# Internet Based Network Control using Fuzzy Modeling

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**Abstract**: This paper presents the design methodology of digital fuzzy controller(DFC) for the systems with time-delay. We propose the fuzzy feedback controller whose output is delayed with unit sampling period and predicted. The analysis and the design problem considering time-delay become easy because the proposed controller is syncronized with the sampling time. The stabilization problem of the digital fuzzy system with time-delay is solved by linear matrix inequality(LMI) theory. Convex optimization techniques are utilized to solve the stable feedback gains and a common Lyapunov function for designed fuzzy control system. To show the effectiveness the proposed control scheme, the network control example is presented.

**Keywords:** Fuzzy control, Digital fuzzy system, Time-delay, Linear matrix inequality, Network control

## 1. INTRODUCTION

The control problems for delayed systems have attention over the last few decades since the time-delay is frequently a source of instability and encountered in various engineering systems. Extensive research has already been done in the conventional control to find the solutions [1][2]. However, for fuzzy control systems, there are few studies on the stabilization problem for especially systems with time-delay[3][4]. A linear controller like PID controllers has a short time-delay in calculating the output since its algorithm is so simple. However, in the case of a complex algorithm like fuzzy or neural networks, a considerable time-delay can occur because so many calculations are needed to get the output. Nevertheless, the most conventional discrete time fuzzy controllers are the ideal controllers in which the time-delay is not considered. Recently, to deal with the time-delay, the design methods of the fuzzy control systems with higher order have been proposed in [5]. However the structure of the control system is very complex because the design of higher order fuzzy rule-base is highly difficult.

In this paper, the digital fuzzy control system considering a time delay is developed and its stability analysis and design method are proposed. We use the discrete Takagi-Sugeno(TS) fuzzy model and parallel distributed compensation(PDC) conception for the controller[6-9]. And we follow the linear matrix inequality(LMI) approach to formulate and solve the problem of stabilization for the fuzzy controlled systems with time-delay. The analysis and the design of the discrete time fuzzy control systems by LMI theory are considered in [10-12].

If the system has a considerable time-delay the analysis and the design of the controller are very difficult since the time-delay makes the output of the controller not synchronized with the sampling time. We propose the PDC-type fuzzy feedback controller whose output is delayed with unit sampling period and predicted using current states and the control input to the plant at previous sampling time. The analysis and the design of the controller are very easy because the output of the proposed controller is synchronized with the sampling time. Therefore, the proposed control system can be designed using the conventional methods for stabilizing the discrete time fuzzy systems and the feedback gains of the controller can be obtained using the concept of the LMI feasibility problem. The proposed DFC is applied to network control through internet to verify the validity and the effectiveness of the control scheme.

## 2. DISCRETE TS MODEL BASED FUZZY CONTROL

In the discrete time TS fuzzy systems without control input, the dynamic properties of each subspace can be expressed as the following fuzzy IF-THEN rules[6].

$$\begin{array}{l}\text{Rule } i: \text{ If } x_1(k) \text{ is } M_{i1} \cdots \text{and } x_n(k) \text{ is } M_{in} \\ \qquad \text{THEN } \mathbf{x}(k+1) = \mathbf{G}_i\mathbf{x}(k)\end{array} \quad i = 1, 2, \cdots, r \tag{1}$$

where $\mathbf{x}(k) = \begin{bmatrix} x_1(k) & x_2(k) & \cdots & x_n(k)\end{bmatrix}^T \in \Re^n$ denotes the state vector of the fuzzy system, $r$ is the number of the IF-THEN rules, and $M_{ij}$ is fuzzy set.

If the state $\mathbf{x}(k)$ is given, the output of the fuzzy system expressed as the fuzzy rules of Eq. (1) can be inferred as follows.

$$\mathbf{x}(k+1) = \frac{\sum_{i=1}^{r} w_i(k)\mathbf{G}_i\mathbf{x}(k)}{\sum_{i=1}^{r} w_i(k)} = \sum_{i=1}^{r} h_i(k)\mathbf{G}_i\mathbf{x}(k) \tag{2}$$

where $w_i(k) = \prod_{j=1}^{n} M_{ij}(x_j(k))$, $h_i(k) = \dfrac{w_i(k)}{\sum_{i=1}^{r} w_i(k)}$

A sufficient condition for ensuring the stability of the fuzzy system(2) is given in **Theorem 1**.

**Theorem 1** : The equilibrium point for the discrete time fuzzy system (2) is asymptotically stable in the large if there exists a common positive definite matrix **P** satisfying the following inequalities.

$$\mathbf{G}_i^T\mathbf{P}\mathbf{G}_i - \mathbf{P} < \mathbf{0} \quad , \quad i = 1, 2, \cdots, r \tag{3}$$

**Proof** : The proof can be given in [7].

In the discrete time fuzzy system with control input to the plant, the dynamic properties of each subspace can be expressed as the following fuzzy IF-THEN rules.

Rule $i$ : If $x_1(k)$ is $M_{i1}$ $\cdots$ and $x_n(k)$ is $M_{in}$
THEN $\mathbf{x}(k+1)=\mathbf{A}_i\mathbf{x}(k)+\mathbf{B}_i\mathbf{u}(k)$ $\quad i=1,2,\cdots,r$ (4)

where $\mathbf{x}(k)=[x_1(k) \quad x_2(k) \quad \cdots \quad x_n(k)]^T\in\Re^n$ denotes the state vector of the fuzzy system.

$\mathbf{u}(k)=[u_1(k) \quad u_2(k) \quad \cdots \quad u_m(k)]^T\in\Re^m$ denotes the input of the fuzzy system.

$r$ is the number of the fuzzy IF-THEN rules, and $M_{ij}$ is the fuzzy set.

If the set of $(\mathbf{x}(k),\mathbf{u}(k))$ is given the output of the fuzzy system (4) can be obtained as follows.

$$\mathbf{x}(k+1)=\frac{\sum_{i=1}^{r}w_i(k)\{\mathbf{A}_i\mathbf{x}(k)+\mathbf{B}_i\mathbf{u}(k)\}}{\sum_{i=1}^{r}w_i(k)}$$

$$=\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i\mathbf{x}(k)+\mathbf{B}_i\mathbf{u}(k)\} \tag{5}$$

where

$$w_i(k)=\prod_{j=1}^{n}M_{ij}(x_j(k)), \text{ and } h_i(k)=\frac{w_i(k)}{\sum_{i=1}^{r}w_i(k)}.$$

In PDC, the fuzzy controller is designed distributively according to the corresponding rule of the plant[9]. Therefore, the PDC for the plant (4) can be expressed as follows.

Rule $j$ : If $x_1(k)$ is $M_{j1}$ $\cdots$ and $x_n(k)$ is $M_{jn}$
THEN $\mathbf{u}(k)=-\mathbf{F}_j\mathbf{x}(k)$ $\quad j=1,2,\cdots,r$ (6)

The fuzzy controller output of Eq. (6) can be inferred as follows.

$$\mathbf{u}(k)=-\frac{\sum_{i=1}^{r}w_i(k)\,\mathbf{F}_i\mathbf{x}(k)}{\sum_{i=1}^{r}w_i(k)}=-\sum_{j=1}^{r}h_j(k)\,\mathbf{F}_j\mathbf{x}(k) \tag{7}$$

where $h_j(k)$ is the same function in Eq. (5).

Substituting Eq. (7) into Eq. (5) gives the following closed loop discrete time fuzzy system.

$$\mathbf{x}(k+1)=\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i\mathbf{x}(k)-\mathbf{B}_i\sum_{j=1}^{r}h_j(k)\mathbf{F}_j\mathbf{x}(k)\}$$

$$=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(k)\,h_j(k)\{\mathbf{A}_i-\mathbf{B}_i\mathbf{F}_j\}\mathbf{x}(k) \tag{8}$$

Defining $\mathbf{G}_{ij}=\mathbf{A}_i-\mathbf{B}_i\mathbf{F}_j$ , the following equation is obtained.

$$\mathbf{x}(k+1)=\sum_{i=1}^{r}h_i(k)\,h_i(k)\,\mathbf{G}_{ii}\mathbf{x}(k)$$

$$+2\sum_{i<j}^{r}h_i(k)\,h_j(k)\{\frac{\mathbf{G}_{ij}+\mathbf{G}_{ji}}{2}\}\mathbf{x}(k) \tag{9}$$

Applying **Theorem 1** to analyze the stability of the discrete time fuzzy system (9), the stability condition of **Theorem 2** can be obtained.

**Theorem 2** : The equilibrium point of the closed loop discrete time fuzzy system (9) is asymptotically stable in the large if there exists a common positive definite matrix P which satisfies the following inequalities for all *i* and *j* except the set $(i,j)$ satisfying $h_i(k)\cdot h_j(k)=0$ .

$$\mathbf{G}_{ii}^T\mathbf{P}\mathbf{G}_{ii}-\mathbf{P}<\mathbf{0} \tag{10a}$$

$$(\frac{\mathbf{G}_{ij}+\mathbf{G}_{ji}}{2})^T\mathbf{P}(\frac{\mathbf{G}_{ij}+\mathbf{G}_{ji}}{2})-\mathbf{P}\le\mathbf{0}, \quad i<j \tag{10b}$$

**Proof** : The proof can be given in [7].

If $\mathbf{B}=\mathbf{B}_1=\mathbf{B}_2=\cdots=\mathbf{B}_r$ in the plant (5) is satisfied, the closed loop system (8) can be obtained as follows.

$$\mathbf{x}(k+1)=\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i\mathbf{x}(k)-\mathbf{B}\sum_{j=1}^{r}h_j(k)\mathbf{F}_j\mathbf{x}(k)\}$$

$$=\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i-\mathbf{B}\mathbf{F}_i\}\mathbf{x}(k)=\sum_{i=1}^{r}h_i(k)\mathbf{G}_i\mathbf{x}(k) \tag{11}$$

where $\mathbf{G}_i=\mathbf{A}_i-\mathbf{B}\mathbf{F}_i$

Hence, **Theorem 1** can be applied to the stability analysis of the closed loop system (11).

## 3. LMI APPROACH FOR FUZZY SYSTEM DESIGN

To prove the stability of the discrete time fuzzy control system by **Theorem 1**, the common positive definite matrix **P** must be solved. LMI theory can be applied to solving **P** [13]. LMI theory is one of the numerical optimization techniques. Many of the control problems can be transformed into LMI problems and the recently developed Interior-point method can be applied to solving numerically the optimal solution of these LMI problems[14].

**Definition 1** : Linear matrix inequility can be defined as follows.

$$\mathbf{F}(\mathbf{x})=\mathbf{F}_0+\sum_{i=1}^{m}x_i\mathbf{F}_i>\mathbf{0} \tag{12}$$

where $\mathbf{x}=[x_1 \quad x_2 \quad \cdots \quad x_m]^T$ is the parameter, the symmetric matrices $\mathbf{F}_i=\mathbf{F}_i^T\in\Re^{n\times n}, i=0,\cdots,m$ are given, and the inequality symol " $>\mathbf{0}$ " means that $\mathbf{F}(\mathbf{x})$ is the positive definite matrix.

LMI of Eq. (12) means the convex constraints for **x**. Convex constraint problems for the various **x** can be expressed as LMI of Eq. (12). LMI feasibility problem can be described as follows.

***LMI feasibility problem*** : The problem of finding $\mathbf{x}^{\text{feasp}}$ which satisfies $\mathbf{F}(\mathbf{x}^{\text{feasp}})>\mathbf{0}$ or proving the unfeasibility in the case that LMI $\mathbf{F}(\mathbf{x})>\mathbf{0}$ is given.

If the design object of a controller is to guarantee the stability of the closed loop system (2), the design of the PDC fuzzy controller(4) is equivalent to solving the following LMI feasibility problem using Schur complements[13].

***LMI feasibility problem equivalent to the PDC design problem*** : The problem of finding $\mathbf{X}>\mathbf{0}$ and $\mathbf{M}_1,\mathbf{M}_2,\cdots,\mathbf{M}_r$ which satisfy the following equations.

$$\begin{bmatrix}\mathbf{X} & \{\mathbf{A}_i\mathbf{X}-\mathbf{B}_i\mathbf{M}_i\}^T\\ \mathbf{A}_i\mathbf{X}-\mathbf{B}_i\mathbf{M}_i & \mathbf{X}\end{bmatrix}>\mathbf{0} \qquad i=1,2,\cdots,r$$

where $\mathbf{X}=\mathbf{P}^{-1}$, $\mathbf{M}_1=\mathbf{F}_1\mathbf{X}$ , $\mathbf{M}_2=\mathbf{F}_2\mathbf{X}$ ,$\cdots$,and $\mathbf{M}_r=\mathbf{F}_r\mathbf{X}$ .

The feedback gain matrices $\mathbf{F}_1,\mathbf{F}_2,\cdots,\mathbf{F}_r$ and the common positive definite matrix **P** can be given by the LMI solutions, **X** and $\mathbf{M}_1,\mathbf{M}_2,\cdots,\mathbf{M}_r$ , as follows.

$\mathbf{P}=\mathbf{X}^{-1}$ , $\mathbf{F}_1=\mathbf{M}_1\mathbf{X}^{-1}$ , $\mathbf{F}_2=\mathbf{M}_2\mathbf{X}^{-1}$ , $\cdots$, and $\mathbf{F}_r=\mathbf{M}_r\mathbf{X}^{-1}$

## 4. DIGITAL FUZZY CONTROL SYSTEM CONSIDERING TIME-DELAY

In a real control system, a considerable time-delay can occur due to a communication and a controller. Let $\tau$ be defined as the sum of all this time-delay. In the case of the real system, the ideal fuzzy controller of Eq. (3) can be described as follows due to the time-delay.

Rule $j$ : If $x_1(kT)$ is $M_{j1} \cdots$ and $x_n(kT)$ is $M_{jn}$

THEN $\mathbf{u}(kT+\tau) = -\mathbf{F}_j \mathbf{x}(kT)$

$$j = 1, 2, \cdots, r \tag{13}$$

Because the time-delay makes the output of controller not synchronized with the sampling time, **Theorem 1** can not be applied to this system. Therefore the analysis and the design of the controller are very difficult. In this paper, DFC which has the following fuzzy rules is proposed to consider the time-delay of the fuzzy plant (1).

Rule $j$ : If $x_1(k)$ is $M_{j1} \cdots$ and $x_n(k)$ is $M_{jn}$

THEN $\mathbf{u}(k+1) = \mathbf{D}_j \mathbf{u}(k) + \mathbf{E}_j \mathbf{x}(k)$ $\quad j = 1, 2, \cdots, r \quad (14)$

The output of DFC (14) is inferred as follows.

$$\mathbf{u}(k+1) = \frac{\sum_{j=1}^{r} w_j(k)\,\{\mathbf{D}_j \mathbf{u}(k) + \mathbf{E}_j \mathbf{x}(k)\}}{\sum_{j=1}^{r} w_j(k)}$$

$$= \sum_{j=1}^{r} h_j(k)\,\{\mathbf{D}_j \mathbf{u}(k) + \mathbf{E}_j \mathbf{x}(k)\} \tag{15}$$

The output timing of a ideal controller, a delayed controller, and the proposed controller is shown in the Fig. 1. In the ideal controller, it is assumed that there is no time-delay. If this controller is implemented in real systems the time-delay $\tau$ is added like Eq. (13). The analysis and the design of this system with delayed controller are very difficult since the output of controller is not syncronized with the sampling time.

On the other hand, the analysis and the design of the proposed controller are very easy because the controller output is syncronized with the sampling time delayed with unit sampling period.

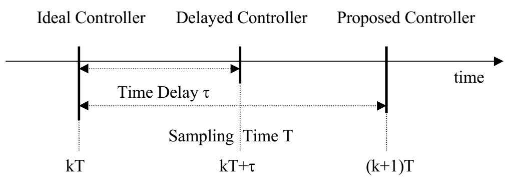



Fig. 1. Output Timing of the Controllers

Combining the fuzzy plant (2) with the DFC (15), the closed loop system is given as follows.

$$\begin{bmatrix} \mathbf{x}(k+1) \\ \mathbf{u}(k+1) \end{bmatrix} = \sum_{i=1}^{r} h_i(k) \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{E}_i & \mathbf{D}_i \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{u}(k) \end{bmatrix} \tag{16}$$

Defining the new state vector as $\mathbf{w}(k) = \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{u}(k) \end{bmatrix}$, the closed loop system (16) can be modified as

$$\mathbf{w}(k+1) = \sum_{i=1}^{r} h_i(k) \mathbf{G}_i \mathbf{w}(k) \tag{17}$$

where $\mathbf{G}_i = \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{E}_i & \mathbf{D}_i \end{bmatrix}$

Hence, the stability condition of the closed loop system (17) becomes the same as the sufficient condition of **Theorem 1** and the stability can be determined by solving ***LMI feasibility problem about the stability condition of Theorem 1***. Also, the design problem of the DFC guaranteeing the stability of the closed loop system can be transformed into ***LMI feasibility problem***. To do this, the design problem of the DFC is transformed into the design problem of the PDC fuzzy controller.

***PDC design problem equivalent to DFC design problem* :**

The problem of designing the PDC fuzzy controller $\mathbf{v}(k) = -\sum_{j=1}^{r} h_j(k)\, \overline{\mathbf{F}}_j \mathbf{w}(k)$ in the case that the fuzzy plant $\mathbf{w}(k+1) = \sum_{i=1}^{r} h_i(k) \{\overline{\mathbf{A}}_i \mathbf{w}(k) + \overline{\mathbf{B}} \mathbf{v}(k)\}$ is given.

where $\overline{\mathbf{A}}_i = \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$, $\overline{\mathbf{B}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}$, and $\overline{\mathbf{F}}_j = -\begin{bmatrix} \mathbf{E}_j & \mathbf{D}_j \end{bmatrix}$

Therefore, using the same notation in section 3, the design problem of the DFC can be equivalent to the following **LMI feasibility problem**.

***LMI feasibility problem equivalent to DFC design problem* :**

The problem of finding $\mathbf{X} > \mathbf{0}$ and $\mathbf{M}_1, \mathbf{M}_2, \cdots, \mathbf{M}_r$ which satisfy following equation.

$$\begin{bmatrix} \mathbf{X} & \{\overline{\mathbf{A}}_i \mathbf{X} - \overline{\mathbf{B}}\ \mathbf{M}_i\}^T \\ \overline{\mathbf{A}}_i \mathbf{X} - \overline{\mathbf{B}}\ \mathbf{M}_i & \mathbf{X} \end{bmatrix} > \mathbf{0}, \quad i = 1, 2, \cdots, r$$

where $\mathbf{X} = \mathbf{P}^{-1}$ , $\mathbf{M}_1 = \overline{\mathbf{F}}_1 \mathbf{X}$ , $\mathbf{M}_2 = \overline{\mathbf{F}}_2 \mathbf{X}$ , $\cdots$, and $\mathbf{M}_r = \overline{\mathbf{F}}_r \mathbf{X}$

The feedback gain matrices $\overline{\mathbf{F}}_1, \overline{\mathbf{F}}_2, \cdots, \overline{\mathbf{F}}_r$ and the common positive definite matrix $\mathbf{P}$ can be given by the LMI solutions, $\mathbf{X}$ and $\mathbf{M}_1, \mathbf{M}_2, \cdots, \mathbf{M}_r$, as follows.

$$\mathbf{P} = \mathbf{X}^{-1},\ \overline{\mathbf{F}}_1 = \mathbf{M}_1 \mathbf{X}^{-1},\ \overline{\mathbf{F}}_2 = \mathbf{M}_2 \mathbf{X}^{-1}, \cdots, \overline{\mathbf{F}}_r = \mathbf{M}_r \mathbf{X}^{-1} \tag{18}$$

Therefore, the control gain matrices $\mathbf{D}_1, \cdots, \mathbf{D}_r, \mathbf{E}_1, \cdots, \mathbf{E}_r$ of the proposed DFC can be obtained from the feedback gain matrices $\overline{\mathbf{F}}_1, \overline{\mathbf{F}}_2, \cdots, \overline{\mathbf{F}}_r$ .

## 5. APPLICATIONS TO NETWORK CONTROL VIA INTERNET

We have shown an analysis technique of the proposed DFC under the condition that time-delay exists in section 4. We apply the controller to network control system with time-delay caused by internet communication. The overall control configuration is shown in Fig. 2. The block diagram of the control system is shown by Fig. 3. In this figure, the actuator that we will control is a linear permanent magnet brushless DC motor (LPMBDCM) with 3 phase.

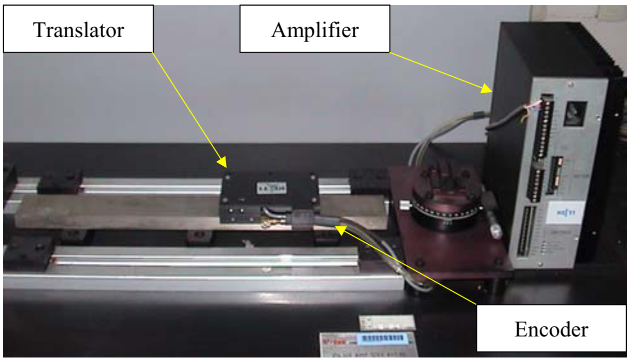



Fig. 2 Linear permanent magnet brushless DC motor system

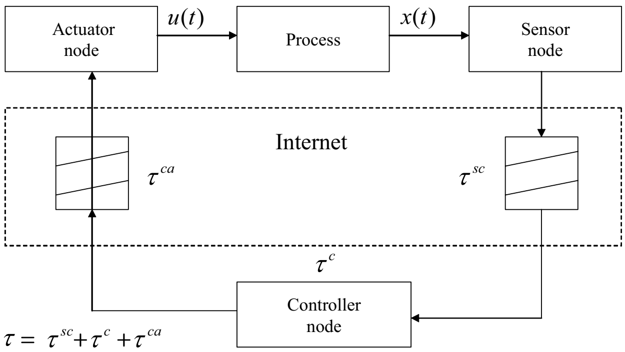



Fig. 3 Block diagram of control system

The state variable model for the LPMBDCM is given by (19) [16].

$$
\begin{aligned}
\frac{dx_p}{dt} &= \dot{x}_p \\
\frac{d\dot{x}_p}{dt} &= \frac{-\beta}{M}\dot{x}_p + \frac{\pi}{M\tau}(L_d - L_q)i_q i_d + \sqrt{\frac{3}{2}\frac{\pi}{M\tau}}\lambda_{\max} i_q - \frac{F_L}{M} \\
&\quad - \sum_{i=1}^{4}\frac{F_{ri}}{M}\sin\frac{6\pi i(x_p)}{\tau} \\
\frac{di_q}{dt} &= -\frac{r}{L_q}i_q - \frac{\pi}{\tau}\frac{L_d}{L_q}\dot{x}_p i_d - \sqrt{\frac{3}{2}\frac{\pi}{\tau}}\frac{\lambda_{\max}}{L_q}\dot{x}_p + \frac{v_q}{L_q} \\
\frac{di_d}{dt} &= -\frac{r}{L_d}i_d + \frac{\pi}{\tau}\frac{L_q}{L_d}\dot{x}_p i_q + \frac{v_d}{L_d}.
\end{aligned} \tag{19}
$$

Table 1. Parameters of LPMBDCM

| | | | |
|---|---|---|---|
| $x_p$ | traslational position | $\tau$ | pole pitch |
| $x_{1d}$ | desired translational position | $L_q$ | quadrature-axis inductance |
| $\dot{x}_p$ | translator speed | $L_d$ | direct-axis inductance |
| $x_{2d}$ | desired translational speed | $L_{ls}$ | direct-axis inductance |
| $i_q$ | quadrature-azis current | $\lambda_{\max}$ | maximum value of flux linkages |
| $x_{3d}$ | desired quadrature-axis current | $r$ | phase resistance |
| $i_d$ | direct-axis current | $F_{ri}$ | value of the i th cogging force |
| $x_{4d}$ | desired direct-axis current | $F_L$ | load force |
| $M$ | translator mass | $v_q$ | quadrature-axis voltage |
| $\beta$ | damping coefficient | $v_d$ | direct-axis voltage |

The problem of position, speed, or current control becomes one of regulating error in the state variables to the origin. These state variable deviations are defined by

$$
\begin{aligned}
&x_1 = x_p - x_{1d}, \quad x_2 = \dot{x}_p - x_{2d} \\
&x_3 = i_q - x_{3d}, \quad x_4 = i_d - x_{4d}.
\end{aligned} \tag{20}
$$

Since we are interested in the position control, the desired speed must be zero, and for maximum output force per ampere, the desired direct-axis current should alse be zero as

$$x_{2d} = x_{4d} = 0.$$

For the sake of simplified notation, we will set

$$
\begin{aligned}
&k_1 = \frac{\pi}{M\tau}(L_d - L_q);\ k_2 = \sqrt{\frac{3}{2}\frac{\pi}{M}}\lambda_{\max}; \\
&k_3 = -\frac{\beta}{M};\ k_4 = -\frac{r}{L_q};\ k_5 = -\frac{\pi}{\tau}\frac{L_d}{L_q}; \\
&k_6 = -\sqrt{\frac{3}{2}\frac{\pi}{\tau}}\frac{\lambda_{\max}}{L_q};\ k_7 = -\frac{r}{L_d};\ k_8 = \frac{\pi}{\tau}\frac{L_q}{L_d}; \\
&k_{9i} = -\frac{F_{ri}}{M};\ k_{10} = -\frac{F_L}{M};\ \omega_i = \frac{6\pi i(x_1 + x_{1d})}{\tau};
\end{aligned} \tag{21}
$$

Solving (20) for $x_p, \dot{x}_p, i_q$, and $i_d$ and substituting into (21) yields the new nonlinear state equations for LPMBDCM in the new coordinates in the following form

$$
\dot{\mathbf{x}}(t) = f(\mathbf{x}(t)) + \sum_{i=1}^{2}u_i(t)g_i
$$

$$
= \begin{bmatrix} x_2 \\ k_3x_2 + k_1x_3x_4 + k_2(x_3 + x_{3d}) + k_1x_{3d}x_4 + \sum_{i=1}^{4}k_{9i}\sin\omega i + k_{10} \\ k_4x_3 + k_5x_2x_4 + k_6x_2 \\ k_7x_4 + k_8x_2x_3 + k_8x_2x_{3d} \end{bmatrix}
+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} k_4x_{3d} + \dfrac{V_q}{L_q} \\ \dfrac{V_d}{L_d} \end{bmatrix} \tag{22}
$$

Since the state variable $x_1(t)$ represents the translational position error, the problem of position control of the motor is equivalent to regulating the states of (22) to the origin. Futher, with $x_{1d}$ known, $x_{3d}$ can be found from the solution of (22) in steady state form to yield

$$x_{3d} = \sum_{i=1}^{4} -\frac{k_{9i}}{k_2}\sin\left(\frac{6\pi i x_{1d}}{\tau}\right) + \frac{k_{10}}{k_2}.$$

To design fuzzy controller, we change nonlinear model for LPMBDCM to T-S fuzzy model. The nonlinear equation representing the LPMBDCM can be converted into a linear form at the operating points, $\mathbf{x_0} = \{x_{10}\ \ x_{20}\ \ x_{30}\ \ x_{40}\}$ as (23).

$$\dot{\mathbf{x}} = \begin{bmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x_1} & \frac{\partial f_1(\mathbf{x})}{\partial x_2} & \frac{\partial f_1(\mathbf{x})}{\partial x_3} & \frac{\partial f_1(\mathbf{x})}{\partial x_4} \\ \frac{\partial f_2(\mathbf{x})}{\partial x_1} & \frac{\partial f_2(\mathbf{x})}{\partial x_2} & \frac{\partial f_2(\mathbf{x})}{\partial x_3} & \frac{\partial f_2(\mathbf{x})}{\partial x_4} \\ \frac{\partial f_3(\mathbf{x})}{\partial x_1} & \frac{\partial f_3(\mathbf{x})}{\partial x_2} & \frac{\partial f_3(\mathbf{x})}{\partial x_3} & \frac{\partial f_3(\mathbf{x})}{\partial x_4} \\ \frac{\partial f_4(\mathbf{x})}{\partial x_1} & \frac{\partial f_4(\mathbf{x})}{\partial x_2} & \frac{\partial f_4(\mathbf{x})}{\partial x_3} & \frac{\partial f_4(\mathbf{x})}{\partial x_4} \end{bmatrix}_{\mathbf{x}=\mathbf{x_0}} \mathbf{x} + \begin{bmatrix} 0\,0 \\ 0\,0 \\ 1\,0 \\ 0\,1 \end{bmatrix} \mathbf{u} \qquad (23)$$

In the experiments, the operating points are selected as $x_{10} \in \{-0.03\ 0\ 0.03\}$, $x_{20} \in \{-0.5\ 0\ 0.5\}$, $x_{30} \in \{-0.02\ 0.02\}$ and $x_{40} \in \{-0.001\ \ 0.001\}$.

Hence, the number of linearized models at the operating points is 36, which comprise the consequent part of each T-S fuzzy rule base (24) for the fuzzy model of LPMBDCM. In (24), the system and input gain matrices, $A_{id}$ and $B_d$ are obtained by discretizing each of linearized models and the membership functions are shown in Fig. 4.

Rule $i$: If $x_1(k)$ is $M_{i1}$ and $x_2(k)$ is $M_{i2}$ and $x_3(k)$ is $M_{i3}$ and $x_4(k)$ is $M_{i4}$

THEN $\mathbf{x}(k+1) = \mathbf{A}_{id}\mathbf{x}(k) + \mathbf{B}_d\mathbf{u}(k)$

$i = 1, 2, \cdots, 36$ (24)

The inferred in-out fuzzy model is

$$\mathbf{x}(k+1) = \frac{\sum_{i=1}^{36} w_i(k)\{\mathbf{A}_{di}\mathbf{x}(k) + \mathbf{B}_d\mathbf{u}(k)\}}{\sum_{i=1}^{36} w_i(k)}$$

$$= \sum_{i=1}^{36} h_i(k)\{\mathbf{A}_{di}\mathbf{x}(k) + \mathbf{B}_d\mathbf{u}(k)\} \qquad (25)$$

The following table presents the value of the physical parameters used in the experiment.

Table 2. Physical parameters

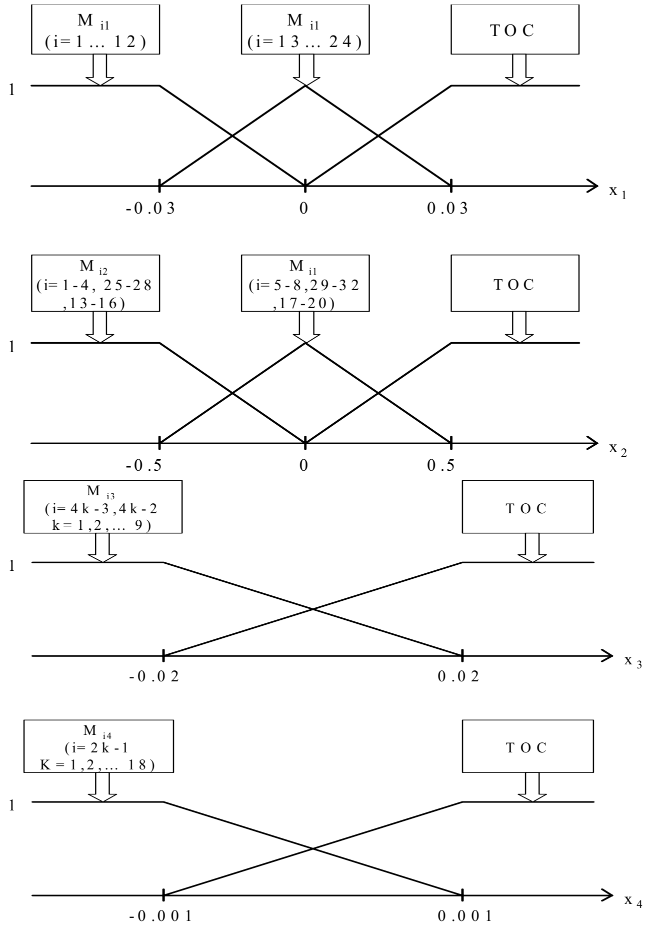
| Parameters | Value | Unit |
|---|---|---|
| $M$ | 2.09 | kg |
| $\tau$ | 0.01 | M |
| $L_d$ | 0.006845 | H |
| $L_q$ | 0.00385 | H |
| $\lambda_{\max}$ | 0.000058 | Wb |
| $\beta$ | 0.02 | kg/s |
| $r$ | 1.2 | Ω |

The fuzzy rules for the controller is designed as

Rule $i$: If $x_1(k)$ is $M_{i1}$ and $x_2(k)$ is $M_{i2}$ and $x_3(k)$ is $M_{i3}$ and $x_4(k)$ is $M_{i4}$

THEN $\mathbf{u}(k+1) = \mathbf{D}_i\mathbf{u}(k) + \mathbf{E}_i\mathbf{x}(k)$

$i = 1, 2, \cdots, 36$ (26)




TOC : The Other Cases

Fig. 4 Membership function

Then, the controller can be inferred as

$$\mathbf{u}(k+1) = \frac{\sum_{i=1}^{36} w_i(k)\{\mathbf{D}_i\mathbf{u}(k) + \mathbf{E}_i\mathbf{x}(k)\}}{\sum_{i=1}^{36} w_i(k)}$$

$$= \sum_{i=1}^{36} h_i(k)\{\mathbf{D}_i\mathbf{u}(k) + \mathbf{E}_i\mathbf{x}(k)\} \qquad (27)$$

The problem of obtaining the control gain matrices, $D_1, D_2, \cdots D_{36}, E_1, E_2, \cdots E_{36}$ guaranteeing the stability of the closed loop system is equivalent to the following LMI based problem.

***The problem of finding*** $\mathbf{X} > \mathbf{0}$ ***and*** $\mathbf{M}_1, \mathbf{M}_2, \cdots, \mathbf{M}_{36}$ ***which satisfy following equation:***

$$\begin{bmatrix} \mathbf{X} & \{\overline{\mathbf{A}}_i\mathbf{X} - \overline{\mathbf{B}}\,\mathbf{M}_i\}^T \\ \overline{\mathbf{A}}_i\mathbf{X} - \overline{\mathbf{B}}\,\mathbf{M}_i & \mathbf{X} \end{bmatrix} > \mathbf{0},$$

where $\overline{\mathbf{A}}_i = \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$ and $\overline{\mathbf{B}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}$, $i = 1, 2, \cdots, 36$,

$\mathbf{X} = \mathbf{P}^{-1}$, $\mathbf{M}_1 = \overline{\mathbf{F}}_1\mathbf{X}$, $\mathbf{M}_2 = \overline{\mathbf{F}}_2\mathbf{X}$, $\cdots$, $\mathbf{M}_{36} = \overline{\mathbf{F}}_{36}\mathbf{X}$, $\overline{\mathbf{F}}_i = [\mathbf{E}_i \quad \mathbf{D}_i]$ (28)

The matrices, $\mathbf{X}$ and $\mathbf{M}_1, \mathbf{M}_2, \cdots, \mathbf{M}_{36}$ in LMI's are determined using a convex optimization technique offered by [15] and the control gains can be obtained from the our method.

$$\mathbf{X} = \mathbf{P}^{-1} = \begin{bmatrix} 157.0056 & 61.9680 & -1.6565 & 220.7727 & 56.9821 & 52.4982 \\ 61.9680 & 50.4822 & 69.8423 & 53.4329 & 124.5211 & -10.5246 \\ -1.6565 & 69.8423 & 489.4416 & -2.3866 & 52.8435 & 25.0084 \\ 220.7727 & 53.4329 & -2.3866 & 442.6866 & 10.5892 & 20.1458 \\ 56.9822 & 124.5211 & 52.8435 & 10.5892 & 62.2894 & -96.2511 \\ 52.4982 & -10.5246 & 25.0084 & 20.1458 & -96.2511 & 154.2584 \end{bmatrix}$$

Figure 5 shows the experiment results without considering the network delay. The dotted line denotes the referenced signal and the solid line denotes the controlled LPMBDCM position. In this case, the controller based on the exact linearization of the model which has been presented in [16] was utilized.

From the figures, one can see that the system was oscillating and the fuzzy controller can not accomplish the tracking control effectively. Figure 6 shows the control results using the proposed controller. As can be seen in these figures, the control is accomplished successfully.

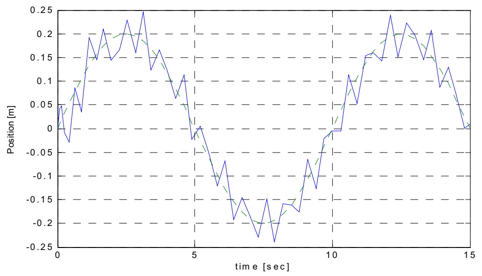



Fig. 5 Control results using conventional PDC controller (Position of LPMBDCM)

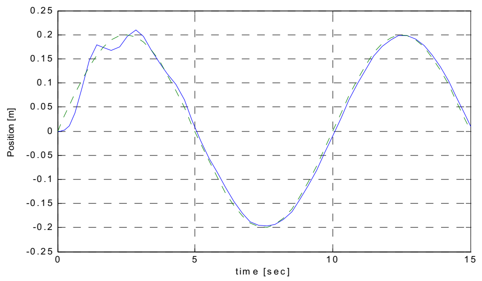



Fig. 6 Control results using proposed controller (Position of LPMBDCM)

## 6. CONCLUSIONS

In this paper, we have developed a DFC framework for a class of systems with time-delay. Because the proposed controller was syncronized with the sampling time delayed with unit sampling period and predicted, the analysis and the design problem considering time-delay could be very easy. Convex optimization technique based on LMI has been utilized to solve the problem of finding stable feedback gains and a common Lyapunov function. Therefore the stability of the system was guaranteed in the existence of time-delay and the real-time control processing could be possible. To show the effectiveness and feasibility of the proposed controller we have developed a digital fuzzy control system for network control of LPMBDCM with time-delay induced by internet traffic. Through the experiments, we have shown that the proposed DFC could achieve the position control of a LPMBDCM successfully although a considerable network time-delay existed.

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