A New Online Calibration Algorithm for Array Antenna using Independent Component Analysis

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Abstract: This paper proposes a new online calibration algorithm for the array antenna system. As you know, the several previous calibration methods for the mutual coupling did not estimate but measure mutual coupling effect at the real or test-bed system directly. Therefore we suggest some idea to compensate the calibration errors due to mutual coupling effect and mismatch in cables and electronic modules without the off-line calibration. In this work, we can calibrate the array antenna system under the operation of the system using Independent Component Analysis(ICA). This is what is called an online calibration. As you know, the ICA method has permutation and scaling problems. However, we solve problems of the ICA method and apply it to the calibration of an array antenna. The method simultaneously estimates the DOA(Direction of Arrival) of the signals, and calibrates the array for that specific angle. The proposed algorithm is evaluated by computer simulation and its behavior is illustrated by a numerical example.

Keywords: Online Calibration, Independent Component Analysis(ICA), Mutual Coupling, Array Antenna

1. INTRODUCTION

The problem of estimating signal parameters from data collected by an array of spatially located antenna has received much attention. In particular, deviations from the actual array manifold, resulting from mutual coupling, mismatch in the array channels and antenna position errors, can seriously degrade their performance. For example, in a typical phased array where the element spacing is small, mutual coupling distorts the beam patterns of the elements so that the elements have no longer identical patterns. Hence, to achieve high-resolution performance, the array calibration is often necessary. A common method of calibrating an array antenna is to store samples of the array manifold over the desired field of view. However, such a calibration procedure can be tedious and time consuming. In addition to the problem of initial array calibration, there is the problem of maintaining array calibration. Many factors contribute to changing the response of the array antenna over time: gradual changes in the behavior of the antenna itself and discordances the electronic circuitry between the antenna and the output of the digitizer due to thermal effect, aging of components, and changes in the location of the antenna elements. In this paper, we consider two calibration errors due to mutual coupling effect and mismatch in cables and electronic modules. The purpose of this work is to develop the online calibration algorithm in the presence of mutual coupling using the ICA method. We can find the important meaning that this work makes it possible to calibrate through online in the presence of mutual coupling. When the mutual coupling is properly accounted for, the performance of system can be stored to the ideal level.

2. SYSTEM MODEL

We consider an array antenna system with m antenna elements. We assume that the number of the signal sources (n) and the length of transmitting symbols (L) are known. When the array is impinged by the signals from the directions $\theta_i (i = 1,2,\cdots,n)$, the complex envelop of the received signal matrix $\mathbf{X}$ can be written as

$$\mathbf{X} = \mathbf{MDBS} = \mathbf{AS}$$

Let $\mathbf{A}, \mathbf{M}, \mathbf{D}$ and $\mathbf{B}$ call the mixing matrix, mutual coupling matrix, calibration matrix, and array manifold matrix respectively.

The mixing matrix $\mathbf{A}$ which is composed of the mutual coupling, calibration and array manifold matrix is an unknown $m \times n$ matrix with full rank.

The matrix $\mathbf{S}$ means the transmitting signal. The signal $\mathbf{S}$ has nongaussian probability distribution and is independent each other. It is an important precondition to apply the ICA to this problem. Generally, the stated above precondition is reasonable in the real environment.

The received signal from the i th path at time $j(j = 1,2,\cdots,L)$ is given by

$$\mathbf{x}_j = [x_i(1) \cdots x_i(l) \cdots x_i(m)]^T$$

Where, $x_i(l)(l = 1,2\cdots,m)$ is the complex base-band signal at the l th antenna element. Therefore the total received signals are $m \times L$ complex matrix and are expressed by

$$\mathbf{X} = [\mathbf{x}_1 \cdots \mathbf{x}_j \cdots \mathbf{x}_L]$$

The mutual coupling matrix $\mathbf{M}$ means the mutual coupling effect between antenna elements. As you know, because the mutual coupling implies the quantity of signal power transferred from the adjacent element, $\mathbf{M}$ is a $m \times m$ real and symmetric matrix.

$$\mathbf{M} = \begin{bmatrix} 1 & c_{12} & \cdots & c_{1m} \\ c_{21} & \ddots & c_{pq} & \vdots \\ \vdots & c_{qp} & \ddots & \vdots \\ c_{m1} & \cdots & \cdots & 1 \end{bmatrix}$$

Where c_{pq} denotes the quantity of mutual coupling between the p th and q th element. It equals c_{qp} because of reciprocity. As a matter of fact, the mutual coupling effect depends on the several factors such as the antenna geometry, kinds of antenna, DOA(Direction of Arrival) of the signal, the feeding method of antenna and so forth. So it is very difficult and complicated to model the mutual coupling effect. We assume that the mutual coupling effect is related to the only function of distance among antenna elements and has no concern with DOA of the signal. Also, the diagonal of mutual coupling matrix is 1 as denoted the quantity of self coupling. Actually, it is difficult to simply represent the number of parameters to estimate. The number of mutual coupling parameters depends on the antenna geometry and distance between antenna elements and is classified several cases. Here we define $f(m)$ that means the function of the number of antenna element. We show the several cases that denote the number of mutual coupling parameters to estimate.

(1) If the antenna geometry is linear and the distance of the adjacent element is equal, $f(m)$ is $m-1$.
(2) If the antenna geometry is circular and the adjacent element is located the same distance, $f(m)$ is $\lfloor \frac{m}{2} \rfloor$.
(3) If the antenna geometry is a square, $f(m)$ is $\lfloor \frac{m}{2} \rfloor + 1$. In other word, the number of antenna elements is m^2 and maintains the same distance each other.
(4) When the antenna geometry is rectangular and the number of antenna elements is m, $f(m)$ is $m-2$.
(5) In case of the arbitrary geometry, $f(m)$ is $\frac{m(m-1)}{2}$.

Aside from the above mentioned, there can be many cases.

Now, we consider other calibration errors which are caused by the mismatch of receiver hardware such as RF/IF circuits and cables. All independent modules connected with each antenna element must be maintained the equal property to support the high performance of the system. Because the radiating elements of an antenna array are fed with independently, the calibration matrix $\mathbf{D}$ is a diagonal matrix with complex value meaning gain and phase responses. Let us select the reference element. We suppose that the mismatch error of the each receiver hardware is a relative value to the reference element. In other words, each mismatch error of the receiver hardware is normalized by the reference element d_1 ($d_1 = 1$). This is the assumption to reduce the number of parameters to estimate and the general suggestion. The calibration matrix model can be expressed as:

$$\mathbf{D} = \begin{bmatrix} d_1 & & 0 \\ & \ddots & \\ 0 & & d_m \end{bmatrix}$$

Where $d_l (l = 1,2,\cdots,m)$ means the mismatch error of receiver hardware connected the l th antenna element.

The proposed algorithm estimates the mismatch error of receiver hardware and the DOA of the signals simultaneously. The array manifold matrix $\mathbf{B}$ is denoted.

$$\mathbf{B} = \left[\beta_1 \mathbf{b}(\theta_1) \ \cdots \ \beta_k \mathbf{b}(\theta_k) \ \cdots \ \beta_n \mathbf{b}(\theta_n)\right]$$

Where $\mathbf{b}(\theta_k)(k = 1,2,\cdots,n)$ is an array response vector of the k th received signal at the j th antenna element and is represented as follows.

$$\mathbf{b}(\theta_k) = [e^{i\alpha(x_1 \sin\theta_k + y_1 \cos\theta_k)} \ \cdots \ e^{i\alpha(x_j \sin\theta_k + y_j \cos\theta_k)} \ \cdots \ e^{i\alpha(x_m \sin\theta_k + y_m \cos\theta_k)}]^T$$

Where $\{x_j, y_j\}$ is the Cartesian coordinate to express position of the j th antenna element in an arbitrary geometry and β_k means amplitude distortion factor of the k th signal under the fading environment. The amplitude distortion factor $\beta_k (k = 1,2,\cdots,n)$ has the important meaning in this paper. As you know, because the ICA method has permutation and scaling problems, is restricted by the limit of application. However, because we deal with calibration matters in time domain, the scaling and the permutation problems are not critical factor in this work. But the permutation problem can be solved by estimating the parameter β. This makes it possible to apply the ICA method to this work.

3. CALIBRATION ALGORITHM

3.1 Independent Component Analysis (ICA)

Independent Component Analysis(ICA) is a signal processing technique whose goal is to express a set of random variables as linear combinations of statistically independent component variables. Two representative and interesting applications of ICA are blind source separation and feature extraction. Intuitively speaking, the key to estimate the ICA model is nongaussianity. The starting point for ICA is the very simple assumption that the components are statistically independent and the independent components must have statistically nongaussian distributions. Actually, without nongaussianity the estimation is not possible at all [3].

The proposed calibration algorithm is based on two procedures. First, by using ICA method, we estimate the mixing matrix $\mathbf{A}$ which is composed of the mutual coupling, calibration and array manifold matrix. Next, we find the mutual coupling, mismatch error of the receiver hardware and the DOA of the signals by using one of the optimization method such as Steepest descent method. The above two procedures are iterated until the cost function converges to previously defined threshold value.

3.2 Estimation of the mixing matrix $\mathbf{A}$

We introduce a very efficient method that is suited for this task. By using the received signal matrix $\mathbf{X}$, we perform the ICA algorithm that uses only the independent and nongaussian signal to estimate the mixing matrix. Without loss of generality, we can assume that both the mixture variables and the independent components have zero mean. The independent components are latent variables, meaning that they cannot be directly observed. All we observe is the received matrix $\mathbf{X}$, we must estimate both $\mathbf{A}$ and $\mathbf{S}$ using it. To begin with, we shall show the one-unit version of FastICA. By a "unit" we refer to a computation unit. Eventually an artificial neuron, having a weight vector $\mathbf{w}$ that the neuron is able to update by a learning rule. The FastICA learning rule finds a direction, i.e. a unit vector $\mathbf{w}$ such that the projection $\mathbf{w}^T\mathbf{x}$ maximizes nongaussianity. The nongaussianity is here measured by the approxomation of negentropy $J(\mathbf{w}^T\mathbf{x})$ in this paper. Recall that the variance of $\mathbf{w}^T\mathbf{x}$ must here to constrained to unity; or whitened data this is equivalent to constraining the norm of $\mathbf{w}$ to be unity. Especially, we consider that one of the several ICA algorithms is the fixed-point method. The FastICA is based on a fixed-point iteration scheme for finding a maximum of the nongaussianity of $\mathbf{w}^T\mathbf{x}$. We shall summarize the fixed-point algorithm procedure below [2].

Procedure 1: The fixed-point algorithm

(1) Center the data to make its mean zero.

(2) Whiten the data to give $\mathbf{x}$.

(3) Choose n, the number of independent components to estimate.

(4) Choose initial values for the $w_i, i=1,\cdots n,$ each of unit norm. Orthogonalize the matrix $\mathbf{W}$ as in step (6) below.

(5) For every $i=1,\cdots,n,$ let

$$\mathbf{w}_i \leftarrow E(\mathbf{x}g(\mathbf{w}_i^T\mathbf{x})\} - E\{g'(\mathbf{w}_i^T\mathbf{x})\} - E\{g'(\mathbf{w}_i^T\mathbf{x})\}\mathbf{w}$$

where g is any non-quadratic function.

(6) Do a symmetric orthogonalization of the matrix $\mathbf{W}=(\mathbf{w}_1,\cdots,\mathbf{w}_n)^T$ by

$$\mathbf{W} \leftarrow (\mathbf{W}\mathbf{W}^T)^{-1/2}\mathbf{W}$$

(7) If not converged, go back to step (5)

Let the estimated matrix of mixing matrix $\mathbf{A}$ by procedure 1 be the unmixing matrix. First, using the ICA method, we estimate the unmixing matrix $\mathbf{W}$ and its inverse can be represented as

$$\mathbf{W}^{-1} = \mathbf{MD}\widetilde{\mathbf{B}}$$

Where $\widetilde{\mathbf{B}}$ is the array manifold matrix which means column permutated and scaled. The ICA method permutes and scales only the array manifold matrix $\mathbf{B}$, not $\mathbf{M}$ and $\mathbf{D}$. Therefore we can utilize ICA method to solve the problem of an array calibration. It is a key-point in this work.

3.2 Estimation of the state vector $\mathbf{v}$

We define the state vector $\mathbf{v}$ to estimate from the estimated unmixing matrix and denote it as follows:

$$\mathbf{v} = [c_1,\cdots,c_{f(m)},d_2,\cdots,d_m,\theta_1,\cdots\theta_n,\beta_1,\cdots,\beta_n]$$

Where $f(m)$ means the function of the number of antenna elements. The state vector $\mathbf{v}$ includes the several parameters such as the mutual coupling effect, the mismatch error of the receiver hardware, the DOA of the signals and the amplitude distortion factor of the signals. The number of parameters in the state vector $\mathbf{v}$ depends on the array antenna geometry. Here, we are supposed to explain the total number $F(m,n)$ to estimate by procedure 2. The function $F(m,n)$ is related to the number of antenna elements (m) and signals (n).

(1) If the antenna geometry is linear and the distance between elements is equal, $F(m,n)$ is $2(m+n-1)$.

(2) If the antenna geometry is circular and the distance between elements is equal, $F(m,n)$ is $\lfloor \frac{m}{2} \rfloor + m + 2n - 1$.

(3) If the number of antenna elements is m^2 and the location of antenna elements maintains a same distance each other, $F(m,n)$ is $\lfloor \frac{m}{2} \rfloor + m + 2n$.

(4) When the antenna geometry is rectangular and the number of antenna elements is m, $F(m,n)$ is $2m+2n-3$.

(5) When the antenna geometry is arbitrary, $F(m,n)$ is $\frac{m(m-1)}{2} + m + 2n - 1$.

With the inverse of unmixing matrix $(\mathbf{W}^{-1})$ obtained from the procedure 1, we can find the state vector $\mathbf{v}$ by using the Steepest descent method. We define the cost function as $J(\mathbf{v})$.

$$J(\mathbf{v}) = \left\| \mathbf{W}^{-1}(\mathbf{v}) - \mathbf{MD}\widetilde{\mathbf{B}}(\mathbf{v}) \right\|_{\mathbf{F}}^2$$

Where $\|\mathbf{P}\|_{\mathbf{F}}$ means the Frobenious norm of the matrix $\mathbf{P}$. We shall summarize the estimation procedure of the state vector $\mathbf{v}$ using Steepest descent method.

Procedure 2: The Steepest descent method

(1) Choose the initial state vector $\mathbf{v}_0$.

(2) Compute $\left[\frac{\partial J}{\partial \mathbf{v}}\right]$.

$$\left[\frac{\partial J}{\partial \mathbf{v}}\right] = \left[\frac{\partial J}{\partial c_1},\cdots,\frac{\partial J}{\partial c_{f(m)}},\frac{\partial J}{\partial d_2},\cdots,\frac{\partial J}{\partial d_m},\frac{\partial J}{\partial \theta_1},\cdots,\frac{\partial J}{\partial \theta_n},\frac{\partial J}{\partial \beta_1},\cdots,\frac{\partial J}{\partial \beta_n}\right]$$

Where $f(m)$ varies with the antenna geometry as previously stated

(3) Perform the following computation.

$$\left[\frac{\partial J}{\partial \mathbf{v}}\right]_{k+1} = \left[\frac{\partial J}{\partial \mathbf{v}}\right]_0 + e\left[\frac{\partial J}{\partial \mathbf{v}}\right]_k$$

Where e denotes the iteration step size.

(4) If not converged, go back to step (3).

3.3 Numerical Example

We show the numerical example to help your understanding about this problem. We assume that the structure of antenna geometry is an equilateral triangle as shown Fig 1.

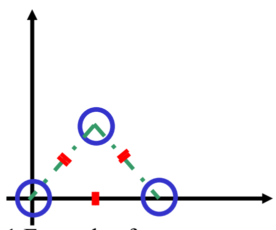

Fig.1 Example of antenna geometry

Because the mutual coupling is only the function of distance and the number of antenna elements and signals is 3 respectively in this example, we can express the mutual coupling, calibration and array manifold matrix individually.

$$\mathbf{M} = \begin{bmatrix} 1 & c & c \\ c & 1 & c \\ c & c & 1 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \beta_1 e^{i\alpha(x_1 \sin\theta_1 + y_1 \cos\theta_1)} & \beta_2 e^{i\alpha(x_1 \sin\theta_2 + y_1 \cos\theta_2)} & \beta_3 e^{i\alpha(x_1 \sin\theta_3 + y_1 \cos\theta_3)} \\ \beta_1 e^{i\alpha(x_2 \sin\theta_1 + y_2 \cos\theta_1)} & \beta_2 e^{i\alpha(x_2 \sin\theta_2 + y_2 \cos\theta_2)} & \beta_3 e^{i\alpha(x_2 \sin\theta_3 + y_2 \cos\theta_3)} \\ \beta_1 e^{i\alpha(x_3 \sin\theta_1 + y_3 \cos\theta_1)} & \beta_2 e^{i\alpha(x_3 \sin\theta_2 + y_3 \cos\theta_2)} & \beta_3 e^{i\alpha(x_3 \sin\theta_3 + y_3 \cos\theta_3)} \end{bmatrix}$$

3.4 Simulation Results

Computer simulations have been conducted to evaluate the performance of the proposed algorithm. To test the algorithm, we consider an array with three antenna elements separated by half a wavelength. Because we use the ICA algorithm for calibration problem, the probability distribution of signals is uniform and independent. It is common for the practical system. The signals are used after centering and whitening processes to increase performance of the estimator. Especially, we use the Fast-fixed point algorithm to estimate the mixing matrix.

We assume that the number of signals is known $(n = 3)$ and the DOA of the signals is $[30^\circ\ 15^\circ\ 60^\circ]$. For simple test, the structure of antenna element is modeled the form of an equilateral triangle as shown an example. The true of calibration matrix $\mathbf{D}$ by mismatch of hardware and mutual coupling matrix $\mathbf{M}$ are represented, respectively.

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5616 + 1.7742i & 0 \\ 0 & 0 & 1.8041 + 1.0416i \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 1 \end{bmatrix}$$

The value of each matrix is nominal. We start the iteration with the initial state as shown Table 1 and perform iterations with step size $(e = 0.005)$. The Fig.2, Fig.3, and Fig.4 show the error performance of parameters. As shown Figures, the error of the parameters becomes smaller according to iterations. The converged state values are given in Table 2, respectively. We see that the proposed algorithm has a good performance.

Table 1 Initial state value of parameters

Parameter	Initial State
DOA	$[20.913^\circ, 10.502^\circ, 41.998^\circ]$
Calibration	$[0.393 + 1.242i, 1.263 + 0.729i]$
Mutual coupling	[0.35]

Table 2 Converged state value of parameters

Iterations	Parameter	Converged state
25000	DOA	$[30.428^\circ, 15.29^\circ, 60.237^\circ]$
25000	Calibration	$[0.59 + 1.77i, 1.82 + 1.02i]$
25000	Mutual coupling	[0.4984]

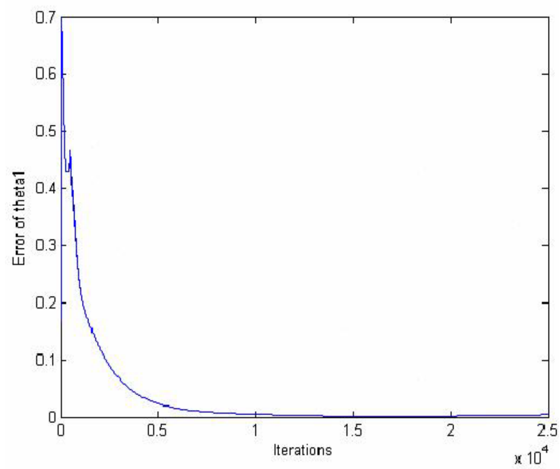

Fig.2 Estimation Error of DOA

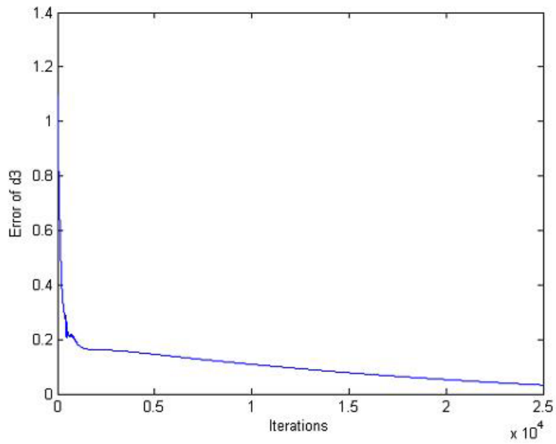

Fig.3 Estimation Mismatch Error of receiver hardware

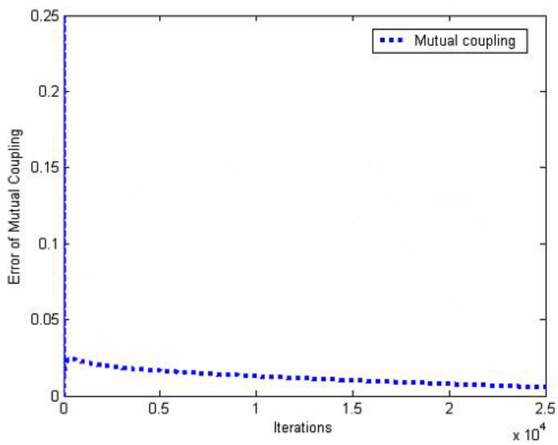

Fig.4 Estimation Error of mutual coupling effect

4. CONCLUSION

In practical array antenna systems, the calibration errors can seriously degrade the performance of estimating the DOA of the signals for beamforming. The gain and phase responses of the receiver hardware vary according to temperature and humidity changes from day to day, and therefore the online calibration is preferable. In this paper, we have presented an array calibration technique which estimates the unknown array parameters consisting of the mutual coupling effect, the DOA of the signals and complex mismatch error of the receiver hardware. The proposed estimator for the mixing matrix is based on the Independent Component Analysis(ICA). The other estimator is derived from the Steepest descent method. They have a wide acquisition range and a good performance. In these estimators, the exact signal information is not required for the calibration algorithm. This is an advantage of the proposed algorithm. The proposed algorithm only uses nongaussian and independent signals. The performance of the estimator has been tested by simulation. A future extension of this work will be the more general case about the unknown array parameters which are composed of mutual coupling and the complex gains by the mismatch of hardware.

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