Digital Watermarking Technique for Images with Perspective Distortion

Nopporn Chotikakamthorn*, and Wiyada Yawai**

Faculty of Information Technology & Research Center for communications and Information Technology, King Mongkut's Institute of Technology Ladkrabang, Chalonkrung Road, Bangkok 10520, Thailand

* (Tel: +66-2-7372551; E-mail: kcnoppor@kmitl.ac.th)

** (Tel: +66-2-7372551; E-mail: wiyadayw@hotmail.com)

Abstract: In this paper, a problem of geometrically distorted images is considered. In particular, the paper discusses the detection of a watermark from a photographed image of the watermarked picture. The image is possibly obtained by using a digital camera. This watermark detection problem is made difficult by various geometric distortions added to the original picture through the printing and photographing processes. In particular, the paper focuses on the geometric distortion due to a projective transformation, as part of a camera 3D-to-2D imaging process. It is well-known that a cross ratio of collinear points is invariant under a perspective projection. By exploiting this fact, a projective-invariant digital watermarking technique is developed. By detecting the picture's corners, and the image center point at the intersection of two main diagonal lines, predefined cross ratios are used to compute the watermark embedded locations. From those identified embedding pixel locations, a watermark can be detected by performing a correlation between a watermark pattern and the image over those pixels. The proposed method does not require an inverse transformation on the distorted image, thus simplifying the detection process. Performance of the proposed method has been analyzed through computer experiments

Keywords: Digital watermarking, Perspective distortion, Cross ratio, Collinear points

1. INTRODUCTION

In recent papers [1-2], watermarking techniques are developed such that they are robust to 2-D geometrical transformation (RST- Rotation, Scale and Translation). One of those methods is developed by Ó Ruanaidh and Pun, and described in [1]. They introduced a watermarking scheme that exploits certain properties of the Fourier transform. In this scheme, watermark embedding is performed in a RST (rotation, scaling, and translation) invariant domain. By applying the Fourier transform to the host image after the log-polar mapping, the Fourier coefficient magnitudes form a translation, rotation, and scaling-invariant domain. This transform is also called the Fourier-Mellin transform. In addition, a similar RST invariant scheme is developed by Lin et al., and is described in [2].

Those methods as described are applicable to 2-D affine transformed images. However, for images taken by a digital camera, the original pictures are projective transformed onto the resulting images. The resulting perspective distortion is different from that is due to 2-D affine-type transformations. In the context of digital watermarking applications, such situation may arise when verification of a printed image's authentication or copyright information must be performed on the move. Another example that requires watermark detection from a perspective-distorted image, is when a watermark is used to link physical media (printed material such as a poster) with an electronic media (such as a web site). By embedding a watermark in the printed picture, the watermarked information may be later recovered from the photographed image of its printed version. The recovered information may be used to provide a link to an appropriate Internet resource location. Use of a watermarking technique under these scenarios is challenging due to the existence of various distortions in the image acquisition process.

In this paper, the problem of digital watermarking in the presence of perspective distortion is considered. The paper proposes a watermarking technique for images that is invariant under a perspective projection. The technique makes use of the well-known invariance of the cross-ratio of four collinear points. The proposed method avoids the need to perform inverse transformation operation. In addition, no image-dependent feature detection is required. Its application is thus

independent to image content.

The paper is organized as follows: In section 2, we describe background invariance theory of the cross-ratio. In section 3, the proposed projective-invariant digital watermarking technique, based on the cross-ratio principle, is detailed. In Section 4, experiment results are reported. Finally, Section 5 provides some conclusion remarks and future work.

2. BACKGROUND THEORY OF THE CROSS-RATIO

The cross-ratio is a basic invariance in projective geometry (i.e., all other projective invariance can be derived from it). Here brief introduction to the cross-ratio invariance property is given.

Let A, B, C, D be four collinear points (Three or more points A, B, C, ..., are said to be collinear if they lie on a single straight line[3]) as shown in Fig. 1. Their cross-ratio is defined as the "double ratio" in Eq. (1).

$$(ABCD) = CA/CB : DA/DB$$
 (1)

where all the segments are thought to be signed. The cross-ratio does not depend on the selected direction of the line *ABCD*, but does depend on the relative position of the points and the order in which they are listed. Based on a fundamental theory, any homography preserves the cross-ratio. Thus central projection, linear scaling, skewing, rotation, and translation preserve the cross-ratio [4].



Fig. 1 Collinear points A, B, C, and D

3. DIGITAL WATERMARKING TECHNIQUE FOR PERSPECTIVE DISTORED IMAGES

To apply the cross-ratio to digital image watermarking, three reference points are required. In this section, a method for deriving such reference points is detailed.

3.1 Embedding Scheme

Let's start by considering the embedding part. The method is described algorithmically below.

- 1. Predefine the set of cross-ratio values, to be used in subsequent steps.
- 2. Find the image center, as denoted by D_c , by using the line intersection formula [5] (two diagonal lines of the image) as described by Eqs.(2)~(7) below.

$$x_{c} = x_{t}/x_{b} \tag{2}$$

$$y_{c} = y_{t}/y_{b} \tag{3}$$

$$x_{t} = \begin{vmatrix} a & c \\ b & d \end{vmatrix} \tag{4}$$

$$x_{b} = \begin{vmatrix} c & e \\ d & f \end{vmatrix}$$
 (5)

$$y_{t} = \begin{vmatrix} a & e \\ b & f \end{vmatrix} \tag{6}$$

$$y_{b} = \begin{vmatrix} c & e \\ d & f \end{vmatrix} \tag{7}$$

where
$$a = \begin{vmatrix} x_1 & y_1 \\ x_4 & y_4 \end{vmatrix}$$
, $b = \begin{vmatrix} x_3 & y_3 \\ x_2 & y_2 \end{vmatrix}$, $c = \begin{vmatrix} x_1 & 1 \\ x_4 & 1 \end{vmatrix}$, $d = \begin{vmatrix} x_3 & 1 \\ x_2 & 1 \end{vmatrix}$,

 $e = \begin{vmatrix} y_1 & 1 \\ y_4 & 1 \end{vmatrix}, f = \begin{vmatrix} y_3 & 1 \\ y_2 & 1 \end{vmatrix}$. In addition, (x_i, y_i) is the coordinate of

the point C_i , i = 1, ..., 4 (see Fig. 2).

From the above equations, x_c is the x-axis value of the point D_c of line intersection, and y_c is the y-axis value of the same point and | | denotes a determinant operator.

3. Find each of the primary-level watermark embedding points ($D_{LU,i}$ and $D_{LD,i}$) on the left diagonal line (see Fig. 2(a)). Those points can be identified by using two corner points of the left diagonal line (C_1 and C_4), in combination with the image center point D_c and the predefined cross-ratio values.

$$x_{LU.i} = x_1 + PD * (x_4 - x_1)$$
 (8)

$$y_{LU,i} = y_1 + PD * (y_4 - y_1)$$
 (9)

$$x_{LD,i} = x_1 + PD * (x_4 - x_1)$$
 (10)

$$y_{LD,i} = y_1 + PD * (y_4 - y_1)$$
 (11)

where

PD = BA / AD

$$BA = (AD * TR) / (1 + TR)$$

TR = (AC/CD)/Cr

 D_c = the center point of the image.

 $A = (x_1, y_1)$

 $B = (x_2, y_2)$

 $C = (x_3, y_3)$

 $D = (x_4, y_4)$

AD = Euclidean distance between the point A and D

AC = Euclidean distance between the point A and C

CD = Euclidean distance between the point C and D

Cr = Cross-ratio value

From the above equations, $(x_{LU,i}, y_{LU,i})$, $i = 1, M_{LU}$, is the coordinate of the point $D_{LU,i}$, $A = C_1$, $B = D_{LU,i}$,

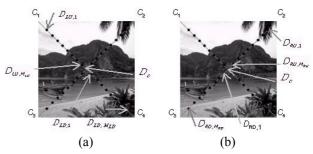


Fig. 2 a) Notations of collinear points on the left diagonal line of the image. b) Notations of collinear points on the right diagonal line of the image.

 $C = D_c$ and $D = C_4$. In addition, $(x_{LD,i}, y_{LD,i})$, $i = 1, \ldots$ M_{LD} , is the coordinate of the point $D_{LD,i}$, $A = C_1$, $B = D_{LD,i}$, $C = D_c$ and $D = C_4$.

4. Find each of the watermark embedding points ($D_{RU,i}$) and $D_{RD,i}$) on the right diagonal line (see Fig. 2(b)) by following the steps and equation similar to those detailed in Step 3. However, now the point A in Eqs. (12) \sim (15) represents the point C_2 while the point B now represents the point C_3 . By using these substitutions, those embedding points are given by

$$x_{RUi} = x_1 + PD * (x_4 - x_1)$$
 (12)

$$y_{RUi} = y_1 + PD * (y_4 - y_1)$$
 (13)

$$x_{RD,i} = x_1 + PD * (x_4 - x_1)$$
 (14)

$$y_{RD i} = y_1 + PD * (y_4 - y_1)$$
 (15)

where $(x_{RU,i}, y_{RU,i})$, $i = 1, ..., M_{RU}$, is the coordinate of the point $D_{RU,i}$, $A = C_2$, $B = D_{RU,i}$, $C = D_c$ and $D = C_3$. In addition, $(x_{RD,i}, y_{RD,i})$, $i = 1, ..., M_{RD}$, is the coordinate of the point $D_{RD,i}$, $A = C_2$, $B = D_{RD,i}$, $C = D_c$ and $D = C_3$.

5. For each pair of $D_{LU,i}$ and $D_{RD,k}$, in combination with C_1 and C_3 , compute watermark embedding points $E_{VL,i,k} = (x_{VL,i,k}, y_{VL,i,k})$ (see Fig. 3) by applying the line intersection point principle using C_1 , $D_{LU,i}$, C_3 and $D_{RD,k}$, as shown in Eqs. (16)~(21)

$$x_{VL,i,k} = x_t/x_b (16)$$

$$y_{VL,i,k} = y_t/y_b \tag{17}$$

$$x_{t} = \begin{vmatrix} a & c \\ b & d \end{vmatrix}$$
 (18)

$$x_{b} = \begin{vmatrix} c & e \\ d & f \end{vmatrix}$$
 (19)

$$y_{t} = \begin{vmatrix} a & e \\ b & f \end{vmatrix}$$
 (20)

$$y_{b} = \begin{vmatrix} c & e \\ d & f \end{vmatrix}$$
 (21)

where
$$a = \begin{vmatrix} x_{C,1} & y_{C,1} \\ x_{RD,k} & y_{RD,k} \end{vmatrix}$$
, $b = \begin{vmatrix} x_{C,3} & y_{C,3} \\ x_{LU,i} & y_{LU,i} \end{vmatrix}$, $c = \begin{vmatrix} x_{C,1} & 1 \\ x_{RD,k} & 1 \end{vmatrix}$,

 $d = \begin{vmatrix} x_{C,3} & 1 \\ x_{LU,i} & 1 \end{vmatrix}$, $e = \begin{vmatrix} y_{C,1} & 1 \\ y_{RD,k} & 1 \end{vmatrix}$, $f = \begin{vmatrix} y_{C,3} & 1 \\ y_{LU,i} & 1 \end{vmatrix}$, $x_{VL,i,k}$ is the x-axis value of the point $E_{VL,i,k}$ of line intersection, and $y_{VL,i,k}$ is the y-axis value of the same point.

6. For each pair of $D_{RU,i}$ and $D_{LD,k}$, in combination with C_2 and C_4 , compute watermark embedding points $E_{VR,i,k} = (x_{VR,i,k}, y_{VR,i,k})$ (see Fig. 3) by applying the line intersection point principle using $D_{RU,i}$, C_2 , $D_{LD,k}$ and C_4 in a similar manner to that of $E_{VL,i,k}$ as shown in Eqs. (22)~(27).

$$x_{VR,i,k} = x_t / x_b \tag{22}$$

$$y_{VR,i,k} = y_t / y_b \tag{23}$$

$$x_{t} = \begin{vmatrix} a & c \\ b & d \end{vmatrix}$$
 (24)

$$x_{b} = \begin{vmatrix} c & e \\ d & f \end{vmatrix} \tag{25}$$

$$y_{t} = \begin{vmatrix} a & e \\ b & f \end{vmatrix}$$
 (26)

$$y_{b} = \begin{vmatrix} c & e \\ d & f \end{vmatrix}$$
 (27)

where
$$a = \begin{vmatrix} x_{RU,i} & y_{RU,i} \\ x_{C,4} & y_{C,4} \end{vmatrix}, b = \begin{vmatrix} x_{LD,k} & y_{LD,k} \\ x_{C,2} & y_{C,2} \end{vmatrix}, c = \begin{vmatrix} x_{RU,i} & 1 \\ x_{C,4} & 1 \end{vmatrix},$$

$$d = \begin{vmatrix} x_{LD,k} & 1 \\ x_{C,2} & 1 \end{vmatrix}, e = \begin{vmatrix} y_{RU,i} & 1 \\ y_{C,4} & 1 \end{vmatrix}, f = \begin{vmatrix} y_{LD,k} & 1 \\ y_{C,2} & 1 \end{vmatrix}, x_{VR,i,k} \text{ is the}$$

x-axis value of the point $E_{VR,i,k}$ of line intersection and $y_{VR,i,k}$ is the y-axis value of the same point.

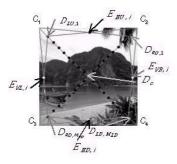


Fig. 3 Example of line intersection points in the image.

- 7. Repeat Steps 5-6 for the case of the embedding point of line intersection point principle using C_1 , C_2 , $D_{LU,i}$ and $D_{RU,k}$ to obtain watermark embedding points $E_{HU,i,k} = (x_{HU,i,k}, y_{HU,i,k})$ and the embedding point of line intersection point principle using C_1 , C_2 , $D_{LD,i}$ and $D_{RD,k}$ to obtain watermark embedding points $E_{HD,i,k} = (x_{HD,i,k}, y_{HD,i,k})$, respectively.
- 8. From all watermark embedding points, embed the watermark patterns by means of a spread-spectrum principle [6] using the following equations

Given the set of watermark embedding points $E_k = (x_k, y_k)$, k = 1, ..., M, and each of the watermarking pattern bits w_k , $w_k \in \{1, -1\}$, k = 1, ..., M, each watermarking pattern bit is

embedded to the original image by using the following Eq. (28)

$$I_e(x_m^k, y_n^k) = I(x_m^k, y_n^k) + \alpha w_k$$
 (28)

where $x_m^k = x_k + m$, m = -P, ..., P, $y_n^k = y_k + n$, n = -Q, ..., Q and $\alpha =$ strength of watermark.



Fig. 4 Line intersection points used for embedding watermark pattern bits.

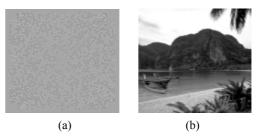


Fig. 5 a) Watermark embedding points, b) The watermarked image.

Fig. 4 shows the lines along which the watermark embedding points are located. Fig. 5(a) shows an example of the watermark pattern and Fig. 5(b) shows an example of the image after embedding the watermark pattern onto the original image.

3.2 Detection Scheme

To detect a watermark from the (possibly perspective-distorted) image I_e , the four image corner points must first be detected. This can be achieved, for example, by using any of the existing corner detection algorithms. Once the four corner points are detected watermark embedding points must be identified. Each point can be calculated by using the method similar to that of the embedding stage (see Section 3.1 for detail). By extracting the values of the pixels corresponding to those watermark embedding points, denoted by $I_e(x_k, y_k)$, a watermark can be detected by using any of the existing watermark detectors. Here, we adopt the correlation coefficient detector [6]. The correlation coefficient value is computed by the following equation.

$$Z_{cc}(I_e, w_k) = \frac{(\widetilde{I}_e \bullet \widetilde{W}_k)}{\sqrt{(\widetilde{I}_e \bullet \widetilde{I}_e)(\widetilde{W}_k \bullet \widetilde{W}_k)}}$$
(29)

where
$$\widetilde{I}_e = I'_e - \overline{I}_e, \widetilde{W}_k = W_k - \overline{W}_k$$

Watermark is detected if the correlation coefficient value is greater than a detection threshold. For example, in the experiment that follows, the detection threshold is 0.06.

4. EXPERIMENT

In this section, results from the computer simulation experiment are reported. Each of five grayscale images of size 512x512 pixels was used to add 20 different watermark pattern of length 2,000 bits, α is 3, and block size of watermark 9x9 pixels/watermark pattern bit. The cross-ratio values used for watermark embedding and detection are given in Table 1.

Table 1 Values of the cross-ratio used for watermark embedding and detection

19.3641	13.5469	10.3151	8.2585	6.8347	5.7906
4.9922	4.3618	3.8516	3.4300	3.0759	2.7743
2.5143		2.0888	1.9125	1.7553	1.6142
1.4869	1.3714	1.2661	1.1699	1.0815	0.9247
0.8548	0.7899	0.7293	0.6728	0.6198	0.5701
0.5233	0.4793	0.4377	0.3984	0.3613	0.3260
0.2925	0.2607	0.2304	0.2016	0.1740	0.1477
0.1226	0.0985	0.0755	0.0534		

The watermark embedded images were then perspective distorted. The distortion was performed along the horizontal, vertical, and both directions. Examples of the distorted images are shown in Fig. 6. The result of the experiment is shown in Table 2. Plots of the correlation coefficient values, as calculated from the original (no watermark) and the watermarked images, are shown in Fig. 7.



Fig. 6 a) Horizontally distorted watermarked image. b) Vertically distorted watermarked image. c) Watermarked image distorted along both directions.

Table 2 Number of images where the embedded watermark were successfully detected. (β = angle of rotation in degrees)

Watermark images	Horizontal distortion		Vertical distortion		Horizontal and vertical distortion	
	β=	β=	β=	β=	β=	β=
	15	20	15	20	15	20
House.tif	19	19	19	19	18	17
Relief.tif	19	19	20	20	18	19
Hongkong.tif	20	20	20	20	20	20
Kopipi.tif	16	18	18	18	19	17
Dragon.tif	20	20	19	20	20	20
Total (100)	94	96	96	97	95	93

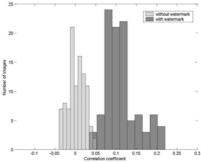


Fig. 7 Correlation coefficient values as calculated from distorted images with and without watermark embedded.

5. CONCLUSION

In this paper, we have presented a watermark technique for images with perspective distortion. Projective invariance is achieved by applying the cross ratio principle. The method developed is capable for detecting a watermark from a perspective-distorted image. This is achieved without the need to perform inverse transformation of the distorted image. Future work includes extension the experiment to cover a real data obtained by photographing a picture using a digital camera.

REFERENCES

- [1] J. J. K. Ó Ruanaidh and T. Pun, "Rotation, scale and translation invariant spread spectrum digital image watermarking," *Signal Processing*, vol. 66, no. 3, pp. 303-317, May 1998.
- [2] C. Y. Lin, J. A Bloom, I. J. Cox, M. L. Miller, and Y. M. Lui, "Rotation-scale, and Translation-resilient public watermarking of images," *Proc. of SPIE*, pp. 90-98, 2000.
- [3] Coxeter, H. S. M. and Greitzer, S. L. "Collinearity and Concurrence." *Geometry Revisited*, Ch. 3, Math. Assoc. Amer., pp. 51-79, 1967.
- [4] R. Mohr and L. Morin, "Relative Positioning from Geometric Invariants," Proceedings of the Conference on Computer Vision and Pattern Recognition, pp. 139-144, 1991.
- [5] Antonio, F. "Faster Line Segment Intersection. Graphics Gems III, Ch. IV.6, Academic Press, pp. 1999-202 and 500-501, 1992.
- [6] I. J. Cox, M. L. Miller, and J. A. Bloom, *Digital Watermarking*, Morgan Kaufmann Publishers, 2002.