Generalised Non Error-Accumulative Quantisation Algorithm with feedback loop

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Abstract: This paper presents a new quantisation algorithm which has the closed-loop form and guarantees the boundness of accumulative error. This algorithm is particularly useful for mobile robot navigation that is usually implemented on embedded systems. If wheel commands of the mobile robot are given by velocity or positional increment at every control instant and quantised due to finite word length of controller's CPU, the quantisation error gets accumulated to causes large position error. Such an error accumulative characteristic is fatal for non wheeled mobile robots or autonomous vehicles with non-holonomic constraint. To solve this problem, we propose a non-error accumulative quantisation algorithm with closed-loop form. We also show it can be extend to a generalized form corresponding to the n-th order accumulation. The boundness of the accumulative quantisation error is investigated by a series of computer simulation. The proposed method is particularly effective to precise navigation control the autonomous mobile robots.

Keywords: Accumulative Quantisation Error, Velocity Control, Embedded Control Systems, Autonomous Robot Control, Remote Control Systems

1. INTRODUCTION

In the field of mobile robot control, many studies have been reported with issues on motion planning of mobile robots [1,2,3]. Mobile robots, driven by wheels on the planar surface are subject to nonholonomic constraints[4,5]. In motion planning, the simple truncation in numeric operations causes infinite accumulative error that is particularly critical for mobile robot navigation [6,7]. The quantisation usually comes from the finite word length of CPU or the limited sensor resolution such as in rotary encoders. The quantisation error may be tolerable in case of directly using them. However, since the quantised values are velocity commands or incremental position commands, the accumulative error introduces large positional error which restricts path tracking of a planned path.

It is also important for embedded real-time systems in which the integer computation is usually performed [8,9]. The hierarchical control structure of mobile robots requires an accumulative quantisation error reduction algorithm. In previous studies, issues on quantisation error reduction had been presented in image processing to reduce image distortion [10] and not focused on control engineering to reduce such an accumulative quantisation error. In a previous study [11], the overall requirements for mobile robot architecture are outlined in terms of control, modularity, software engineering, and run-time performance. Traditionally the architectures of the mobile robot were of a hierarchical form [12] in which high-rate servo control loop is performed in lower level controller and low-rate path tracking controller is performed in higher level controller. The wheel velocity commands are precisely computed by the path tracking controller based on floating point operations and transferred to wheel servo controller in which quantisation of the commands occurs due the finite word length of the embedded microprocessor.

To reduce the accumulative error resulted from quantisation, we propose a new simple quantisation algorithm with closed-loop form. After all, it grants us more advantages than just use of floating command. Basically, since the algorithm is based on the closed-loop form, numerical overflow problem in internal computation can be completely eliminated. We can also extend it to a generalized form corresponding up to the n-th order command. The mathematical error boundness of the algorithm can be referred to [12]. To show the validity of the algorithm, a series of computer simulations are performed and the results show that the presented algorithms works well.

2. ACCUMULATIVE QUANTISATION ERROR REDUCTION ALGORITHM

Let a conventional quantisation function $Q_0(\cdot)$ be

$$x_0 = Q_0(x), \qquad x_0 \le x < x_0 + \delta \tag{1}$$

where x_0 is quantised value and δ is quantisation unit. The quantisation error is bounded by δ . For the path tracking control in wheeled mobile robot navigation system, let us assume the wheel velocity command $v(k)$ at every control instant is computed and its quantised value is transferred to wheel servo controller in which a velocity control loop is performed for the quantised velocity command. At this instant, the position error occurs due to the quantisation process. We can imagine that the small errors in the wheel velocity control cause large positional error due to accumulative error that is particularly critical for non-holonomic mobile robot such as wheeled vehicles. Now, we investigate the error characteristic of the conventional quantisation $Q_0(\cdot)$. Assuming no quantisation process to happen, the wheel position can be computed $p(k)$ at $k-th$ control instant by simple accumulation of the velocity command $v(k)$ as follows:

$$p(k) = p(k-1) + v(k) . \tag{2}$$

Now, we consider the velocity command. The accumulative error due to the quantisation of $Q_0(\cdot)$ goes to infinity as the number of iteration increases.

$$\lim_{k\to\infty}(p(k)-\sum_{n=0}^{k}Q_0(v(n))) \qquad (3)$$
$$=\lim_{k\to\infty}\sum_{n=0}^{k}((v(n)-Q_0(v(n)))<\lim_{k\to\infty}k\delta=\infty .$$

In order to prevent the infinity problem of the conventional quantisation algorithm of $Q_0(\cdot)$, let us consider a new quantisation algorithm of $Q_1(\cdot)$. At each step, $v(k)$ is quantised by the following quantisation function,

$$v_1(k)=Q_0(p(k))-Q_0(p(k-1))\equiv Q_1(v(k)) \cdot \qquad (4)$$

Fig. 1 shows eqn. 4 as block diagram in digital control system. Using eqns. 2 and 4 the accumulative error due to the quantisation of Fig. 1 is bounded.

$$\lim_{k\to\infty}(p(k)-\sum_{n=0}^{k}Q_1(v(n)))=\lim_{k\to\infty}(p(k)-Q_0(p(k)))<\delta \cdot \qquad (5)$$

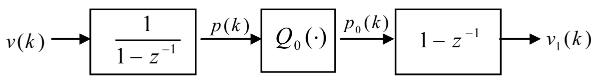

Fig. 1 $Q_1(\cdot)$ algorithm of the open-loop form

However, the open-loop form of $Q_1(\cdot)$ causes the computational overflow which may happen as the number of sequence increases since $p(k)$ is computed by eqn. 2.

To solve this overflow problem of internal computation, we convert $Q_1(\cdot)$ into closed-loop form as follows:

$$v_1(k)=Q_0(z(k)) \qquad (6)$$
$$z(k)=z(k-1)+v(k)-Q_0(z(k-1)) \cdot \qquad (7)$$

This algorithm can be described as control block diagram in Fig. 2.

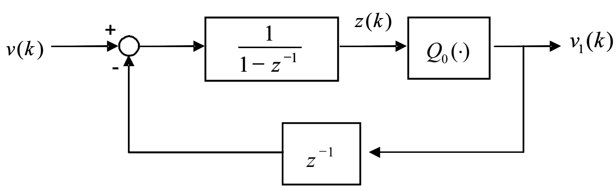

Fig. 2 $Q_1(\cdot)$ algorithm of the closed-loop form

The equivalence of the closed-loop form of $Q_1(\cdot)$ given by eqns. 6 and 7 to the open-loop form of $Q_1(\cdot)$ of eqn. 4 can be seen in [12]. This algorithm also has the following characteristic that it does not cause internal overflow of z(k) as

$$z(k)-v(k)=z(k-1)-Q_0(z(k-1))<\delta . \qquad (8)$$

Therefore, we can solve the overflow problem in computation of eqn. 4 since $z(k)$ in eqn. 7 is bounded within $v(k)+\delta$. And the boundness of accumulative error is proven by verifying the equivalence of the closed form to the open-loop form of $Q_1(\cdot)$. This is a feasible quantisation algorithm in embedded digital control system to reduce accumulative error caused by truncation.

3. GENERALIZED QUANTISATION ERROR REDUCTION ALGORITHM

When acceleration is given as a control command in motion planning, we should consider double accumulative error effect in using the quantisation algorithm. First of all, let us examine the double accumulative error characteristic of $Q_1(\cdot)$. The quantisation error of $Q_1(\cdot)$ is bounded by the quantisation value as

$$x_{1,}=Q_1(x), \qquad |x-Q_1(x)|<\delta , \qquad (9)$$

where x_1 is a natural number and δ is the quantisation unit. For motion planning in digital control system, the acceleration command $a(k)$ at every control instant is double-accumulated as follows:

$$v(k)=v(k-1)+a(k) \qquad (10)$$
$$p(k)=p(k-1)+v(k) . \qquad (11)$$

Then, the double accumulative error of the quantisation of $Q_1(\cdot)$, where $Q_1(\cdot)$ is the closed-loop form of eqns. 6 and 7, goes to infinity as the number of iteration increase as follows:

$$\lim_{k\to\infty}(p(k)-\sum_{n=0}^{k}\sum_{m=0}^{n}Q_1(a(m)))=\lim_{k\to\infty}\sum_{n=o}^{k}(v(n)-Q_0(v(n)))<\lim_{k\to\infty}k\delta=\infty \qquad (12)$$

since $\sum_{m=0}^{n}Q_1(a(m))=Q_0(v(n))$ by using eqn. 4.

To solve this double accumulative infinity problem of the quantisation algorithm of the closed-loop form of $Q_1(\cdot)$, let us consider the double accumulative error reduction algorithm of $Q_2(\cdot)$. By extending $Q_1(\cdot)$, we can easily make the double accumulative error reduction algorithm $Q_2(\cdot)$.

Double Accumulative error Reduction Algorithm: At each step, $a(k)$ is quantised by the following quantisation function $Q_2(\cdot)$,

$$a_2(k)=Q_1(v(k))-Q_1(v(k-1))\equiv Q_2(a(k)) \cdot \qquad (13)$$

Fig 3 shows eqn. 13 as block diagram in digital control system.

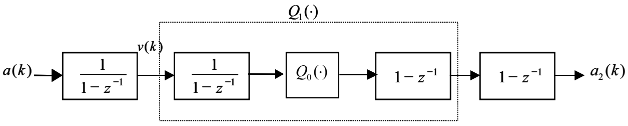

Fig.3 $Q_2(\cdot)$ algorithm of the open-loop form

The double accumulative error due to the quantisation of Fig. 3 can be written by using eqn. 13.

$$\lim_{k\to\infty}(p(k)-\sum_{n=0}^{k}\sum_{m=0}^{n}Q_2(a(m)))=\lim_{k\to\infty}(p(k)-\sum_{n=o}^{k}Q_1(v(n)))\cdot \quad (14)$$

And eqn. 14 yields eqn. 15 since $\sum_{n=0}^{k}Q_1(v(n))=Q_0(p(k))$ using eqn. 4.

$$\lim_{k\to\infty}(p(k)-\sum_{n=0}^{k}\sum_{m=0}^{n}Q_2(a(m)))=\lim_{k\to\infty}(p(k)-Q_0(p(k))<\delta\cdot \quad (15)$$

Therefore, the double accumulative error of $Q_2(\cdot)$ is bounded by δ regardless of iteration number, but the computational overflow may happen as the number of sequence increases since $v(k)$ is computed by eqn. 10.

To solve this overflow problem of the internal value, we change $Q_2(\cdot)$ of eqn.16 into the closed-loop form structure as follows:

$$a_2(k)=Q_1(y(k)) \quad (16)$$

$$y(k)=y(k-1)+a(k)-Q_1(y(k-1)), \quad (17)$$

where $Q_1(\cdot)$ is given as the closed-loop form of eqns. 6 and 7. Including the inner block of Fig. 3, the closed-loop form of $Q_2(\cdot)$ algorithm can be descibed in digital control block diagram as Fig. 4.

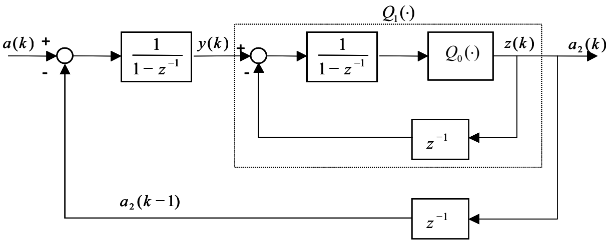

Fig.4 $Q_2(\cdot)$ algorithm of the closed-loop form

This is just equivalent to the open-loop form of $Q_2(\cdot)$ while it has the bounded double accumulative error characteristic. The equivalence of the closed-loop form quantisation given by eqns. 16 and 17 to the open-loop form of $Q_2(\cdot)$ of eqn. 13 can be also seen in [12].

The algorithm given by eqns. 16 and 17 has the closed-loop form and the error is bounded within quantisation unit since it is equivalent to the open-loop form of $Q_2(\cdot)$ of eqn. 16. And by following equations, we can easily show that internal variable $y(k)$ of eqn. 17 is bounded.

$$y(k)-a(k)=y(k-1)-Q_1(y(k-1))$$

$$|y(k)-a(k)|=|y(k-1)-Q_1(y(k-1))|<\delta\cdot \quad (18)$$

Therefore, the overflow problem in computation is solved since $|y(k)|$ is bounded within $|a(k)|+\delta$. It is a feasible quantisation algorithm for the acceleration control system. By using the feature of internal recursive structure, one can easily extend this algorithm to n-th order quantisation algorithm $Q_n(\cdot)$ as

$$x_n(k)=Q_{n-1}(y(k)) \quad (19)$$

$$y(k)=y(k-1)+x(k)-Q_{n-1}(y(k-1)), \quad (20)$$

which reveals n-th order bounded accumulative error.

4. COMPUTER SIMULATION

To show the validity of the proposed quantization algorithm, a series of the computer simulation were performed for an acceleration profile of mobile robot. In mobile robot navigation, the wheel acceleration command is computed by floating-point operation in the main computer and transferred to the wheel servo controller after quantization process [2]. To avoid the slippage between wheel and floor, the acceleration profile is designed in trapezoidal form by considering its maximum jerk as shown in Fig. 5.

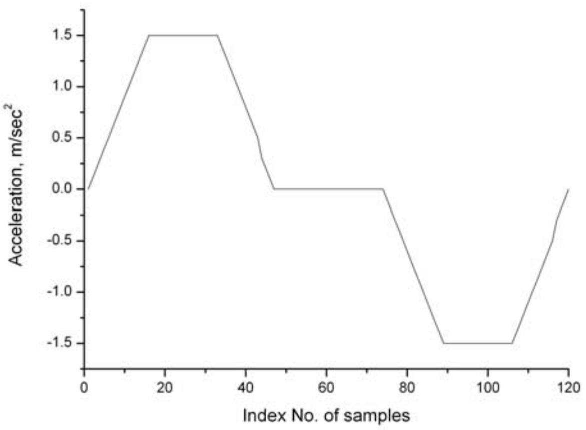

Fig. 5 Acceleration profile used for wheeled mobile robot

Let us assume the acceleration command given in Fig. 5 is quantised by $Q_0(\cdot)$ of eqn. 1 as shown in Fig. 6. Fig. 7 shows that the quantisation of the acceleration command by $Q_0(\cdot)$ of eqn. 1 cause large velocity error during the accumulation of the quantised values. At this time, let us assume that the acceleration command given in Fig. 5 is quantised by the closed-loop form of $Q_1(\cdot)$ of eqns. 6 and 7 as in Fig. 8. The simulation results of the open-loop form of $Q_1(\cdot)$ of eqn. 4 is not describe since the results are same with them of the closed-loop form except the open-loop form has computational overflow in the internal value.

Fig. 9 shows the velocity profile by accumulating the acceleration profile quantised by the closed-loop form of $Q_1(\cdot)$ of eqns. 6 and 7. The velocity profile is the first order accumulation of acceleration command and it has no error.

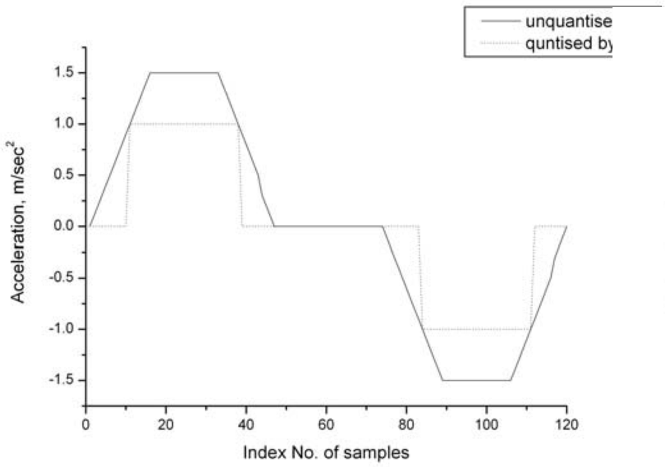

Fig. 6 Acceleration profile quantised by $Q_0(\cdot)$ of eqn. 1

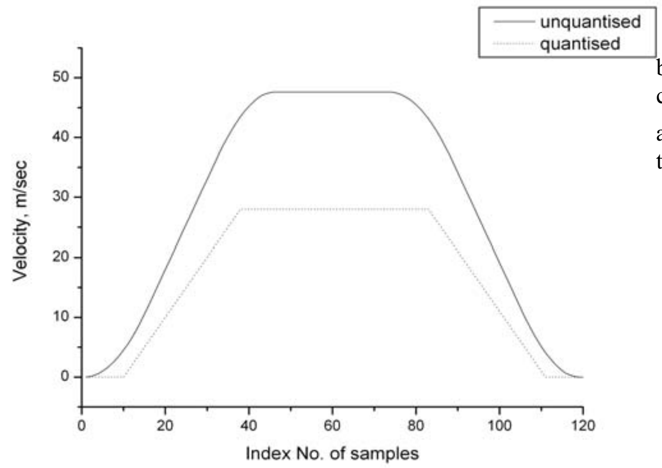

Fig. 7 Velocity profile generated from the acceleration profile quantised by $Q_0(\cdot)$ of eqn. 1

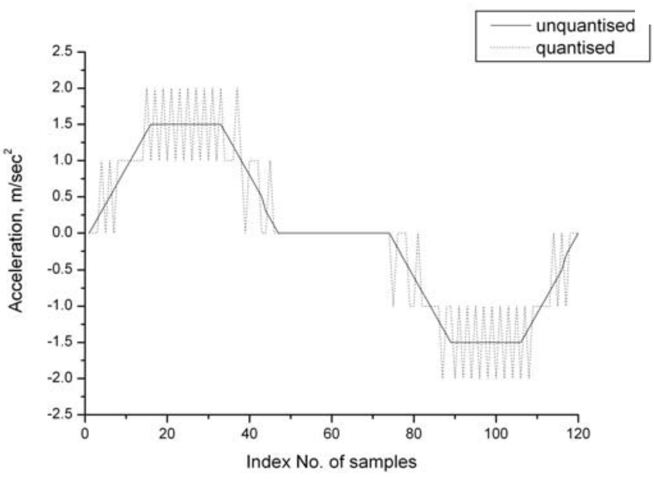

Fig. 8 Acceleration profile Quantised by the closed-loop form of $Q_1(\cdot)$

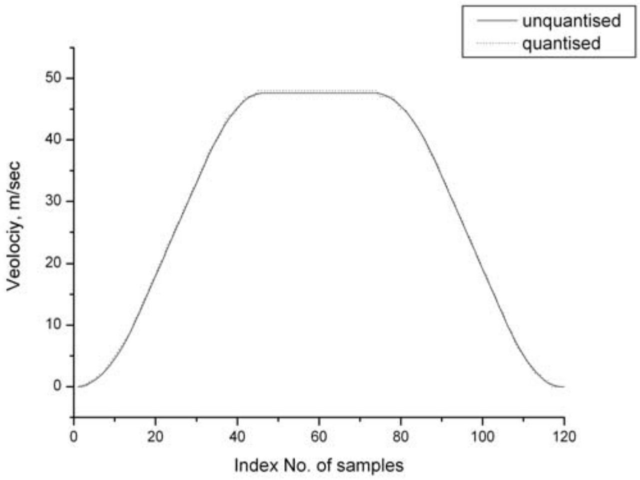

Fig. 9 Velocity profile by accumulating the acceleration profile quantised by the closed-loop form of $Q_1(\cdot)$

In Fig. 10, we depict errors between the velocity generated by accumulating the accelerations quantised by the closed-loop form of $Q_1(\cdot)$ and the velocity generated by accumulating the original acceleration command. Therefore, the velocity error is bounded.

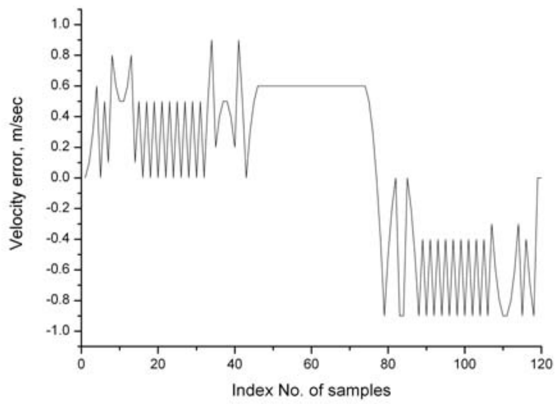

Fig. 10 Velocity error due to the quantised acceleration by the closed-loop form of $Q_1(\cdot)$

Even though the velocity error is bounded as shown in Fig. 10, it can also produce large position error as predicted by eqn. 15 in Section 3. Fig. 11 shows position error due to the double accumulative infinity problem of the quantisation algorithm of $Q_1(\cdot)$ of eqns. 6 and 7.

Now, let us observe the simulation result of applying the $Q_2(\cdot)$ of eqns. 19 and 20 algorithm to the quantization of acceleration command. Fig. 12 is the acceleration profile after applying $Q_2(\cdot)$ algorithm.

The velocity profile by accumulating the acceleration profile quantised by $Q_2(\cdot)$ of eqns. 19 and 20 is described in Fig. 13. The velocity command obtained by the quantised acceleration command reveals bounded error.

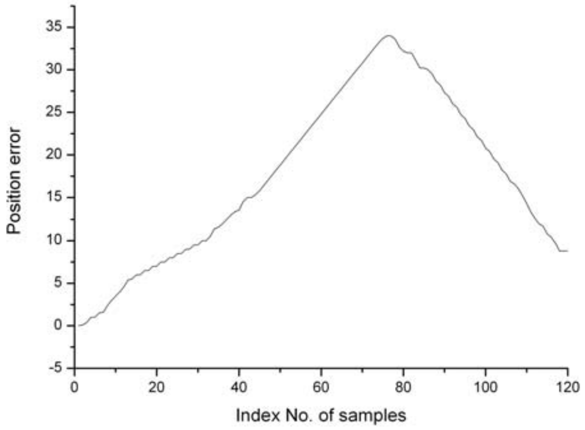

Fig. 11 Position error due to the quantisation of acceleration in the closed-loop form of $Q_1(\cdot)$

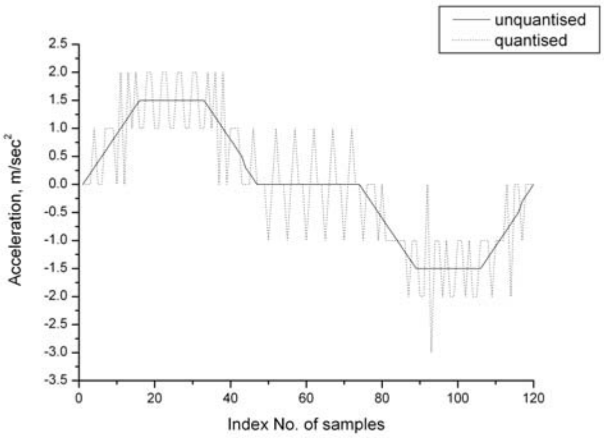

Fig. 12 Acceleration profile Quantised by $Q_2(\cdot)$

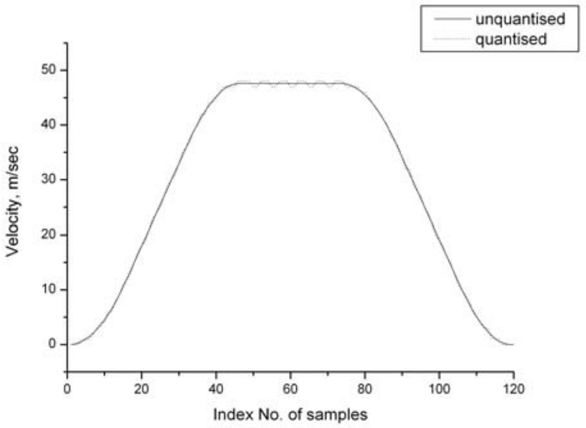

Fig. 13 Velocity profile by accumulating the acceleration profile quantised by $Q_2(\cdot)$

Fig.14 shows errors between the velocity generated by accumulating the acceleration quantised by $Q_2(\cdot)$ of eqns. 19 and 20 in Fig. 12 and the velocity generated by accumulating the unquantised acceleration.

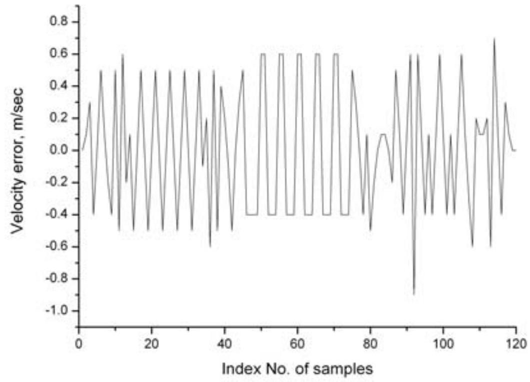

Fig. 14 Velocity error due to the quantised acceleration by $Q_2(\cdot)$ of eqns. 19 and 20

As predicted by eqn. 18 in Section 3, the quantisation of $Q_2(\cdot)$ has the bounded double accumulative error characteristic. Fig.15 shows the boundness of positional error due to quantisation of $Q_2(\cdot)$.

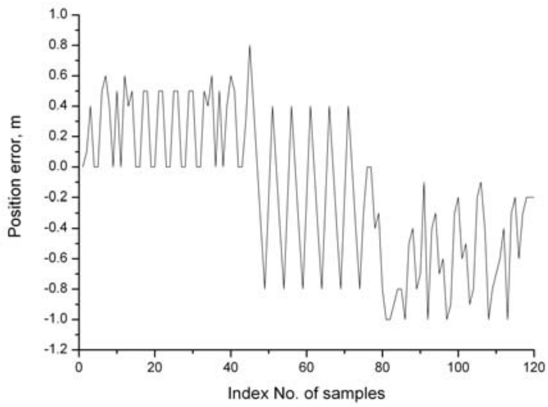

Fig. 15 Position error due to the quantisation acceleration by $Q_2(\cdot)$ of eqns. 19 and 20

5. CONCLUSION

In this paper we proposed quantization error reduction algorithm that is practically meaningful in control system implemented by embedded systems. Since the proposed algorithm has a closed-loop form, it can solve the internal overflow problem. The accumulative error boundness of the proposed algorithm is proven via some mathematical manipulations. We can also generalize the quantization algorithm by showing that the extension of the algorithm to the N-th order guarantees the error boundness of the N-th order accumulation. To show effectiveness of the proposed algorithm, a series of computer simulation were performed for a motion planning in which a acceleration profile is given by trapezoidal form. The simulation results confirm us that the algorithm works well.

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