

Active Suspension System for a One-wheel Car Model Using Single Input Rule Modules Fuzzy Reasoning

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Abstract: This paper presents the construction of an active suspension system of a one-wheel car model by using fuzzy reasoning. The car model is approximately described by a nonlinear two degrees freedom system subject to excitation from a road profile, and the active control force is constructed by actuating a pneumatic actuator, and the degradation of the performance due to the delay of the pneumatic actuator is improved by inserting a compensator. The fuzzy control is obtained by single input rule modules fuzzy reasoning, and the excitation from the road profile is estimated by using a disturbance observer. The experimental result shows that the proposed active suspension system much improves the performance in the vibration suppression of the car model.

Key words: One-wheel car model; Active suspension system; Single input rule modules fuzzy reasoning; Pneumatic actuator; Compensator

1. INTRODUCTION

Active suspension systems for car models are recently much interesting as they more improve the ride comfort of passengers than passive suspension systems in high-speed ground transportation. Generally, linear active suspension systems are derived by the optimal control theory on the assumption that the car model is described by a linear system [1-3]. However, as the car models are practically denoted as a complicated system including non-negligible non-linearity and uncertainty, active suspension systems are derived by nonlinear and/or intelligent approaches, for example, fuzzy reasoning [4-6], neural network [7], and sliding mode control [8]. It is seen from the numerical and experimental results that such active suspension systems provide more satisfactory performance, but necessitate more increasing loads in the controllers, comparing with the linear active suspension systems.

The purpose of this paper is to propose an active suspension system of a one-wheel car model that is approximately described as a nonlinear two degrees of freedom system subject to excitation from a road profile by using fuzzy reasoning and a disturbance observer. The active control for the suspension system to be proposed here is composed of the fuzzy control obtained by single input rule modules (SIRMs) fuzzy reasoning [5-6] including the estimate of the excitation from the road profile. The active control force is constructed by actuating a pneumatic actuator, and the performance degradation due to the delay of the pneumatic actuator is improved by inserting a compensator following the accelerometer to measure the acceleration of the car body [5].

2. ONE-WHEEL CAR MODEL

The experimental apparatus of a one-wheel car model, vertically confined by two polls, due to the active control force generated by the pneumatic actuator is constructed as shown in Fig. 1. The masses of the car body and the wheel are respectively denoted as m_1 and m_2 whose displacements are respectively corresponding to z_1 and z_2 . The restoring force of the suspension part is practically assumed to be nonlinear, and it is constructed by two coil springs with the stiffness k_1 or by four coil springs with the stiffness $k_1 + k_1'$, depending on the suspension ($z_1 - z_2$). Then, the nonlinear restoring force

$f(z_1 - z_2)$ is

$$f(z_1 - z_2) = \begin{cases} (k_1 + k_1')(z_1 - z_2) - ak_1' & \text{for } z_1 - z_2 > a \\ k_1(z_1 - z_2) & \text{for } |z_1 - z_2| \leq a \\ (k_1 + k_1')(z_1 - z_2) + ak_1' & \text{for } z_1 - z_2 < -a \end{cases} \quad (1)$$

where a is a positive constant. The non-linearity of the restoring force means that the restoring force becomes stronger as the suspension deflection does larger. The gravity mainly due to the masses, m_1 and m_2 , is supported by the mass m_3 whose displacement is denoted as z_3 , and the coil spring with the stiffness K . The tire part of the wheel is denoted as the stiffness k_2 , and the excitation from a road profile is assumed the signal generated by the electric vibrator connecting the signal function generator. The damping force of the suspension part is assumed due to the Coulomb damping caused by contact with two polls and the viscous damping caused by the pneumatic cylinder, and it is assumed linear with the damping coefficient c as considered relatively small.

Then, the equations of motion for the car model are

$$m_1 \ddot{z}_1 + c(\dot{z}_1 - \dot{z}_2) + f(z_1 - z_2) = u \quad (2)$$

$$m_2 \ddot{z}_2 - c(\dot{z}_1 - \dot{z}_2) - f(z_1 - z_2) + k_2'(z_2 - z_3) = -u \quad (3)$$

$$m_3 \ddot{z}_3 - k_2'(z_2 - z_3) + Kz_3 = f_e \quad (4)$$

where u is the active control constructed by actuating a pneumatic actuator, and f_e is the exciting force generated by the electric vibrator. Dividing both sides of equation (4) by K , neglecting the resultant first term on the left-hand of equation (4) as considered relatively small compared with other terms, defining that

$$k_2 = k_2'K / (k_2' + K), \quad w = f_e / K$$

$$z_2 - w = (k_2' / k_2)(z_2 - z_3)$$

and substituting the above definitions into equation (3), then the resultant equation becomes

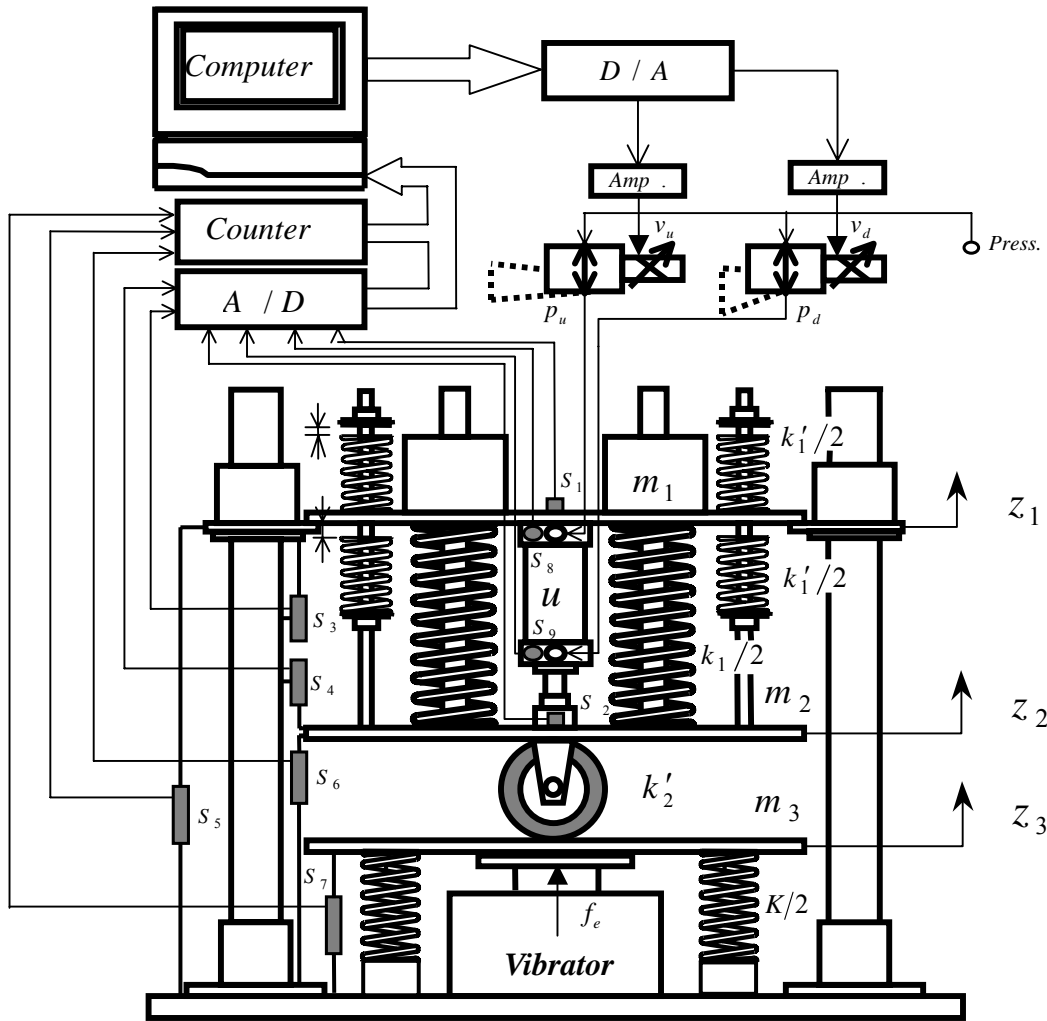


Fig.1 Experimental apparatus of one-wheel car model

$$m_2 \ddot{z}_2 - c(\dot{z}_1 - \dot{z}_2) - f(z_1 - z_2) + k_2(z_2 - w) = -u \quad (5)$$

Therefore, the one-wheel car model described by equations (2) and (5) indicates a nonlinear two degrees of freedom system subject to the excitation from the road profile.

The control part in the one-wheel car model provides the accelerometers (S_1 and S_2), the velocity sensors (S_3 and S_4), and the linear encoders (S_5 , S_6 and S_7). The control signal is calculated by using the personal computer based on the measurement data of the state variables through the A/D converters and the counters. The control valve of the pneumatic actuator is operated by the control signal through the D/A converter and the power amplifier, and the active control u is constructed by actuating the pneumatic actuator as $u = 77.0 v$, that is experimentally obtained, where v denotes the voltage of the control valve.

3. ACTIVE SUSPENSION SYSTEM

The active suspension system to be proposed here is constructed as follows. Firstly, a compensator is constructed to

improve the degradation of the performance in the vibration suppression of the car model due to the delay of the pneumatic actuator and it is inserted by following the accelerometer to measure the acceleration of the car body. Secondly, the active control is derived as the fuzzy control using the SIRMs fuzzy reasoning including the estimate of the excitation from the road profile obtained by using the disturbance observer.

3.1 Compensator

The degradation of the performance in the vibration suppression of the car model due to the delay of the pneumatic actuator is improved by inserting the compensator following the accelerometer to measure the acceleration of the car body. The transfer function for the pneumatic actuator was experimentally identified by the step response as

$$G_a(s) = \frac{P_u(s)}{K_a V_u(s)} = \frac{900}{s^2 + 30s + 900} e^{-0.035s} \quad (6)$$

where $K_a = 90kPa/V$. Assuming that the transfer function for

the compensator is

$$G_c(s) = \frac{s^2 + 2\zeta_b \omega_b s + \omega_b^2}{s^2 + 2\zeta_a \omega_a s + \omega_a^2} \quad (7)$$

then the parameters characterizing $G_c(s)$ are determined from the frequency response function $G_a(j2\pi f)G_c(j2\pi f)$ for the transfer function $G_a(s)G_c(s)$.

3.2 Fuzzy reasoning

The fuzzy control is obtained by using the *SIRMs* fuzzy reasoning [5-6]. It is a well-known fact that the number of fuzzy control rules to infer the defuzzificated values of the variables in the conclusion parts exponentially more increases if the number of the variables in the precondition parts more increases. Therefore, the *SIRMs* fuzzy reasoning is proposed to decrease the number of fuzzy control rules and to reduce the computation load of fuzzy reasoning. The *SIRMs* fuzzy control rules [5-6] are given as

$$\begin{aligned} \text{SIRM} - 1 : & \left\{ \text{if } \alpha_1 \text{ is } A_{1j} \text{ then } \beta_1 \text{ is } B_{1j} \right\}_{j=1}^{m_1} \\ \dots\dots & \\ \text{SIRM} - i : & \left\{ \text{if } \alpha_i \text{ is } A_{ij} \text{ then } \beta_i \text{ is } B_{ij} \right\}_{j=1}^{m_i} \\ \dots\dots & \\ \text{SIRM} - n : & \left\{ \text{if } \alpha_n \text{ is } A_{nj} \text{ then } \beta_n \text{ is } B_{nj} \right\}_{j=1}^{m_n} \end{aligned} \quad (8)$$

where α_i and β_i are respectively the variables in the precondition and the conclusion parts, and A_{ij} and B_{ij} is respectively the fuzzy sets whose membership functions are respectively denoted as $A_{ij}(\alpha_i)$ and $B_{ij}(\beta_i)$. Reducing the inference time to obtain the defuzzificated values of the variables in the conclusion parts, the product sum-gravity method [5-6] is proposed. Measuring the variable α_i as α_i^0 in the precondition part of (8), the degree of fitness ω_{ij} is given by

$$\omega_{ij} = A_{ij}(\alpha_i^0) \quad (9)$$

and the variable in the conclusion part is inferred as

$$\beta_{ij} = B_{ij}^{-1}(\omega_{ij}) \quad (10)$$

Then, the defuzzificated value β_i^0 of β_i in the conclusion part becomes

$$\beta_i^0 = \frac{\sum_{j=1}^{m_i} \omega_{ij} \beta_{ij}}{\sum_{j=1}^{m_i} \omega_{ij}} \quad (11)$$

and the fuzzy control u becomes

$$u = \sum_{i=1}^n g_i \beta_i^0 \quad (12)$$

where g_i is the control gain.

3.3 Disturbance observer

The disturbance observer is constructed to estimate the excitation from the road profile w by assuming that $(\dot{z}_1, z_1, \dot{z}_2, z_2)$ can be directly measured. The minimum-order observer is introduced as the disturbance observer [9]. Approximating the nonlinear restoring force $f(z_1 - z_2)$ as

the linear restoring force $k_1(z_1 - z_2)$, and defining the augmented state vector \mathbf{x} and the measurement vector \mathbf{y} as

$$\mathbf{x} = [\dot{z}_1 \quad z_1 \quad \dot{z}_2 \quad z_2 \quad w]^T$$

$$\underline{\Delta} \mathbf{y} = [w]^T$$

then the state equation is obtained as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \quad (13)$$

where the matrices characterizing (13) are respectively defined as

$$\mathbf{A} = \left[\begin{array}{cccc|c} -c/m_1 & -k_1/m_1 & c/m_1 & k_1/m_1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ c/m_2 & k_1/m_2 & -c/m_2 & -(k_1+k_2)/m_2 & k_2/m_2 \\ 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\underline{\Delta} = \left[\begin{array}{c|c} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{0} & \mathbf{0} \end{array} \right]$$

$$\mathbf{B} = [1/m_1 \quad 0 \quad -1/m_2 \quad 0 \quad 0]^T \underline{\Delta} [\mathbf{B}_1 \quad 0]^T$$

The minimum-order observer [9] as the disturbance observer is derived to obtain the estimate \hat{w} of w as follows. Dividing equation (13) into two equations, then

$$\dot{\mathbf{y}} = \mathbf{A}_{11}\mathbf{y} + \mathbf{A}_{12}w + \mathbf{B}_1u \quad (14)$$

$$\dot{w} = 0 \quad (15)$$

Transforming w into w^* as

$$w^* = -\mathbf{T}\mathbf{y} + w \quad (16)$$

where

$$\mathbf{T} = [T_1 \quad T_2 \quad T_3 \quad T_4]$$

then the estimate \hat{w}^* of w^* is derived from the following equation

$$\hat{w}^* = \hat{\mathbf{A}}\hat{w}^* + \hat{\mathbf{J}}\mathbf{y} + \hat{\mathbf{B}}u \quad (17)$$

where

$$\hat{\mathbf{A}} = -\mathbf{T}\mathbf{A}_{12}, \quad \hat{\mathbf{J}} = -\mathbf{T}(\mathbf{A}_{11} + \mathbf{A}_{12}\mathbf{T}), \quad \hat{\mathbf{B}} = -\mathbf{T}\mathbf{B}_1$$

Therefore, the estimate \hat{w} of w is obtained from

$$\hat{w} = \mathbf{T}\mathbf{y} + \hat{w}^* \quad (18)$$

and the estimate \hat{w} is denoted as the variable in the precondition part.

4. EXPERIMENTAL RESULT

The parameters characterizing $G_c(s)$ are determined from the Bode diagram of $G_a(j2\pi f)G_c(j2\pi f)$. It is seen from Fig. 2 that the frequency response function with compensation more

raises the gain and more leads the phase shift than it without compensation where the parameters characterizing the compensator $G_c(j2\pi f)$ are determined as

$$\zeta_a = 0.7, \quad \zeta_b = 0.7, \quad \omega_a = 28.2 \text{ rad/s}, \quad \omega_b = 22.0 \text{ rad/s}$$

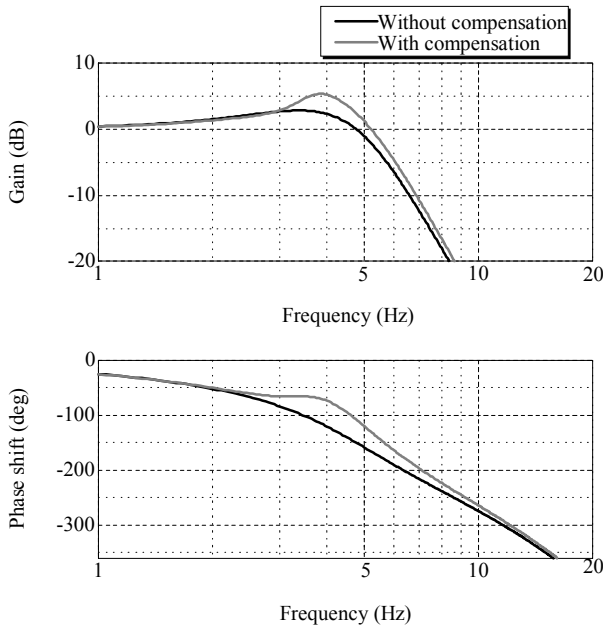


Fig.2 Frequency response functions

The parameters characterizing the experimental apparatus of the car model are given as

$$\begin{aligned} m_1 &= 46.1 \text{ kg}, & m_2 &= 18.1 \text{ kg}, & m_3 &= 1.32 \text{ kg}, \\ k_1 &= 6.8 \text{ kN/m}, & k_1' &= 20 \text{ kN/m}, & k_2' &= 100 \text{ kN/m}, \\ K &= 100 \text{ kN/m}, & c &= 400 \text{ Ns/m}, & a &= 1.4 \text{ mm} \end{aligned}$$

where c is determined by performing the free vibration experiment of the car model. The excitation force generated by the electric vibrator is assumed random with 5Hz bandwidth and the active control u is generated at the sampling instant with the time interval 10ms.

The variables $(\ddot{z}_1, \dot{z}_1, z_1, \ddot{z}_2, \dot{z}_2, z_2, \hat{w})$ are assumed in the precondition part and are respectively normalized as

$$\begin{aligned} \alpha_1 &= [\ddot{z}_1 / c_{1\max}] & \alpha_2 &= [\dot{z}_1 / c_{2\max}] & \alpha_3 &= [z_1 / c_{3\max}] \\ \alpha_4 &= [\ddot{z}_2 / c_{4\max}] & \alpha_5 &= [\dot{z}_2 / c_{5\max}] & \alpha_6 &= [z_2 / c_{6\max}] \\ \alpha_7 &= [\hat{w} / c_{7\max}] \end{aligned}$$

where $C_{1\max} \sim C_{7\max}$ are the scaling factors. The fuzzy sets are respectively assumed as P and N , and their membership functions are given as Fig. 3.

The performance index to determine the parameters characterizing the active control is assumed to be

$$J = E[q_1 \ddot{z}_1^2 + q_2 (z_1 - z_2)^2 + q_3 (z_2 - w)^2 + ru^2] \quad (19)$$

where

$$q_1 = 10, \quad q_2 = 1, \quad q_3 = 1, \quad r = 0.001$$

The three kinds of suspension systems are presented to compare the performance:

- Method A** : Passive suspension system
- Method B** : Active suspension system without disturbance observer
- Method C** : Active suspension system with disturbance observer

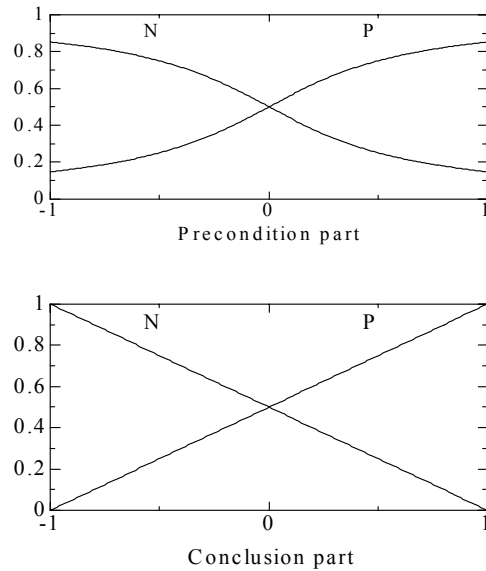


Fig.3 Membership functions

Table 1 shows that the root mean squares (RMS) values of the time responses of the one-wheel car model obtained from three kinds of methods. It is seen from the table that Method C generally is better in the vibration suppression of the car model, specially in the accelerations of the car body and the wheel, than Methods A and B. The spectral density calculated from the time response of the acceleration of the car body is shown in Fig.4. It is seen from the figure that Method C more reduces the peak of the spectral density comparing with Methods A and B. The estimation of w obtained from the proposed disturbance observer is shown in Fig.5, and it indicates that the proposed disturbance observer is very effective as the exact and its estimated values of the excitation from the road profile have good agreement.

5. CONCLUSION

In this paper, a pneumatic active suspension system for a one-wheel car model was constructed by using fuzzy reasoning. The fuzzy control was obtained by single input rule modules fuzzy reasoning where excitation from a road profile was estimated by using as a disturbance observer. The active control force was constructed by actuating a pneumatic actuator, and the degradation of the performance due to the delay of the pneumatic actuator was improved by inserting a compensator following the accelerometer to measure the acceleration of the car body. The experimental results indicated that the proposed active suspension system much improved the performance in the vibration suppression of the car model.

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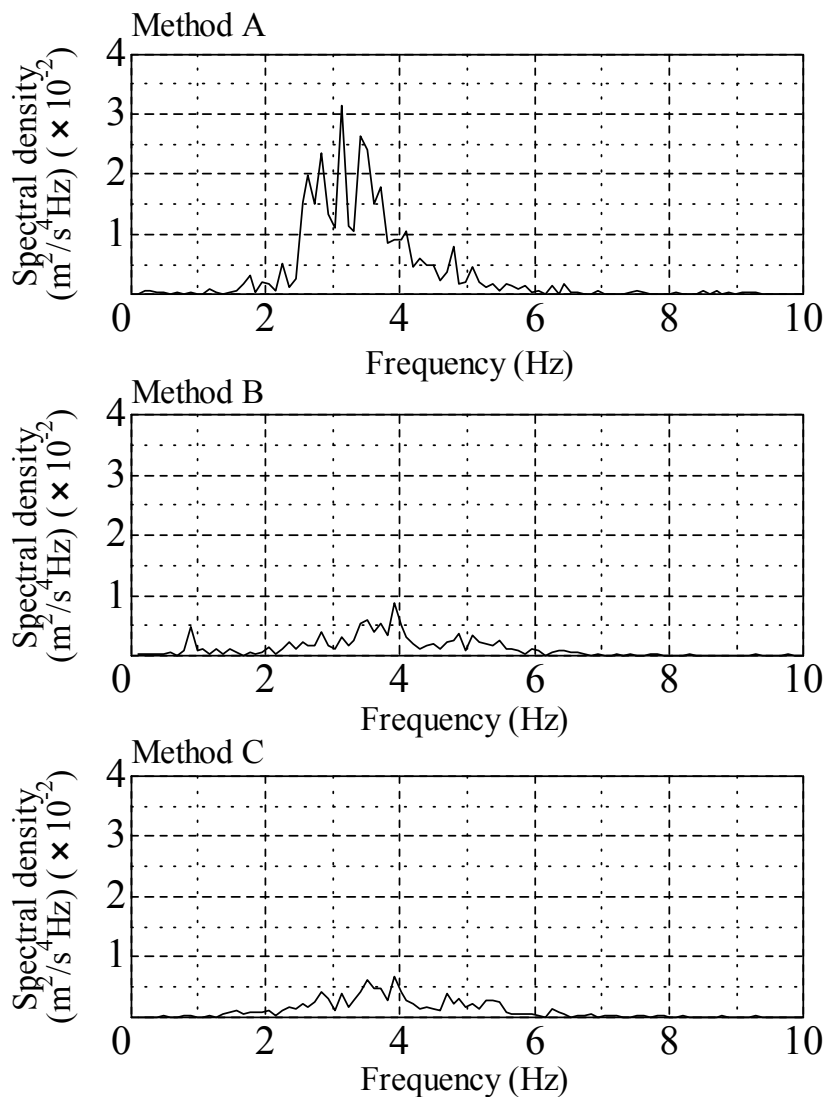


Fig. 4 Spectral density for \ddot{z}_1

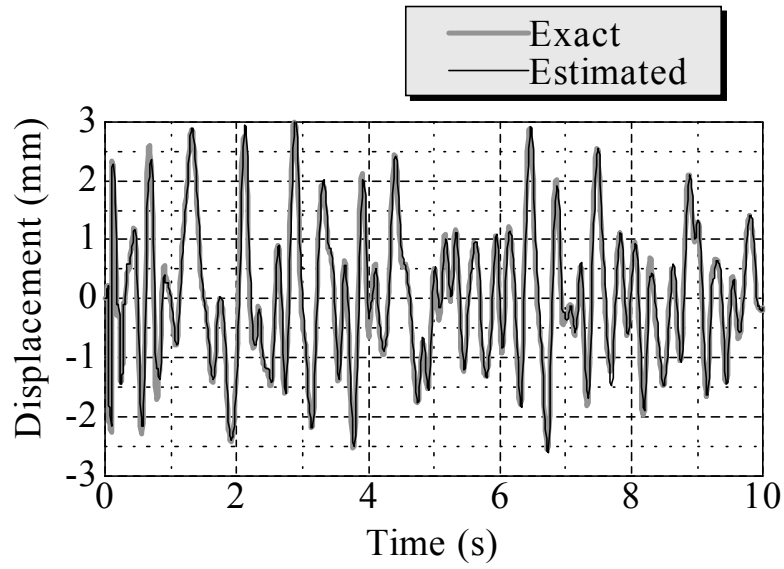


Fig.5 Estimation of the excitation from the road profile

Table 1 RMS values of the time responses

	<i>Method A</i>	<i>Method B</i>	<i>Method C</i>	<i>Unit</i>
$\ddot{z}_1 (\times 10^{-1})$	9.32	5.33	4.94	m/s^2
$\dot{z}_1 (\times 10^{-2})$	3.97	2.03	2.04	m/s
$z_1 (\times 10^{-3})$	2.34	1.48	1.50	m
$\ddot{z}_2 (\times 10^{-1})$	9.95	9.84	8.45	m/s^2
$\dot{z}_2 (\times 10^{-2})$	2.14	2.20	2.14	m/s
$z_2 (\times 10^{-3})$	1.31	1.21	1.20	m
$z_1 - z_2 (\times 10^{-3})$	1.63	1.42	1.45	m
$z_2 - w (\times 10^{-4})$	5.32	2.72	2.73	m
u	0	23.8	24.1	N
J	8.68	2.84	2.40	-