Control of a Flexible Link with Time Delays

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Abstract: This paper presents a control method for time-delay systems and verifies the performance of the designed control system via real experiments. Specifically, the control method is applied to a flexible-link system with time delays. The method combines time- and frequency-domain controllers: linear quadratic optimal controller (or LQR) and lag compensator. The LQR is used to stabilize the system in optimal fashion, whereas the lag compensator is used to compensate time-delay effects by increasing the delay margin of the system. With this methodology, the maximum allowable time delay can be increased significantly. The proposed method is simple but quite practical for time-delay system control as it is based on the conventional loop-shaping method, which gives practical insights on delay-phase relationship. Simulation and experiment results show that the method presented in this paper is feasible and practical.

Keywords: Time delay, flexible link, linear quadratic control, lag compensation.

1. Introduction

Time delays are frequently encountered in many industrial applications where information processing and data transmission capabilities are limited. These time delays often deteriorate the system performance and even destabilize the feedback system. To deal with the problems regarding time-delay effects, there has been much work on time-delay system control; see, e.g., [2], [4] and references therein.

Recently, the use of communication networks for control has attracted much interest and the time-delay effects on performance and stability become one of most extensively researched subjects [9]. The work in this paper is also motivated by the fact that even a small time delay arising in the communication control-loop can significantly degrade the control performance of a flexible link unless time-delay effects are considered in the design phase.

Among time-delay system control theories, state-space approaches based on the Lyapunov theory have been extensively studied for the past years [2]– [4]. In these approaches, the controller (gain) is designed with some conditions that ensure that the candidate Lyapunov function be decreased as time goes by. To obtain such conditions, in general, the well-known Lyapunov-Krasovskii and Lyaunov-Razumikhin theorems are used.

Although some of these approaches give good properties such as robustness, these approach tend to result in conservative controllers in some cases. For example, most of delay-independent controllers based on the Lyapunov theory are criticized for their conservativeness. Infinite delay margins, which are their major characteristics, imply from the frequency-domain viewpoint that the system's speed of response is so slow that any time delay does not affect the system response, which in turn means to be impractical; see [3], [4]. Therefore, there arises the need for practical controllers for time-delay systems that can provide good performance results.

In this paper, we propose a control method for time-delay systems. The proposed method is simple but quite practical. The method combines time- and frequency-domain controllers, that is, linear quadratic optimal controller (or LQR) and lag compensator. The LQR is used to stabilize the system in optimal fashion, whereas the lag compensator is used to compensate time-delay effects by increasing the delay margin of the system.

Since the proposed method adopts the LQR methodology, under mild conditions, it is used for multi-variable systems. Also, since it involves the frequency-domain design and analysis, it provides good practical insights on time-delay control and helps the designer to estimate the maximum allowable time delay in practice.

The organization of the paper is as follows. Section 2 formulates the problem of the paper and reviews some theoretical background results. In Section 3, the proposed controller that combines LQR and lag compensation is presented for time-delay systems, and the design procedures are applied to a flexible-link system with time delays in Section 4. Simulation and experiment results are discussed in Section 5 and finally, conclusions are drawn in Section 6.

2. Problem Formulation and Theoretical Foundation

The system under consideration is given by

$$\dot{x}(t) = Ax(t) + B\,u(t) \tag{1}$$

$$y(t) = Cx(t)$$

with delayed state-feedback control $u(t) = -Kx(t-h)$ and initial conditions $x(t) = 0$ for $t < 0$ and $x(0) = x_0$. In these equations, $x(t) \in \mathbf{R}^n$ is the state, $u(t) \in \mathbf{R}^q$ is the control input, $y(t) \in \mathbf{R}^r$ is the output, and h is a positive constant time delay satisfying $0 \leq h \leq \gamma$. It is assumed that all matrices have appropriate dimensions.

When all states are available and there is no time delay, i.e., $h = 0$, the most preferable controller will be a LQ controller. But, when there is a time delay in the control loop, the performance cannot be maintained by LQ control only. There must be some strategies for dealing with time delay effects.

The problem of this paper is to design a controller that stabilizes the system (1) in the face of a time delay h while maintaining the system performance as close as to that of the delay-free case.

2.1. Linear Quadratic Optimal Control

Linear quadratic optimal control has been one of very important theories in optimal control since the 1960s. In LQ control, the plant is a linear system and the objective function or performance index is formulated as a quadratic function of the system states and inputs. The problem is to minimize a given performance index while satisfying some design constraints.

For a linear system represented by (1) and a performance index (PI) represented by

$$J = \frac{1}{2}\int_0^\infty (x^T Q x + u^T R u)dt,$$

the optimal state-feedback control is given by

$$u(t) = -Kx(t) = -R^{-1}B^T P x(t)$$

where P is a positive definite matrix satisfying the algebraic Riccati equation:

$$A^T P + PA + Q - PBR^{-1}B^T P = 0.$$

The LQ optimal control has good properties. In 1964, Kalman [1] showed that the LQ control (LQR) has good stability margins. When the system under consideration is a SISO minimum-phase system, under mild conditions, the return difference is always larger than unity, i.e.,

$$|1 + L(j\omega)| = |1 + K(j\omega I - A)^{-1}B| \geq 1.$$

This inequality implies that $L(j\omega)$ always stays outside the unit circle centered at $(-1, 0)$ in the Nyquist plot, which corresponds to infinite gain margin and guaranteed 60° phase margin.

However, if there is a time delay in the loop transfer function, this good property is lost since the time delay increases the phase lag unboundedly as frequency increases and thus the delay decreases the phase margin.

2.2. Lag Compensator

Lag compensator design is one of well-taught subjects in many undergraduate control courses. Since it is based on the classical frequency-domain design methodology, the designer can obtain the insight how to 'tune' the system with the help of frequency-domain design tools such as the Bode and/or Nyquist plots.

Lag compensation has the following properties [6]: it 1) adds phase lag, 2) decreases the system gain, 3) decreases steady-state error, and 4) decreases the gain crossover frequency. One interesting thing is that the first property of adding phase lag tends to destabilize the system, whereas the other properties improve both performance and stability. It implies that appropriate trade-offs should be made to obtain good performance results.

A lag compensator in transfer-function form is given by

$$C_{lag}(s) = \frac{1 + \tau s}{1 + \beta\tau s}, \quad \beta > 1, \ \tau > 0,$$

where

$$\beta = \frac{1 + \sin(\phi)}{1 - \sin(\phi)}, \qquad \tau = \frac{1}{\omega_0\sqrt{\beta}}.$$

In these equations, ϕ is the largest phase lag and ω_0 is the target frequency at which the largest phase lag is achieved.

2.3. Delay Margin

Let an open-loop system with the transfer function $G(s) = e^{-hs}G_0(s)$ be strictly proper and $G_0(s)$ be stable. From the classical frequency-domain theory, the phase margin of the system is given by

$$\varphi = (\pi + \angle G_0(j\omega_c))$$

where ω_c is the crossover frequency. From this equation we obtain the following quantity for the maximum allowable time delay or delay margin:

$$h_{\max} = \frac{\varphi}{\omega_c}.$$

If the system has multiple crossover frequencies $\omega_{c1}, \cdots, \omega_{cl}$, then ϕ is defined to have the minimum value as

$$\varphi = \min\{\varphi_i : 1 \leq i \leq l\},$$

where $\varphi_i = (\pi + \angle G_0(j\omega_{ci}))$. In this case, $h_{\max}$ is given by

$$h_{\max} = \min\{\frac{\varphi_i}{\omega_{ci}}\}.$$

Lemma 1: *Assume that $G(s) = e^{-hs}G_0(s)$ is strictly proper and $G_0(s)$ is stable. Then, the feedback system is stable for all open-loop transfer functions $G(s)$ with a constant time delay satisfying $h < h_{\max}$.*

More on delay margin and stability of delay systems can be found in [5].

3. Controller Design for Time-Delay Systems

3.1. Main Idea

Control of multi-variable systems often involves state-feedback. By multi-variable, we mean multi-input multi-output (MIMO) or single-input single-output (SIMO). The linear quadratic (LQ) optimal control is used in this class of systems when full state measurements are available. The LQ control provides good robustness against unmodeled dynamics as it guarantees infinite gain margin and at least 60° phase margin in minimum-phase linear systems.

In general, the classical transfer-function method is not appropriate for multi-variable systems because it only considers single-input single-output (SISO) properties. However, the transfer-function method is appropriate for analysis and design of robust control systems. Moreover, it is

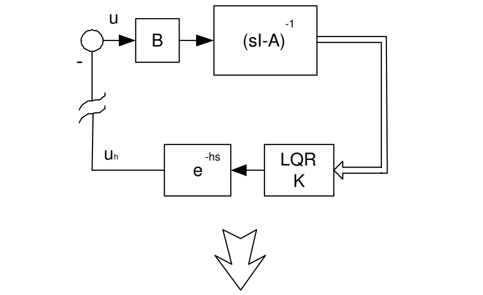

Fig. 1. Loop transfer function.

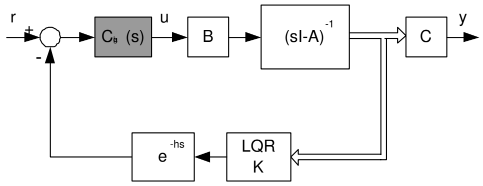

Fig. 2. Closed-loop system.

appropriate for time-delay systems. Examples include stability analysis using the Nyquist stability theory and controller design using the Smith predictor or lag compensator.

The basic idea behind the controller design of the paper is to combine the transfer-function and LQ control methods for a multi-variable time-delay system.

The LQ control is used for stabilizing the delay-free system. This control guarantees that all closed-loop poles lie in the open right-half plane (RHP). Then, breaking the control loop at the controller-output node as shown in Fig. 1, we obtain a SISO system where the LQ controller output is regarded as the open-loop system output. In fact, the transfer function of this open-loop system is the loop transfer function. Since the system under consideration is a SISO system, it is possible to apply the transfer-function method to this system for time-delay compensation.

Time delay effects are compensated by a loop-shaping method such as lag compensation. That is, the time-delay margin is increased with this methodology. When the lag compensator is well designed, the delay margin is increased while the control performance is maintained. Therefore, one can assess that with lag compensation, the original LQ control performance is not degraded significantly in delay-free cases while the robustness is improved in delay cases. The resulting closed-loop control system is depicted in Fig. 2.

3.2. Controller Design

It is assumed that the number of control input is one. This assumption is needed for recasting a multi-variable system to a SISO system.

To design the LQ controller, we assume that there is no time delay. The procedure is same as the conventional LQR design. In fact, the success of LQR design heavily relies on the choice of weighting matrices Q, R. Therefore, the designer may repeat the weighting selection until satisfactory

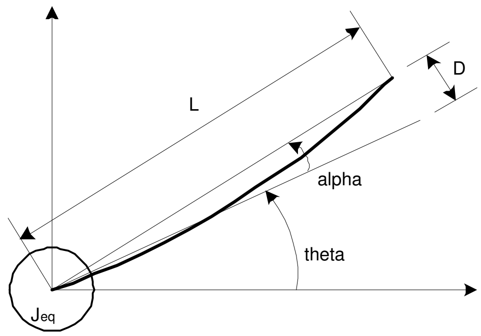

Fig. 3. Flexible link.

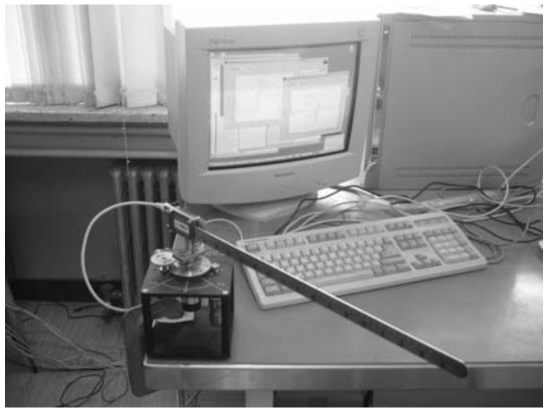

Fig. 4. Flexible link experimental setup.

results are obtained. Since computer-aid design tools such as MATLAB provide solvers for the ARE, the LQR design is straightforward. Note that the LQR gain affects stability of the resulting time-delay system. The larger the gain is, the smaller the delay margin is. However, it is not recommended to reduce the gain too much to obtain large delay margins, since the reduced gain results in slow responses.

To design the lag compensator for the LQ control system with time delays, after breaking the loop of the system (1) as shown in Fig. 1, we obtain the following transfer function:

$$G(s) = e^{-hs}G_0(s) = e^{-hs}K(sI - A)^{-1}B \qquad (2)$$

The lag compensator design in this paper is different from that of the traditional lag compensator design. Instead of choosing the target frequency ω_0 as the desired crossover frequency, our design chooses ω_0 less than the desired crossover frequency such that the resulting crossover frequency is decreased and the phase lag at the crossover frequency is not varied too much. This results in the decreased bandwidth and increased delay margin. Note that the lag compensator should minimize the added phase lag at the resulting crossover frequency so that the resulting delay margin is maximized.

However, it is not recommended to decrease the bandwidth too much. The more the bandwidth is decreased, the slower the system response is. Therefore, to obtain satisfactory performance results, trade-offs should be made between the bandwidth and delay margin, that is, between the speed of response and the stability margin of the time-delay system.

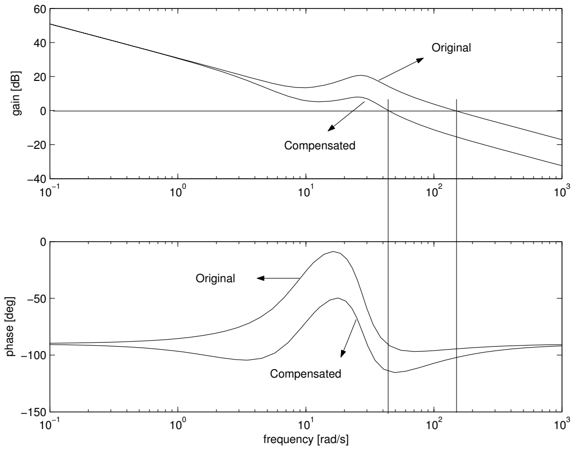

Fig. 5. Bode plots for uncompensated and compensated delay-free systems.

The lag compensator design is summarized as follows:

Step 1. Given Bode plots of $G(j\omega)$, determine the phase margin (φ_{old}) and crossover frequency (ω_{c-old}).

Step 2. Determine the largest phase lag (ϕ) due to lag compensation.

Step 3. Calculate β from the formula

$$\beta = \frac{1+\sin(\phi)}{1-\sin(\phi)}.$$

Step 4. Determine the target frequency (ω_0). ω_0 should be less than ω_{c-new}.

Step 5. Calculate $\tau = 1/(\omega_0\sqrt{\beta})$.

Step 6. From the Bode plot of $G(j\omega)C_{lag}(j\omega)$, check the resulting phase margin to see if the delay system will be stable.

Repeat until satisfactory responses are obtained.

4. Simulations and Experiments with a Flexible Link with Time Delays

4.1. Flexible Link Modeling

Fig. 3 illustrates the flexible-link model under consideration. It consists of a strain gauge, a thin stainless steel flexible link, and a hub connected to a geared DC motor. The strain gauge is mounted at the end of the flexible link where the link is clamped on the hub. Due to this configuration, only horizontal vibration is possible in this flexible link. The experimental flexible-link system is shown in Fig. 4.

The strain gauge with proper calibration measures the deflection angle approximated by

$$\alpha = \frac{D}{L}$$

where D is the end-point displacement and L is the length of the link. The output of the strain gauge is a voltage signal that is proportional to the deflection angle of the link.

The hub (and flexible link) is rotated horizontally by the DC motor which is equipped with an encoder for the hub's angular position measurement.

Equations governing the motion of the flexible-link system are [7]

$$J_{eq}\ddot{\theta} + J_{link}(\ddot{\theta}+\ddot{\alpha}) = T - B_{eq}\dot{\theta} \tag{3}$$

$$J_{link}(\ddot{\theta}+\ddot{\alpha}) + K_{stiff}\,\alpha = 0 \tag{4}$$

where θ is the angle of the hub, α is the deflection angle of the link, J_{eq} is the equivalent inertia of the hub and motor, J_{link} is the inertia of the link, B_{eq} is the viscous friction coefficient of the hub and motor, $K_{stiff} = \omega_n^2\ J_{link}$ is the stiffness coefficient of the link with ω_n being the natural frequency of the link, and T is the torque output from the motor.

The torque generated by the geared DC motor is obtained as

$$T = \frac{\eta K_t K_g (V_m - K_g K_{emf}\ \dot{\theta})}{R_m} \tag{5}$$

where η is energy transfer efficiency, K_t is the motor torque constant, K_g is the gear ratio, K_{emf} is the motor back-EMF constant, R_m is the motor resistance, and V_m is the input voltage applied to the motor.

Eqs. (3)–(5) are combined to yield the following state equation:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & a_{32} & a_{33} & 0 \\ 0 & a_{42} & a_{43} & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b_3 \\ b_4 \end{bmatrix} V_m \tag{6}$$

where

$$a_{32} = \frac{K_{stiff}}{J_{eq}},$$

$$a_{33} = \frac{-\eta K_t K_{emf} K_g^2 + B_{eq} R_m}{J_{eq} R_m},$$

$$a_{42} = \frac{-K_{stiff}(J_{eq}+J_{link})}{J_{eq} J_{link}},$$

$$a_{43} = \frac{\eta K_t K_{emf} K_g^2 + B_{eq} R_m}{J_{eq} R_m},$$

$$b_3 = \frac{\eta K_t K_g}{J_{eq} R_m},$$

$$a_4 = \frac{-\eta K_t K_g}{J_{eq} R_m}.$$

The flexible link used for experiments is made by Quanser Consulting Inc. [8]. Substituting the parameters of (6) with given values results in the following (delay-free) equation:

$$\dot{x}(t) = Ax(t) + Bu(t)$$

where $x^T = [\theta,\ \alpha,\ \dot{\theta},\ \dot{\alpha}]^T$, $u = V_m$,

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 673.53 & -31.9744 & 0 \\ 0 & -1028.8 & 31.9744 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ 56.2361 \\ -56.2361 \end{bmatrix}.$$

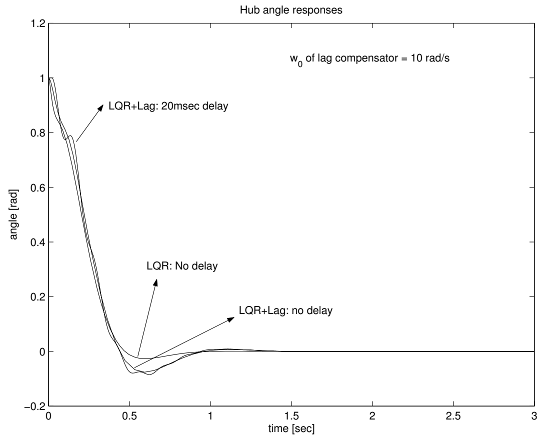

Fig. 6. Simulation results: hub angle responses. LQR only: delay-free case; LQR+lag: delay-free and 20ms delay cases.

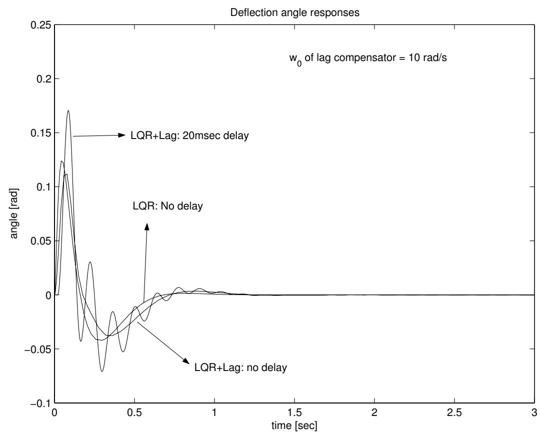

Fig. 7. Simulation results: tip deflection angle responses. LQR only: delay-free case; LQR+lag: delay-free and 20ms delay cases.

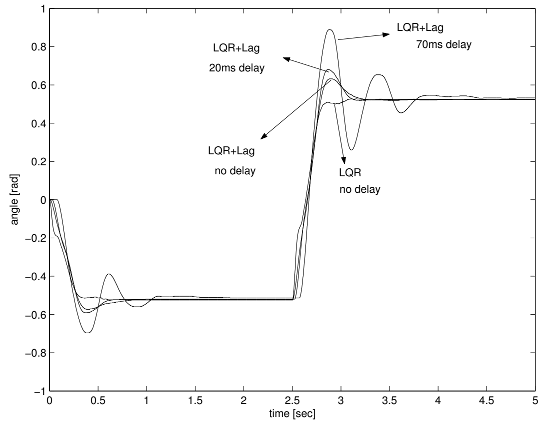

Fig. 8. Experiment results: hub angle responses. LQR only: delay-free case; LQR+lag: delay-free, 20ms and 70ms delay cases.

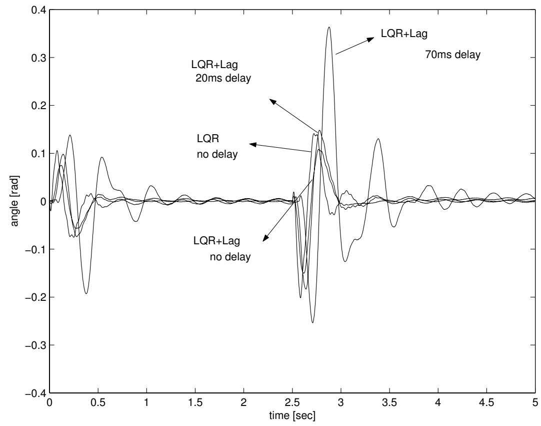

Fig. 9. Experiment results: tip deflection angle responses. LQR only: delay-free case; LQR+lag: delay-free, 20ms and 70ms delay cases.

5. Simulation and Experiment Results

In this paper, we assume that there is a time delay in the control loop. For experiments, we intentionally impose a time delay to the flexible-link control system by making control signals delayed by a prescribed value.

Simulations are carried out using MATLAB/Simulink. The time delay is simulated by the time-delay block in Simulink. All simulations are done in continuous time mode.

For experiments, we use MATLAB/Simulink/Real-Time Workshop for design and WinCon/RTX with a Q8 data acquisition board for real-time control. An 800MHz Pentium-3 PC is used for implementation. The control algorithms run at 1KHz sampling rate.

The controller parameters are as follows. The LQR gain is obtained as $K = [17.3205\ -24.7388\ 1.7164\ 0.5007]$ with the design parameters $Q = \mathrm{diag}(400,\ 10000,\ 3,\ 2)$ and $R = 1$. The lag compensator parameters are $\omega_0 = 10$rad/s and $\phi = 45^\circ$, which results in the following transfer function:

$$C_{lag}(s) = \frac{1 + 0.0414s}{1 + 0.2414s}.$$

Fig. 5 shows the Bode plots of the lag-compensated and uncompensated system models both of which are delay-free. In this figure, the target frequency of the lag compensator is less than the resulting crossover frequency. This results in the decreased bandwidth and increased delay margin. Note that even the resulting phase margin is decreased in the case of no delay, the resulting delay margin is increased because the amount of the decreased bandwidth is more than the amount of the decreased phase.

Fig. 6 and Fig. 7 show simulation results of hub angle and tip deflection angle responses, respectively. The LQR and the proposed method (LQR+Lag) are compared. The figures indicate that the proposed method, when the lag compensator is properly designed, provides good robustness as well as control performance in the face of time delays. In fact, although it is not depicted in the figures, the LQR cannot stabilize the system with time delays greater than 11ms. Note that the proposed method provides similar performance as the LQR in the delay-free case while maintaining stability in delay-cases.

Fig. 8 and Fig. 9 show experimental results of hub angle and tip deflection angle responses, respectively. The reference signal is a square wave of 30° amplitude with 0.2Hz frequency. This signal is applied to the hub. The figures indicate that as expected the proposed method provides good robustness and control performance in the face of time delays. In the experiments, the LQR cannot stabilize the system with 15ms. However, our method outperforms in robustness the LQR as it stabilizes the delay system with only small performance deterioration. Our method even stabilizes the system with a large time delay of 70ms.

From the simulation and experimental results, we can assess that the classical phase margin concept is a very important factor for time-delay system control.

6. Conclusions

This paper has presented a time-delay system controller and verified the performance of the designed control system via real experiments. Specifically, the controller is applied to a flexible-link system with time delays.

The method combines time- and frequency-domain controllers: linear quadratic optimal controller (or LQR) and lag compensator. The LQR is used to stabilize the system in optimal fashion, whereas the lag compensator is used to compensate time-delay effects by increasing the phase margin of the system. With this methodology, the maximum allowable time delay can be increased significantly.

The proposed method is simple but quite practical for time-delay system control as it is based on the conventional loop-shaping method, which gives practical insights on delay-phase relationship.

From simulations and experiments, it is found that the proposed method outperforms the LQR in various time-delay cases. For example, the proposed method stabilizes the system delayed by 20ms and gives good performance as if there is no time delay, whereas the LQR cannot stabilize the same system. Moreover, the proposed method can extend the allowable time delay range significantly. The simulation and experiment results confirm that the method presented in this paper is feasible and practical.

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