of the vehicle, shown in Fig. 1, are as follows

$$\dot{v} = \frac{-F_x}{m} \tag{1}$$

$$\dot{\omega} = \frac{rF_x - T_b}{j_\omega} \tag{2}$$

where, m is mass of the vehicle, v is vehicle speed, ω is angular speed of the wheel, F_x is tyre friction force, T_b is brake torque, r is wheel radius and j_ω is wheel inertia. F_z is vertical force as follows:

$$F_z = mg \tag{3}$$

The tire friction force F_x is given by

$$F_x = F_z \mu(\lambda) \tag{4}$$

where the friction coefficient μ is a nonlinear function of λ longitudinal tyre slip. For various road conditions, the $\mu(\lambda)$

curves have different peak values and slopes. Here, the following function is used as friction coefficient

$$\mu(\lambda) = \frac{2\mu_p\lambda_p}{\lambda_p^2 + \lambda^2} \quad (5)$$

It is used for a nominal curve, where μ_p and λ_p are the peak values. For various road conditions, the curves have different peak values and slopes. The adhesion coefficient characteristics are also influenced by operational parameters such as speed and vertical load. The peak value for the friction coefficient usually has values between 0.1 (icy road) and 0.9 (dry asphalt and concrete). Figure 2 shows a typical curve $\mu(\lambda)$ for different road conditions.

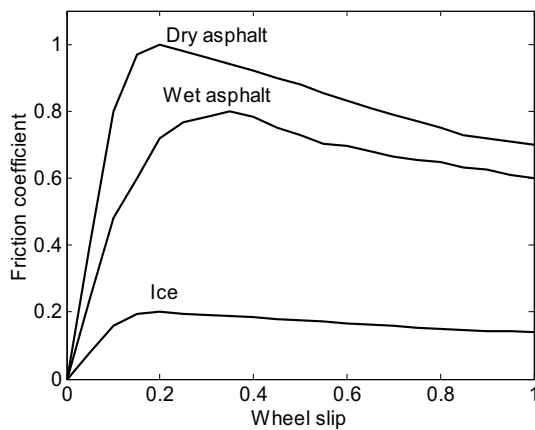


Fig. 2 $\mu(\lambda)$ curves for different road conditions

Mathematically λ is defined as

$$\lambda = 1 - \frac{r\omega}{v} \quad (6)$$

and describes the normalized difference between the vehicle speed v and the speed of the wheel perimeter ωr . The slip value of $\lambda = 0$ characterizes the free motion of the wheel where no friction force F_x is exerted. If the slip attains the value $\lambda = 1$, then the wheel is locked ($\omega = 0$).

By derivation of Eq. (6) and with substituting (3) and (4) in (1), the following state variables can be found:

$$\dot{v} = -\frac{\frac{1}{2}g\mu_p\lambda_p}{\lambda_p^2 + \lambda^2} \quad (7)$$

$$\dot{\lambda} = \frac{\frac{1}{2}\mu_p\lambda_p\lambda^2 + \frac{1}{2}\mu_p\lambda_p\left(-g - \frac{r^2mg}{j_\omega}\right)\lambda}{v(\lambda_p^2 + \lambda^2)} - \frac{r}{v}T_b \quad (8)$$

3. CONTROLLER DESIGN PROCEDURE

To design a sliding mode controller, a sliding surface should be defined, where it is described by

$$s = \lambda - \lambda_r \quad (9)$$

Where λ_r and λ are the slip set point and the output slip value, respectively.

In the sliding surface the following condition should be satisfied:

$$\frac{1}{2} \frac{d}{dt} s^2 \leq -\eta|s| \quad (10)$$

where η is a positive constant that guarantees the system trajectories hit the sliding surface in finite time [5]. So according to [17] it is assumed

$$\dot{s}(t) = -u_q^2 \text{sign}(s) \quad (11)$$

According to Eqs. (8), (9) and (11) the control signal can be found as

$$T_b = \left(\frac{\frac{1}{2}\mu_p\lambda_p\lambda^2 + \frac{1}{2}\mu_p\lambda_p\left(-g - \frac{r^2mg}{j_\omega}\right)\lambda}{r(\lambda_p^2 + \lambda^2)} + \frac{v}{r}u_q^2 \text{sign}(s) \right) \quad (12)$$

Now u_p should be considered to minimize the following cost function

$$J = \int s^2(t) dt \quad (13)$$

With substituting (12) in (8)

$$\dot{\lambda} = -u_q^2 \text{sign}(\lambda - \lambda_r) \quad (14)$$

It can be rewritten as

$$\frac{d\lambda}{-u_q^2 \text{sign}(\lambda - \lambda_r)} = dt \quad (15)$$

With substituting (15) in (13)

$$J = \int (\lambda - \lambda_r)^2 \frac{d\lambda}{-u_q^2 \text{sign}(\lambda - \lambda_r)} \quad (16)$$

In order to minimize the cost function, it is obvious that u_p should be increased as much as possible. So the control law can be obtained.

4. SIMULATION RESULTS

In this section the proposed sliding mode controller is evaluated. For simulation, the system parameters shown in

Table 1 are used.

Table 1 System parameters.

M	1208 kg
j_w	2.11 kg-m ²
R	0.30 m
g	9.8 m/s ²
v_0	20 m/s
μ_p	0.9
λ_p	0.2
λ_r	0.2
T_{bmax}	1000 N-m ²

In Figs. 3, 4 and 5 the system responses in the presence of fifty percent uncertainty in mechanical parameters (j_w , m and r) are compared. As it is seen in Fig. 3, the slip response is settled very well in less than 0.02 sec. In Fig. 4 the vehicle speed is shown. It is obvious in Figs. 3 and 4 that the proposed controller is very robust again uncertainties. The cost function value is depicted in Fig. 5. As you see the uncertainties have not deeply affected the controller performance.

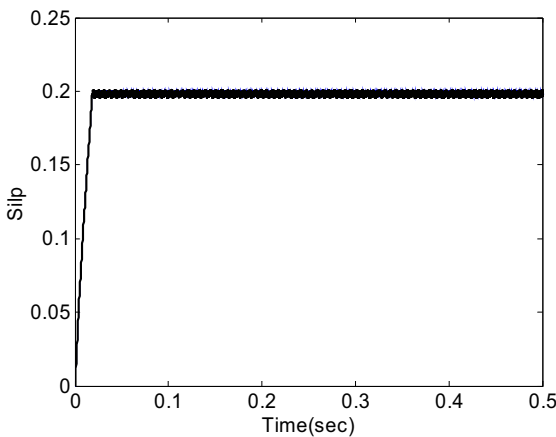


Fig. 3 Slip-ratio response for proposed controller

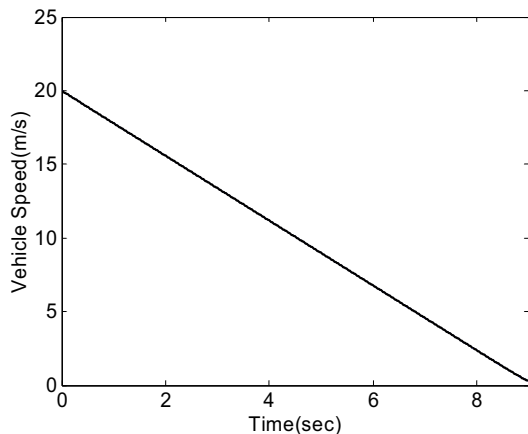


Fig. 4 Vehicle speed response using optimal sliding mode controller

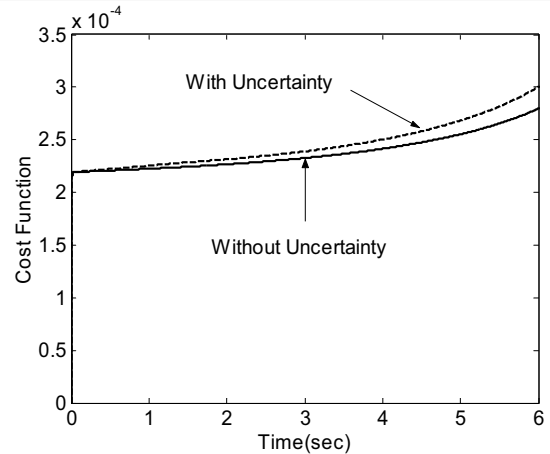


Fig. 5 Cost function for system with and without uncertainty

5. CONCLUSION

In this paper, an optimal sliding mode controller was applied to an Anti-Lock Brake System. The Sliding mode controller was designed based on minimizing a cost function. The cost function is itself based on sliding surface parameters. Simulation results showed the robustness of the proposed sliding mode controller.

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