The 2-DOF Control system design for Quadruple-Tank process using Root Locus Technique.

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Abstract: The control system design of 2-DOF for SISO process by root locus technique is not complicated and efficiently. It can design the control system to have the transient and steady state responses, and do not adjust the gain of process controller later. However, due to control system design for MIMO process, by root locus technique, there is not exact method. This paper is presents the control system design method for Quadruple-Tank Process, by using root locus technique for the structure of 2-DOF control system. The design procedures are first decentralized then using the relative gain array, and finally 2-DOF controller design is applied.

Keywords: Quadruple-Tank Process, MIMO, PID Control.

1. Introduction

Recently, the new model process represents on multivariable control is very interesting [1], [2]. The presentation process model is restricted to the systems with two inputs and two outputs because such systems are common. Typical examples are boilers, machine direction moisture and basis weight control in paper machines, distillation columns, heat exchangers, and air-conditional systems [3]. The Quadruple-tank process has two operation modes by changing the value of valve resistance. This paper focuses on the design of the controllers by using the Root Locus Technique discovered by W. Evans in 1948 and was mathematically formulated in 1950 in his famous paper (Evans, 1950)[4]. The main idea behind the root locus technique is hidden in equation

$$G_1(s)H_1(s) = -\frac{1}{K} \quad (1)$$

which is an algebraic equation involving complex numbers. It actually represents two equations (for real and imaginary parts, or for magnitudes and phase angles).

This paper presents the 2-DOF control system design for the Quadruple-Tank process by using Root Locus Technique, where the decentralized and Relative Gain Array (RGA) for testing the level of their interaction [5] is employed. The experimental results of 1-DOF and 2-DOF controller for minimum phase system are compared.

The outline of the paper is as follows: The formulation of the quadruple process is described in detail in section 2, the nonlinear physical model is discussed in the transfer matrix model, and introduction of linearization for linear model, including the parameter values for minimum phase system. In Section 3, the 2-DOF controller design problems consist of the control system structure and the design procedure of controller are presented. The verification via simulation is shown in section 4 for both of the 1-DOF and 2-DOF controls scheme.

2. Dynamical of the Quadruple-Tank Process

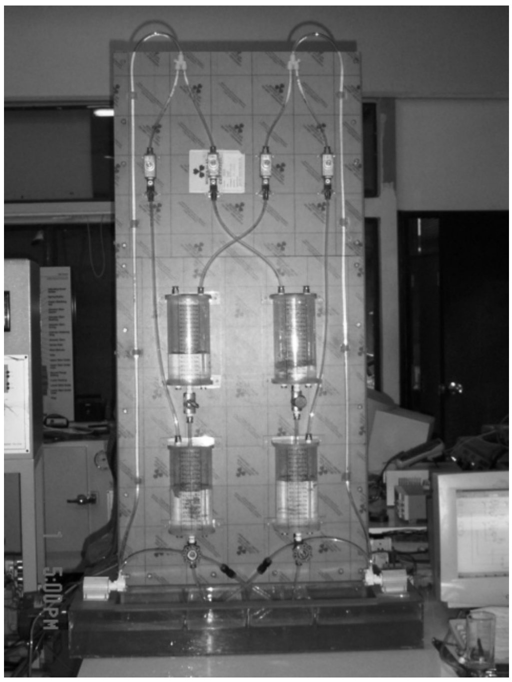

Fig. 1 The Quadruple-Tank Process

2.1. Nonlinear physical model

The quadruple-tank process is shown in Fig.1, this process has been used for demonstration in various issues in multivariable control [6]. The objective of employing the process is to controls the level of the lower two tanks with two pumps. The process inputs are u_1 and u_2 (input voltages to the pumps) and the outputs are y_1 and y_2 (voltages from level measurement devices). The state and output equations of each tank are written as

$$A\frac{dh}{dt} = -q_{out} + q_{in}, \qquad (2)$$
$$q_{out} = a\sqrt{2gh}.$$

The out flow rate from each pump which proportion to it's applied voltage v is split on two ways by the ratio γ as the in flow rate q_L to the lower tank and q_U to the cross couple upper tank, respectively. These relations can be expressed as

$$q_L = \gamma k v, \quad q_U = (1-\gamma)kv, \quad \gamma \in [0,1]. \qquad (3)$$

$$\left.\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1,\\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2,\\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2,\\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1,
\end{aligned}\right\} \qquad (4)$$

where a_i is the cross-sectional area of the outlet tube, A_i is the cross-sectional area of the tank, h_i is the height of the water level in tank, and g is the acceleration due to gravity. The voltage applied to the pump i is v_i and the corresponding flow is $k_i v_i$. The parameters $\gamma_1, \gamma_2 \in [0,\ 1]$ are the ratio of in flow rate of each the cross couple tanks.

2.2. Linearization of physical Model

The variables $x_i := h_i - h_i^0$ and $u_i := v_i - v_i^0$ are first introduced. The linearization state space equations are then given by

$$\frac{d}{dt}x(t) = \begin{pmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{pmatrix} x(t) + \begin{pmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{pmatrix} u(t), \qquad (5)$$
$$y(t) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} x(t).$$

Where the time constants are

$$T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}},\ i = 1,\dots,4 \qquad (6)$$

When the parameter is set as $1 < \gamma_1 + \gamma_2 \le 2$, this model is minimum phase model, and when the parameter is set as $0 < \gamma_1 + \gamma_2 \le 1$, the model is non-minimum phase.

2.3. Parameters of the Quadruple-Tank Process Model

The parameters of the physical model of the quadruple –tank process are given as

A_1, A_3	$[\mathrm{cm}^2]$	69
A_2, A_4	$[\mathrm{cm}^2]$	69
a_1, a_3	$[\mathrm{cm}^2]$	0.093,0.072
a_2, a_4	$[\mathrm{cm}^2]$	0.081,0.099
g	$[\mathrm{cm/s}^2]$	981

And the parameters of the minimum phase operating point $G_{\min}^-$ is initially set as follows

Operating point	$G_{\min}^-$
(h_1^0, h_2^0) $[cm]$	$(11.1, 11.6)$
(h_3^0, h_4^0) $[cm]$	$(0.74, 0.44)$
(v_1^0, v_2^0) $[V]$	$(5.00, 5.00)$
(k_1, k_2) $[cm^3/Vs]$	$(2.697, 2.395)$
(γ_1, γ_2)	$(0.791, 0.772)$

Hence, this model can be written to matrix form as

$$G_{\min}^- = \begin{bmatrix} \frac{3.4353}{(111.7417s+1)} & \frac{0.8792}{(34.2382s+1)(111.7417s+1)} \\ \frac{1.0707}{(131.8047s+1)(18.4475s+1)} & \frac{3.5115}{(131.8047s+1)} \end{bmatrix} \qquad (7)$$

3. The 2-DOF Controller Design Problems

3.1 The Control System Structure

The RGA analysis is the technique of input-output matching for controller design. In the case of minimum phase, the transfer function $G_{11}(s)$ and $G_{22}(s)$ will be used to design the controller. For the 1-DOF controller, it is differing from the 2-DOF controller only it has no feed-forward controller as show in Fig.2

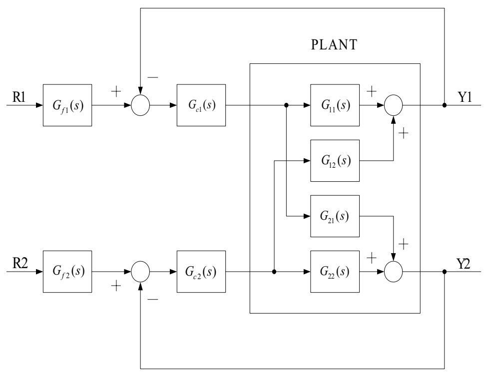

Fig 2 The Structure of 2-DOF for minimum phase

In case of minimum phase system designed by using $G_{11}(s)$ and $G_{22}(s)$. As this process is first order type zero process, therefore the designing of controller are as follows

$$G_{c1}(s) = \frac{Kc_1(s+z_{c1})}{s}, G_{f1}(s) = \frac{z_{c1}}{(s+z_{c1})} \tag{8}$$

$$G_{c2}(s) = \frac{Kc_2(s+z_{c2})}{s}, G_{f2}(s) = \frac{z_{c2}}{(s+z_{c2})}\ . \tag{9}$$

3.2 The design procedure of controllers

The design procedure to meet the transient and steady state response specifications are as follows:

Step 1. The damping ratio (ζ), undamped natural frequency (ω_n) and s_d are determined from the transient response specifications in (1).

$$\left.\begin{aligned} &P.O. = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}\%\ , \\ &t_s = 4/\zeta\omega_n \quad (\pm 2\%)\ , \\ &s_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}\ , \end{aligned}\right\} \tag{10}$$

where $P.O.$ is percent overshoot and t_s is the settling time.

Step 2. Find the sum of the angles at s_d with all of the open-loop poles of $G_c(s)G_p(s)$ by graphical or by numerical computations. Then determine the necessary angle of $\angle(s_d + z_c)$ to be added so that the total sum of the angles satisfies (11).

$$\sum(\theta_z + \theta_{z_c}) - \sum\theta_p = -(2k+1)\pi, \quad k = 0,1,2,\ldots \tag{11}$$

When the angle of z_c or θ_{z_c} is known, then the location of z_c can be obtained from

$$z_c = \frac{|\mathrm{Im}(S_d)|}{tan(\theta_{z_c})} - |\mathrm{Re}(S_d)| \tag{12}$$

Step 3. Find the gain K_c of the controller from magnitude condition of root locus method.

$$K_c = K_{sd} = \frac{1}{|G(s_d)H(s_d)|} \tag{13}$$

Step 4. Substitute all parameters into controller equation forms.

$$G_c^{PI}(s) = \frac{K_c(s+z_c)}{s} \tag{14}$$

Step 5. Plot the root loci of $G_c^*(s)G_p(s)$ to assure that the root loci are pass through the locations of s_d, where $G_c^*(s)$ is the corresponding controller.

Step 6. Find the step response of the closed-loop system can meet the specifications to be designed or not, if it is not satisfied then the feed-forward controller in (14) is cascaded to the forgoing closed-loop system as shown in Fig. 2.

$$G_f(s) = \frac{z_c}{(s+z_c)} \tag{15}$$

4. Verification via simulation

In this section, the design procedures of 1-DOF and 2-DOF Controller via root locus method for both cases are verified through the MATLAB. The step responses of the control systems which employ the proposed 1-DOF controller and the 2-DOF controller are also be compared.

Design: Loop 1 (Y1-R1)

$$G_{11}(s) = \frac{3.4353}{(111.7417s+1)} \tag{15}$$

The desired settling time $t_s = 100$ secs, $P.O. = 5\%$ are designed follow the proposed procedure. The results of the 1–DOF and 2-DOF controller design are

$$G_{c1} = \frac{2.4642(s+0.0497)}{s}, G_{f1} = \frac{0.0497}{(s+0.0497)} \tag{16}$$

Design: Loop 2 (Y2-R2)

$$G_{22}(s) = \frac{3.5115}{(131.8047s+1)} \tag{17}$$

The desired settling time $t_s = 100$ secs, $P.O. = 5\%$ are designed follow the proposed procedure. The results of the 1–DOF and 2-DOFcontroller design are

$$G_{c2} = \frac{2.8948(s+0.0488)}{s}, G_{f2} = \frac{0.0488}{(s+0.0488)}, \tag{18}$$

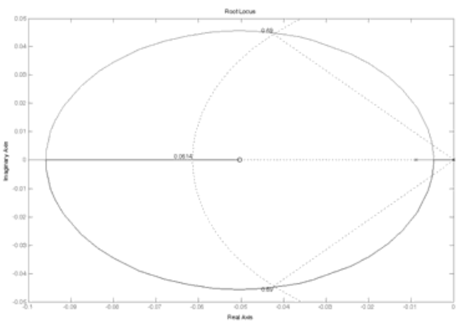

Fig. 3 (a) Root loci of Control of the Loop 1

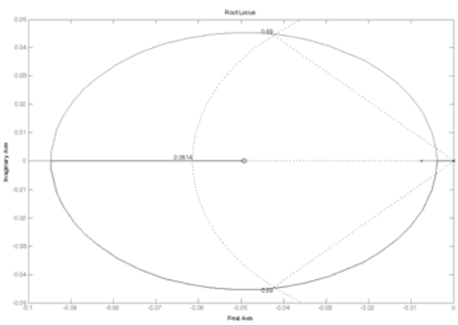

Fig. 3 (b) The Root loci of Control of the Loop 2

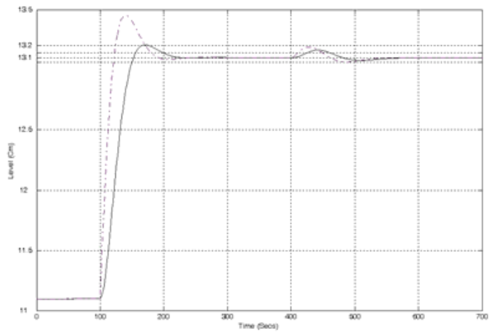

Fig. 4(a) The Step Responses of the Loop 1

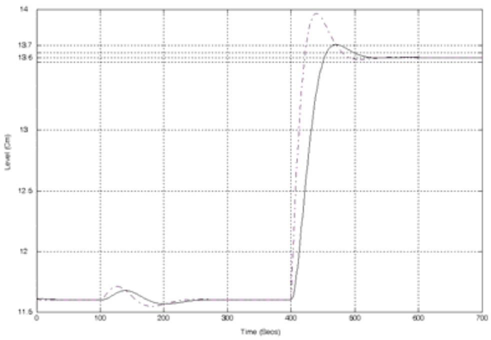

Fig. 4(b) The Step Responses of the Loop 2

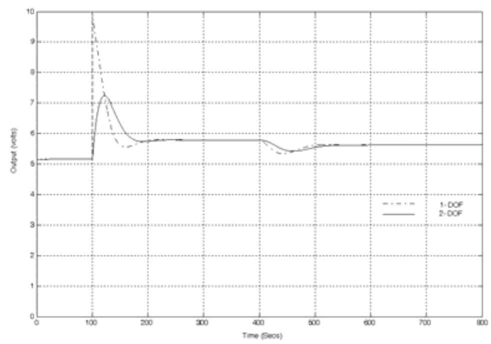

Fig. 4(c) The Output controller of the Loop 1

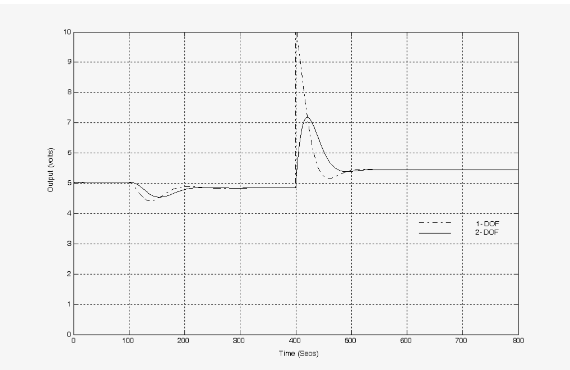

Fig. 4(c) The Output controller of the Loop 2

It is evident that both of control loops have the same manner. Here, the outputs of the controllers in case of 1-DOF are higher than that of the 2-DOF case, while the desired output response can not be achieved too. This contrast to the case of 2-DOF controllers that meet the desired responses to be designed and the outputs of the controllers are lower as well.

5. Conclusions

Since a Quadruple-tank process is quite complicate because of it is MIMO system. It is known that in order to apply the root locus technique to MIMO system quite difficult. This paper has been shown that the problem of the controllers' design can be reduced via decentralized and relative gain array analysis; such that the control problem looked like SISO system then the root locus technique is easily to be applied. Furthermore, when compare between the 2-DOF controller and the 1-DOF controller, its performance is better at all. For the case of non-minimum phase system is the problem still remained.

References

[1] G.C.Goodwin, "Control system design," *New Jersey*, Prentice Hall, 2001.

[2] M. Grebeck, "A Comparison of controllers for the quadruple tank system," *Technical report, Department of Automatic control, Lund Institute of Technology, Lund, Sweden, 1998*.

[3] W.L. Luyben, "Process Modeling simulation and control for chemical engineers," *New York*, McGraw-Hill,1990.

[4] R.C.Dorf, "Modern Control systems," *California*, Addison-Wesley, 2001.

[5] E.Bristol, "On a new measure of interaction for multivariable process control", *IEEE Transactions on Automatic Control*, 11:133 1966.

[6] K. H. Johansson, "The Quadruple-Tank Process – a multivariable process with an adjustable zero," *IEEE Transactions on Control Systems Technology,* vol. 1, pp. 456-465, May 2000.