# Implementation of Position Control of PMSM with FPGA 

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#### Abstract

This paper presents of position control of Permanent Magnet Synchronous Motor (PMSM) the implementation with Field Programmable Gate Array (FPGA) is proposed. Cascade control with inner loop as a current control and an outer loop as a position control is chosen for simplicity and fast response. FPGA is a single chip (single processing unit), which will perform the following tasks: receive and convert control signal, create a reference current signal, control current and create switch signal and act as position controller in a addition of zero form. The 10 kHz sampling frequency and 25 bit of floating point data are defined in this implementation. The experimental results show that the performance of FPGA based position control is comparable with the hardware based position control, with the advantage of control algorithm flexibility


Keywords: FPGA, Position Control, Permanent Magnet Synchronous Motor, PMSM

## 1. INTRODUCTION

The implementation of position control of PMSM with FPGA is proposed. This approach is considered suitable for FPGA rapid prototyping. The function blocks, configured by the FPGA, can be treated as the concurrent and programmable hardware modules, gaining both efficiency and flexibility in system design. The program, designed by an end-user, will form logic gates within FPGA into a digital loop. FPGA works as a hardware format that allows the user to define the work procedure. FPGA provides more flexibility than microprocessor.

The transfer function of a typical PMSM has the general form that relates $\theta / i^{*}$. This transfer function have pole at $\mathrm{s}=$ 0 , -pole1, -pole 2 thus system that is stable for small gain but unstable for large gain. The addition of a zero to the transfer function by compensator has the effect of pulling the root locus to the left, tending to make the system more stable and to speed up the settling of the response. (Physically, the addition of a zero in the feedforward transfer function means the addition of derivative control to the system. The effect of such control is to introduce a degree of anticipation into the system and system and speed up the transient response.) Notice that when a zero is added to the motor system it becomes stable for all value of gain.[1-2]

## 2. BACKGROUND

### 2.1 Modeling of the PMSM motor

The structure of a PMSM-based drive system is shown in Fig. 1. The control inputs take values from the discrete set $-u_{0},+u_{0}$ instead of on-off signals from the discrete set $\{0,1\}$. Let the six on-off signals be $\mathbf{s}_{\mathbf{w}}=\left[\begin{array}{llllll}s_{w 1} & s_{w 2} & s_{w 3} & s_{w 4} & s_{w 5} & s_{w 6}\end{array}\right]^{T}$ with $s_{w 4}=1-s_{w 1} \quad s_{w 5}=1-s_{w 2} \quad s_{w 6}=1-s_{w 3}$ and the current control inputs design be $\mathbf{U}_{\text {gate }}=\left[\begin{array}{lll}u_{1} & u_{2} & u_{3}\end{array}\right]^{T}$, then the following relation holds:
$\mathbf{U}_{\text {gate }}=u_{0} \mathbf{G}_{\mathbf{w}} \mathbf{s}_{\mathbf{w}} \quad$ where $\quad \mathbf{G}_{\mathbf{w}}=\left[\begin{array}{cccccc}1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1\end{array}\right]$


Fig. 1: Structure of a PMSM drive system.
In general, the dynamic model of an AC motor can be established using physical law:

$$
\begin{align*}
U & =R I+\frac{d \psi}{d t}  \tag{2}\\
\psi & =L I+\psi_{M}
\end{align*}
$$

Where U,I and $\psi$ are the voltage vector, the current vector and the flux vector, respectively; $\mathrm{R}, \mathrm{L}$ are the resistance matrix and the inductance matrix, respective; $\psi \mathrm{M}$ is the flux vector caused by the permanent magnet, if applicable.


Fig. 2: Coordinate systems of PMSM

The equation system Eq. (2) is a general description of the electro coordinate system. For PMSMs, three reference frames are normally used to describe the dynamic behavior of a motor Fig.2: the phase frame, i.e. the ( $a, b, c$ ) coordinate frame; the stator frame, i.e. the $(\alpha, \beta)$ coordinate frame; and the field-oriented frame, i.e. the ( $\mathrm{d}, \mathrm{q}$ ) coordinate frame (also called the rotor coordinate frame for PMSMs).

The motor model in the ( $\mathrm{d}, \mathrm{q}$ ) coordinate frame, which rotates synchronously with the motor rotor, can also be obtained by transforming the motor model form the $(\alpha, \beta)$ coordinate frame to $(\mathrm{d}, \mathrm{q})$ coordinate

$$
\begin{align*}
& \frac{d i_{d}}{d t}=-\frac{R}{L} i_{d}+\omega_{e} i_{q}+\frac{1}{L} u_{d}  \tag{3}\\
& \frac{d i_{q}}{d t}=-\frac{R}{L} i_{q}+\omega_{e} i_{d}-\frac{1}{L} \lambda_{0} \omega_{e}+\frac{1}{L} u_{q}
\end{align*}
$$

Where $i_{d}$ and $i_{q}$ are the stator currents in the ( $\mathrm{d}, \mathrm{q}$ ) coordinate frame; ud and uq are the stator voltage in the same coordinate frame. Term $\lambda_{0} \omega_{e}=e_{q}$ is the q-component of the induced EMF generated by the permanent magnet; the d-component of the EMF $e_{d}$ is equal to zero. Note the second equation of Eq. (3). If the current component $i_{d}$ could be made equal to zero, we would get exactly the behavior of a constant-excited DC motor.

Finally, the electrical torque $\tau$ e and the mechanical power P of the motor are given by

$$
\begin{align*}
& \tau_{e}=K_{t} i_{q}  \tag{4}\\
& P=\tau_{e} \omega_{r}
\end{align*}
$$

In which Kt is the torque constant, assumed to be equal to (3/2) $\lambda 0 \mathrm{Nr}$ with Nr being the number of pole pairs of the motor, and $\omega r$ is the mechanical angular speed of the motor rotor. In developing the motor models, we assume there is no reluctance torque in the PMSM motor. Under this assumption, the output torque of the motor is proportional to the q -axis stator current iq. The mechanical motion equation of the motor can be written as

$$
\begin{align*}
& J \frac{d \omega_{r}}{d t}=\tau_{e}-\tau_{l} \\
& \frac{d \theta}{d t}=\omega_{r} \tag{5}
\end{align*}
$$

Where $\tau \mathrm{e}$ and $\theta \mathrm{r}$ denote the load torque and the mechanical angular position of the motor rotor. For the electrical angular position/speed and the mechanical angular position/speed, the following relations hold:

$$
\begin{align*}
& \omega_{e}=N_{r} \omega_{r}  \tag{6}\\
& \theta_{e}=N_{r} \theta_{r}
\end{align*}
$$

Rotor position $\theta \mathrm{r}$ is usually measured; $\omega \mathrm{r}, \theta \mathrm{e}, \omega \mathrm{e}$, are calculated according to Eqs. (5)~ (6)

For position control of a PMSM, a cascaded control structure is usually preferred, with an inner current control loop and an outer position control loop as shown in Fig. 3[2-4]


Fig. 3: Cascades control structure of PMSM

### 2.2 Current Control

The goal of the current control is to design a current controller to track the desired currents $\left(\mathbf{i}^{*}\right)$ which are normally provided by an outer-loop position controller. In the context of current control, the controller has to determine the discontinuous controls $u_{1}, u_{2}, u_{3}$ as well as the on-off signals of the switch. The on-off signals may also be called switching patterns and are defined as [1]
$\mathbf{s}_{123}=\mathbf{i}_{\text {abc }}^{*}-\mathbf{i}_{\text {abc }}$

### 2.3 Addition of zeros

Form open loop control of $\theta / i^{*}$ can be written to Eq. (8). Since load torque $\left(\tau_{l}\right)$ is unknown, it will be set as a disturbance in the system.

$$
\begin{equation*}
\frac{\theta}{i^{*}}=\frac{1.5 k_{t} N_{r} k_{i n v}}{s\left(J L s^{2}+\left(J R+B L+J k_{i n v}\right) s+\left(B R+1.5 k_{t} N_{r}+B k_{i n v}\right)\right)} \tag{8}
\end{equation*}
$$

The transfer function of a system is call type I. The magnitude of the steady-state error to unit step as s approaches zero, for at root-locus plot of a three-pole system is $0,-\mathrm{a},-\mathrm{b}$ as shown in Fig. 4 (a)

The transfer function of a typical PMSM has the general form that relates motor position to input armature current. This transfer function have at $s=0,-a,-b$ thus system that is stable for small gain but unstable for large gain. The addition of a zero to the transfer function by compensator has the effect of pulling the root locus to the left, tending to make the system more stable and to speed up the settling of the response. (Physically, the addition of a zero in the feedforward transfer function means the addition of derivative control to the system. The effect of such control is to introduce a degree of anticipation into the system and system and speed up the transient response.) Notice that when a zero is added to the motor system it becomes stable for all value of gain.

The addition of a zero to the open-loop transfer function has the effect of pulling the root locus to the left, tending to mark the system more stable and to speed up the setting of the response. (Physically, the addition of a zero in the feedforward transfer function means the addition of derivative control system and speed up the transient response.) Fig. 4(a) show the root loci for a system that is stable for small gain but unstable for large gain Fig. 4(b) , Fig. 4(c), Fig. 4(d) show root-locus plots for the system when a zero is added to the open-loop transfer function. Notice that when a zero to the system of Fig. 4(a) it becomes stable for all value of gain.[5]


Fig. 4: effect of addition of zero to the three-pole system.

### 2.4 Position control

For the design of position controller in an outer loop, the current control loop may be treated as an ideal current source, i.e. it can track a given reference current immediately and accurately. This assumption can be true only for systems in which the electrical time constant is much smaller than the mechanical time constant, or for systems where the dynamic response of the position control is not critical. Any control design techniques, linear or nonlinear, may be used for position control loop. When use configuation controller by zero $_{s}$ and $K_{s}$ equation of the control can be writen the sdomain transfer function of analog controller as

$$
\begin{equation*}
G(s)=\text { Ko }_{s}\left(s+\text { zero }_{s}\right) \tag{9}
\end{equation*}
$$

A transformation of s from the s-domain into $G(z)$ in the z domain using Bilinear Transformation ( this technique, also called the Tustin or trapezoidal approximation) gives the relationship Eq. (10), where T is the sampling period.[6]

$$
\begin{equation*}
s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) \tag{10}
\end{equation*}
$$

The z-domain transfer function of discrete controller $G(z)$ obtained from the mathematical modeling can be given in the form of Eq. (11) or (12)

$$
\begin{align*}
& G(z)=\frac{\left(2 K_{0}+K_{0} \text { Tzero }\right) / T+\left(K_{-} \text {zero }-2 K_{0}\right) / T z^{-1}}{1+z^{-1}}  \tag{11}\\
& G(z)=K o_{z}\left(\frac{1+\text { zero }_{z} z^{-1}}{1+z^{-1}}\right) \tag{12}
\end{align*}
$$

## 3. SIMULATION

The motor parameters used to verify the design principle are $N_{r}=4, J_{m}=0.002 \mathrm{kgm}^{2}, B_{m}=0.0002 \mathrm{Nmsrad}^{-1}, k_{t}=0.1665$ $N m A^{-1} R_{s}=0.1 \Omega, L=3 \mathrm{mH}$ en co der $=2048$ pulse $/ \mathrm{rev}$,thus Eq. (8) can be given in the form of Eq. (13). Theus thansfer function have pole at $0,-13$ and -13354 .
$\frac{\theta}{i^{*}}=\frac{6660000}{s\left(s^{2}+13367 s+167840\right)}$

The addition of zero gain of the adaptation controller are zero $_{s}=9.77$ and $K_{0}=10000$ can be give in the form of (12)
$G(s)=10000 \frac{(s+9.77)}{(s+1000000)}$
The simulation results using simulink in matlab this show in Fig. 5. When step reference 16384 position (8 rev).


Fig. 5 step response of position control

## 4. IMPLEMENTATION

In FPGA module, APEX DSP Development Board (starter Version) of Altera Co., Ltd will be used as a controller. The Quartus II Limited Edition development software provides a comprehensive environment for system-on-a-programmablechip (SOPC) design. It controls PMSM. The supplied link voltage is $u_{0}=40 \mathrm{~V}$. The data were being processed with the size of 25 bit of floating point as suggested by Pavle Belanovic,[7], working at sampling frequency 10 kHz .


Fig. 6: Block diagram of PMSM control with FPGA
The block diagram of a position control of a permanent magnet synchronous motor is shown in Fig.6. The FPGA is employed to perform 4 main functions as follows:

### 4.1. Receive speed and current signals

4.1.1. The block diagram of FPGA and associated circuits are shown in Fig. 7. The analog current signals are fed to IC MAX 310 for multiplexing and then converted to digital signal by an analog to digital (A/D) converter. The FPGA will perform the following tasks: 1) Control the multiplexing and de-multiplexing of the input signals. 2) Create $i_{b}$ signal from the currents flow in the three-phase motor windings which are star-connected without neutral. The expression is $i_{b}=-\left(i_{a}+i_{c}\right)$. These functions require about 3,000 logic gates.


Fig. 7: Receiving motor current with FPGA.


Fig.8: Receiving a speed signal with FPGA.
4.1.2. Fig. 8 shows the block diagram of FPGA for position reference and actual position calculation. Three digital signals from encoder are pulse A, pulse B and pulse $Z$ signal. A phase angle difference between pulse A and pulse B is 90 degree. It will set a speed and direction of motor rotation. Pulse Z signal will set a starting point of rotor. This function requires about 1,200 logic gates.

### 4.2. Create current reference signals

Three current reference signals in form of alternating current format can be generated by FPGA as shown in Fig. 9 The magnitude of reference current signal from controller generating from section 4.4 was multiplied by three phase sinusoidal waveform. The sinusoidal waveform was generated by the translation from sin table into sin value. In order to get a synchronizing relationship between current wave and rotor movement, it requires about 12,000 logic gates.


Fig.9: Creating a reference current signal in form of alternating current signal with FPGA.

### 4.3. Control current and create switch signal

The block diagram for switch signal generation and current control are shown in Fig.10.Eq (7) was programmed on FPGA by comparing reference current signal(section 4.2) to measured current signal (section 4.1.1). The outputs of the comparators are switch signals of inverter in Fig.1. D-flip-flop will set a sampling frequency of switching at 10 kHz . This function requires 8,000 logic gates.


Fig.10: Control of current and creating of inverter switch.

### 4.4. Position Controller

Position controller block diagram was written in the form of addition of zeros controlling Eq. (12) on FPGA using 25 bit of floating point format for processing Fig. 11. It requires about 80,000 logic gates.

## 5. EXPERIMENTAL RESULTS

A transformation of trnsfer function Eq. (14) of the sdomain into transfer function in the z-domain using bilinear transform this technique this show in as Eq. (15)


Fig.11. Position Controller in digital format
$G(z)=0.5\left(\frac{1-0.9990 z^{-1}}{1+z^{-1}}\right)$
The performance of position control (step 65536 and 16384 position) is shown in Fig. 12 with the time scale of $200 \mathrm{~ms} / \mathrm{div}$. and Fig.13. with the time scale of $100 \mathrm{~ms} /$ div. The scale for all positions is 512 point/div a).P-controller methode. .b).addition of zero methode in Fig. (4d). It can be seen that the control system responded well with the change of controller methode and over shoot of position respond.

## 6. CONCLUSION

FPGA has been used for controlling PMSM in step response of position control. FPGA is set in the form of IC single chip at sampling frequency 10 kHz . It was programmed to control the motor current and to control the position of the motor by processing the floating point number. The user can design the work procedure and define the value of the parameters without changing there structure such as fixed point of microprocessor. Experimental results showed that FPGA based position control performance in real time is comparable to the hardware based position control in terms of response time and accuracy, with the advantage of flexibility in control scheme implementation. The results

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Fig. 12: position control of step reference 65536 position( 32 rev )


Fig. 13: position control of step reference 16384 position( 8 rev)

