Stability Analysis of a Multi-Link TCP Vegas Model

PooGyeon Park*, Doo Jin Choi*, Yoon Jong Choi*, and Jeong Wan Ko*

*Division of Electrical and Computer Engineering, POSTECH, Pohang, Kyungbuk 790-784, Korea
(Tel: +82-54-279-5588; Fax: +82-54-279-2903; Email:{ppg,chdj,scope75,hrp26}@postech.ac.kr)

Abstract: This paper provides a new approach to analyze the stability of a general multi-link TCP Vegas, which is a kind of feedback-based congestion control algorithm. Whereas the conventional approaches use the approximately linearized model of the TCP Vegas along equilibrium points, this approach models a multi-link TCP Vegas network in the form of a piecewise linear multiple time-delay system. And then, based on the exactly characterized dynamic model, this paper presents a new stability criterion via a piecewise and multiple delay-dependent Lyapunov-Krasovskii function. Especially, the resulting stability criterion is formulated in terms of linear matrix inequalities (LMIs).

Keywords: Network congestion control, TCP Vegas, multi-link, stability, linear matrix inequalities(LMIs).

1 INTRODUCTION

TCP (Transmission Control Protocol), which is a category of essential feedback-based congestion control mechanisms for realizing efficient data transfer services in packed-switched networks, has been widely used in the current Internet. In 1981, RFC(Request for Comments)-793 [1] introduced the basic structure of the TCP, which is a window-based flow control mechanism to pace the transmission of packets. And, RFC-1122 [2] presented the second version of TCP called TCP Tahoe, which includes a congestion avoidance scheme and a fast retransmission additionally in 1989. Furthermore, RFC-2001 [3] presented the third version of TCP called TCP Reno, which includes a fast recovery scheme additionally to the TCP Tahoe in 1997. Now, the TCP Reno is adopted as a standard TCP algorithm by RFC.

In 1994, Brakmo *et.al.* [4] introduced another version of TCP called TCP Vegas, which has the following advantages over the TCP Reno via a new time-out mechanism: an improved congestion avoidance mechanism and and a modified slow-start mechanism [5]. Especially, the TCP Vegas measures an RTT (Round Trip Time), which denotes an elapsed time from a packet transmission to the receipt of its corresponding ACK (Acknowledgement) packet. Based on the RTT, the TCP Vegas uses queueing delay as a measure of congestion, which differs from the other TCP mechanisms. Namely, if the measured RTT is getting large, the source host of the TCP Vegas conjectures that the network is falling into congestion. Then, the window size is throttled. If the measured RTTs become small, on the other hand, the source host recognizes that the network is relieved from the congestion and thus increases the window size again. Consequently, in the TCP Vegas, it is not necessary for the source host to wait for a packet loss in the network to detect congestion. With this mechanism, the window size of a source host is expected to converge to a constant value in steady state. Simulation and experimental results show that the congestion control mechanism of the TCP Vegas leads to 37-71 % higher throughput than that of the TCP Reno.

Some papers [6], [7], [8] have analytically investigated the dynamics of the TCP Vegas using an approximated fluid type of models. Furthermore, associated with AQM (Active Queue Management), Low et. al. [9], [10] proposed a distributed type of dynamic model for the end-to-end TCP Vegas and studied those equilibrium properties. However, they have not investigated at all the stability and the behaviors of their dynamics, even though the TCP Vegas is essentially a feedback congestion control and thus a stable operation of the control mechanism is extremely important.

More recently, some people have tried to analyze the stability of the TCP Vegas. But, since the dynamic TCP Vegas model belongs to a class of nonlinear time-varying systems with time-delay, which are usually extremely hard to analyze, people brought some critical assumptions such as no-delay, simple linearity, and so on, for the dynamic TCP model. And most of them were based on the discrete-time dynamic TCP Vegas model such as [5], [11], and [12], in which they linearized a discrete-time nonlinear dynamic TCP Vegas around a fixed point and considered the stability of that using a stability test such as Jury's test. Especially, an improved stability analysis for the continuous-time version of a distributed-type dynamic TCP Vegas model has been introduced by Choe and Low [13], which is based on the results of Paganini *et.al.* [14], [15] and Vinnicombe [16], [17], [18]. They used a classical stability method for the loop gain of the dynamic TCP Vegas model in the frequency-domain. Furthermore, based on the proposed stability analysis method for the dynamic TCP Vegas, they presented a modified TCP Vegas algorithm. Such an approach, however, has some difficulties. Since they de-

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veloped the new stability analysis method based on an approximated dynamic model of TCP Vegas, in which they approximated the $sgn(\cdot)$ function with a smooth $tan^{-1}(\cdot)$ function, they could not describe a region of attraction for the approximated linear model and the stability of the original nonlinear model.

In this paper, based on the results of [19] in which we represented a single link TCP Vegas model with a piecewise linear time-delay system and provided a new stability analysis criterion associated with a piecewise and delay-dependent Lyapunov-Krasovskii function, we extend the results to a general multi-link TCP Vegas model. This paper models a multi-link TCP Vegas network in the form of a piecewise linear multiple time-delay system. And then, based on the exactly characterized dynamic model, this paper presents a new stability criterion via a piecewise and multiple delay-dependent Lyapunov-Krasovskii function. Especially, this paper formulates the resulting stability criterion as a convex optimization problem in terms of linear matrix inequalities (LMIs).

This paper is organized as follows. Section 2 describes the general multi-link TCP Vegas model. And Section 3 presents a new stability criterion in terms of LMIs. The notation of this paper is fairly standard. In symmetric block matrices, we use $(*)$ as an ellipsis for terms that are induced by symmetry.

2 TCP VEGAS NETWORK MODEL

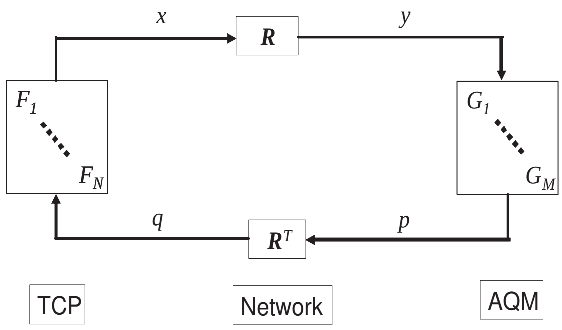

Fig. 1. Multi-link multi-source network

Before presenting a new model of the TCP Vegas, we shall briefly explain the general multi-link TCP Vegas network model. Generally, a network is modeled as a set of M links with finite capacities $c = (c_l, l \in M)$ and the links are shared by a set of N sources indexed by r as shown in Figure 1. Each source r uses a set $M_r \subseteq M$ of links defined by the $M \times N$ routing matrix $R \in \mathcal{R}^{M\times N}$ with the following elements

$$R_{lr} \quad = \quad \begin{cases} 1 & \text{if } l \in M_r \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Associated with each link l is a congestion measure $p_l(t)$ we will call 'price'; $p_l(t)$ is the scaled queueing delay at link l. Each source r maintains a rate $x_r(t)$ in packets/sec. At time t, we assume source r observes the aggregate price in its path

$$q_r(t) \quad = \quad \sum_l R_{lr} p_l(t - \tau_{lr}^b) \tag{2}$$

and link l observes the aggregate source rate

$$y_l(t) \quad = \quad \sum_r R_{lr} x_r(t - \tau_{lr}^f) \tag{3}$$

where τ_{lr}^f and τ_{lr}^b denote the forward and backward RTTs of the equilibrium RTT T_r such as

$$T_r \quad = \quad \tau_{lr}^f + \tau_{lr}^b, \ \ \forall l \in M_r \tag{4}$$

Then, Low *et.al.* [9] models TCP Vegas, with its associated queue management, as the following dynamical system : $r = 1, \cdots, N,\ l = 1, \cdots, M$

$$\dot{x}_r(t) \quad = \quad T_r^{-2}(t) sgn(x_r(t)q_r(t) - \alpha_r d_r) \tag{5}$$

$$\dot{p}_l(t) \quad = \quad \begin{cases} \frac{1}{c_l}(y_l(t) - c_l) & \text{if } p_l(t) > 0 \\ \frac{1}{c_l}(y_l(t) - c_l)^+ & \text{if } p_l(t) = 0 \end{cases} \tag{6}$$

where $(z)^+ = max\{0, z\}$, $sgn(z) = 1$ if $z > 0$, -1 if $z < 0$, and 0 if $z = 0$. Here, α_r is a Vegas protocol parameter, and d_r is the round trip propagation delay of source r. Price $p_l(t)$ is the queueing delay at link l and $q_r(t)$ is the end-to-end queueing delay of source r. Round trip time of source r is defined as

$$T_r(t) \quad = \quad d_r + q_r(t) \tag{7}$$

with equilibrium value T_r defined in (4). An interpretation of TCP Vegas algorithm is that each source r adjusts its rate to maintain $\alpha_r d_r$ number of its own packets buffered in the queues in its path. The link algorithm (6) is automatically carried out by the buffer process. The source algorithm (5) increments or decrements the "window" by 1 packet per round trip time, according as the number $x_r(t)q_r(t)$ of packets buffered in the links is smaller or bigger than $\alpha_r d_r$.

For the simplicity of developing procedures, we consider the multi-link TCP Vegas network in the following vector form with reasonable physical limits $p_l(t) > 0, \forall l$:

$$\dot{x}(t) \quad = \quad D(t) sgn(S) \tag{8}$$

$$q(t) \quad = \quad R^T p_b(t) \tag{9}$$

$$\dot{p}(t) \quad = \quad Cy(t) - 1_M \tag{10}$$

$$y(t) \quad = \quad Rx_f(t) \tag{11}$$

where

$$
\begin{aligned}
x(t) &\triangleq \begin{bmatrix} x_1(t) & \cdots & x_N(t) \end{bmatrix}^T \\
q(t) &\triangleq \begin{bmatrix} q_1(t) & \cdots & q_N(t) \end{bmatrix}^T \\
p(t) &\triangleq \begin{bmatrix} p_1(t) & \cdots & p_M(t) \end{bmatrix}^T \\
y(t) &\triangleq \begin{bmatrix} y_1(t) & \cdots & y_M(t) \end{bmatrix}^T \\
x_f(t) &\triangleq \begin{bmatrix} x_1(t-\tau_{l1}^f) & \cdots & x_N(t-\tau_{lN}^f) \end{bmatrix}^T \\
p_b(t) &\triangleq \begin{bmatrix} p_1(t-\tau_{1r}^b) & \cdots & p_M(t-\tau_{Mr}^b) \end{bmatrix}^T \\
D(t) &\triangleq diag\{-a_1(t), \cdots, -a_N(t)\} \\
S &\triangleq \begin{bmatrix} S_1 & \cdots & S_N \end{bmatrix}^T \\
C &\triangleq diag\{1/c_1, \cdots, 1/c_M\} \\
1_M &\triangleq \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}^T \in \mathcal{R}^M \\
a_r(t) &\triangleq \frac{1}{(d_r + q_r(t))^2}, \ s_r(t) \triangleq x_r(t)q_r(t) - \alpha_r d_r.
\end{aligned}
$$

Consequently, we can get the following dynamical system with two states :

$$
\begin{aligned}
\dot{x}(t) &= D(t)sgn(S) \qquad (12)\\
\dot{q}(t) &= R^T(CRx_T(t) - 1_M) \qquad (13)
\end{aligned}
$$

where

$$
x_T(t) \triangleq \begin{bmatrix} x_1(t-T_1) & \cdots & x_N(t-T_N) \end{bmatrix}^T.
$$

Then, we assume followings for simplicity without loss of generality.

• Assumption 1 : We limit $x_r(t)$ and $q_r(t)$ with any physical bounds $\bar{x}_r$ and $\bar{q}_r$ such as

$$
0 \le x_r(t) \le \bar{x}_r, \ \ 0 < q_r(t) \le \bar{q}_r \qquad (14)
$$

• Assumption 2 : RTT has the following upper and lower bounds

$$
d_r \ \le \ T_r(t) \ \le \ d_r + \bar{q}_r \qquad (15)
$$

In equilibrium, we reach $x_r^* q_r^* = \alpha_r d_r$ and

$$
CRx^* = 1_M \in \mathcal{R}^M \qquad (16)
$$

which results in

$$
\begin{aligned}
q^* &\triangleq \begin{bmatrix} q_1^*, & \cdots, & q_N^* \end{bmatrix}^T \qquad (17)\\
&\triangleq \begin{bmatrix} \alpha_1 d_1/x_1^*, & \cdots, & \alpha_N d_N/x_N^* \end{bmatrix}^T
\end{aligned}
$$

where

$$
x^* \triangleq \begin{bmatrix} x_1^*, & \cdots, & x_N^* \end{bmatrix}^T. \qquad (18)
$$

Remark 1: In the equation (16), we should solve the equilibrium x^* by maximizing each source rate $x_r(t)$ based on fairness [9], [20], and [21]. However, because the fairness is beyond the scope of this work, we do not treat the equilibrium problem relating fairness in this paper. We just use the equilibrium relation of the equation (16). ∎

3 NEW STABILITY ANALYSIS

We shall first redefine the variables $x(t)$ and $p(t)$ with the following variables:

$$
\begin{aligned}
\eta(t) &= x(t) - x^* \ \ \in \mathcal{R}^N, \qquad (19)\\
\mu(t) &= q(t) - q^* \ \ \in \mathcal{R}^N. \qquad (20)
\end{aligned}
$$

In this case, the physical constraints shown in (14) the assumption 1 can be written as for $r = 1, \cdots, N$

$$
m_r^T(t)M_{1r}m_r(t) < 0, \ m_r^T(t)M_{2r}m_r(t) < 0, \qquad (21)
$$

where $m_r(t)$, M_{1r}, and M_{2r} denote

$$
\begin{aligned}
m_r(t) &\triangleq \begin{bmatrix} \eta_r(t) & \mu_r(t) & 1 \end{bmatrix}^T \ \in \mathcal{R}^3 \qquad (22)\\
M_{1r} &\triangleq \begin{bmatrix} 2 & 0 & -\bar{x}_r + 2x_r^* \\ 0 & 0 & 0 \\ -\bar{x}_r + 2x_r^* & 0 & 2(x_r^* x_r^* - \bar{x}_r x_r^*) \end{bmatrix} \qquad (23)\\
M_{2r} &\triangleq \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & -\bar{q}_r + 2q_r^* \\ 0 & -\bar{q}_r + 2q_r^* & 2(q_r^* q_r^* - \bar{q}_r q_r^*) \end{bmatrix}. \qquad (24)
\end{aligned}
$$

Furthermore, we can rewrite the model (12) and (13) with a piecewise linear multiple time-delayed system as follows: for $m(t) \in \Phi_k$ and $k = 1, \cdots, 2^N$

$$
\dot{m}(t) = A_k(t)m(t) - \sum_{r=1}^{N} B_r \int_{t-T_r}^{t} \dot{m}(\alpha)d\alpha, \qquad (25)
$$

where the extended state $m(t)$ and the system matrices $A_k(t)$ are defined as

$$
\begin{aligned}
m(t) &\triangleq \begin{bmatrix} \eta^T(t) & \mu^T(t) & 1_N^T \end{bmatrix}^T \ \in \mathcal{R}^{3N} \qquad (26)\\
A_k(t) &\triangleq \begin{bmatrix} 0 & 0 & D_k(t) \\ R^TCR & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (27)\\
D_k(t) &\triangleq L_k D(t) \\
D(t) &\triangleq diag\{-a_1(t), \cdots, -a_N(t)\} \\
L_k &\triangleq \left\{ L \in \mathcal{L} \,\middle|\, k = 1, \cdots, 2^N \right\} \\
L &\triangleq \{diag\{l_1, \cdots, l_N\}, \mid l_r \in [-1, 1]\} \\
\mathcal{L} &\triangleq \{\{L_1, \cdots, L_k, \cdots, L_{2^N}\} \mid L_i \ne L_j, i \ne j\}
\end{aligned}
$$

and B_r is a $3N \times 3N$ matrix in which the all elements are zero except for the elements of rth column satisfying

$$
B_1 + \cdots + B_N = \begin{bmatrix} 0 & 0 & 0 \\ R^TCR & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.
$$

And the switching zones Φ_k, resulted from the switching functions $x_r(t)q_r(t) - \alpha_r d_r$ for $r = 1, \cdots, N$, denote

$$\Phi_k \triangleq \left\{ m(t) \,\middle|\, m_r^T(t) L_k(r,r) M_{3r} m_r(t) \geq 0, \forall r \right\} \quad (28)$$

where

$$M_{3r} \triangleq \begin{bmatrix} 0 & 1 & q_r^* \\ 1 & 0 & x_r^* \\ q_r^* & x_r^* & 0 \end{bmatrix}. \quad (29)$$

We shall now consider the stability of the above system. Since the system is piecewise and multiple time-delayed, we shall suggest the following piecewise and multiple delay-dependent Lyaunov-Krasovskii functional, which is modified from the functional introduced by Park [22], for $m(t) \in \Phi_k$ and $k = 1, \cdots, 2^N$

$$V(m(t-\alpha), \alpha \in [0, max(T_r)]) = V_1 + V_2 + V_3, \quad (30)$$

where

$$\begin{aligned} V_1 &\triangleq m^T(t) P_k m(t), \\ V_2 &\triangleq \sum_{r=1}^{N} \int_{-T_r}^{0} \int_{t+\beta}^{t} \dot{m}^T(\alpha) Z_r \dot{m}(\alpha) d\alpha d\beta, \\ V_3 &\triangleq \sum_{r=1}^{N} \int_{t-T_r}^{t} m^T(\alpha) Q_r m(\alpha) d\alpha. \end{aligned}$$

For the functional above, we shall impose the following condition for V_1 in order to achieve the radial unboundedness of V_1 with respect to $m(t)$: for any nonzero vector $m(t) \in \Phi_k$ yielding the condition (14) in the assumption 1, or the condition (21),

$$V_1(t) > 0, \quad (31)$$

which can be written as follows: there exist nonnegative Λ_{1k}, Λ_{2k}, and Λ_{3k} such that, for any nonzero $m(t)$,

$$P_k + E^T(\Lambda_{1k} M_1 + \Lambda_{2k} M_2 + \Lambda_{3k} M_3^k) E > 0. \quad (32)$$

where

$$\begin{aligned} \Lambda_{1k} &\triangleq diag\{\lambda_{1k1} I_3, \cdots, \lambda_{1kN} I_3\} \\ \Lambda_{2k} &\triangleq diag\{\lambda_{2k1} I_3, \cdots, \lambda_{2kN} I_3\} \\ \Lambda_{3k} &\triangleq diag\{\lambda_{3k1} I_3, \cdots, \lambda_{3kN} I_3\} \\ M_1 &\triangleq diag\{M_{11}, \cdots, M_{1N}\} \\ M_2 &\triangleq diag\{M_{21}, \cdots, M_{2N}\} \\ M_3^k &\triangleq diag\{L_k(1,1) M_{31}, \cdots, L_k(N,N) M_{3N}\} \end{aligned}$$

and I_3 and E denote a 3×3 identity matrix and a permutation matrix which changes $\begin{bmatrix} m_1^T(t) & \cdots & m_N^T(t) \end{bmatrix}^T$ to $m(t)$.

With this condition, the positive definiteness of Z_r and Q_r, or

$$Q_r > 0, \quad Z_r > 0, \quad (33)$$

says that $V(m(t-\alpha), \alpha \in [0, max(T_r)])$ is radially unbounded with respect to $m(t)$.

As mentioned in the work [23], [24], when one uses a type of piecewise Lyapunov function for the analysis of piecewise continuous-time systems, one should consider the continuously decreasing property of the piecewise Lyapunov function at the switching instant. Similarly, when a *piecewise* Lyapunov-Krasovskii functional is considered, an important condition that should be checked out is the continuous variation of the Lyapunov-Krasovskii functional on the switching surface constructed by the switching functions $(x_r(t)q_r(t) - \alpha_r d_r)$. In this model, there are 2^N switching zones related to $\frac{2^N(2^N-1)}{2}$ switching surfaces and thus we need to have the following contiuity condition:

$$\begin{aligned} &\text{for } i = 1 : (2^N - 1) \\ &\quad \text{for } j = (i+1) : 2^N \\ &\qquad m^T(t) P_i m(t) = m^T(t) P_j m(t) \;\; \text{s.t.} \\ &\qquad m^T(t) M_3^i m(t) = m^T(t) M_3^j m(t) = 0 \\ &\quad \text{end} \\ &\text{end} \end{aligned} \quad (34)$$

which can be written with the existence of $\frac{2^N(2^N-1)}{2}$ real numbers $\omega_{i,j}$ such that

$$\begin{aligned} &\text{for } i = 1 : (2^N - 1) \\ &\quad \text{for } j = (i+1) : 2^N \\ &\qquad P_j = P_i + \omega_{i,j} M_3^i, \;\; \omega_{i,j} \in \mathcal{R}. \\ &\quad \text{end} \\ &\text{end} \end{aligned} \quad (35)$$

For example when $N = 2$, we should check the existence of the following relations:

$$\begin{cases} P_2 = P_1 + w_{1,2} M_3^1 \\ P_3 = P_1 + w_{1,3} M_3^1 \\ P_4 = P_1 + w_{1,4} M_3^1 \end{cases} \quad (36)$$

$$\begin{cases} P_3 = P_2 + w_{2,3} M_3^2 \\ P_4 = P_2 + w_{2,4} M_3^2 \end{cases} \quad (37)$$

$$\left\{ P_4 = P_3 + w_{3,4} M_3^3 \right. \quad (38)$$

which results in

$$P_1 = P \quad (39)$$

$$P_2 = P + w_{1,2} M_3^1 \quad (40)$$

$$P_3 = P + w_{1,2} M_3^1 + w_{2,3} M_3^2 \quad (41)$$

$$P_4 = P + w_{1,2} M_3^1 + w_{2,3} M_3^2 + w_{3,4} M_3^3. \quad (42)$$

Finally what we need for the stability is that its derivative $\dot{V}$ in each switching zone Φ_k should be strictly negative for nonzero $\{m(t-\alpha), \alpha \in [0, max(T_r)]\}$ under the constraints (21). We shall present such conditions in the following theorem.

Theorem 1: Suppose the TCP Vegas satisfies the assumptions 1 and 2. Then the piecewise linear multiple time-delay model (25) with the equilibrium point (x^*, q^*) is stable, if there exist matrices $X_r = X_r^T$, Y_r, $Z_r = Z_r^T$, $P = P^T$, $Q_r = Q_r^T$, and scalar variables Λ_{1k}, Λ_{2k}, Λ_{3k}, Γ_{1k}, Γ_{2k}, Γ_{3k}, $\omega_{i,j}$, for all $k = 1, \cdots, 2^N$, $h = 1, 2$, $r = 1, \cdots, N$, $i = 1, \cdots, N$, $j = 1, \cdots, N$, $j = i+1$ satisfying

$$P_k + E^T(\Lambda_{1k} M_1 + \Lambda_{2k} M_2 + \Lambda_{3k} M_3^k)E > 0, \quad (43)$$

$$\begin{bmatrix} X_r & Y_r \\ Y_r^T & Z_r \end{bmatrix} > 0, \; Q_r > 0, \; Z_r > 0, \quad (44)$$

$$\begin{bmatrix} (1,1)_{kh} & \Pi_1 & \cdots & \Pi_N & A_{kh}^T \sum_{r=1}^{N} T_r Z_r \\ \Pi_1^T & -Q_1 & 0 & 0 & -B_1^T \sum_{r=1}^{N} T_r Z_r \\ \vdots & 0 & \ddots & 0 & \vdots \\ \Pi_N^T & 0 & 0 & -Q_N & -B_N^T \sum_{r=1}^{N} T_r Z_r \\ (*) & (*) & (*) & (*) & -\sum_{r=1}^{N} T_r Z_r \end{bmatrix} < 0, \quad (45)$$

$$\Lambda_{1k} > 0, \; \Lambda_{2k} > 0, \; \Lambda_{3k} > 0, \quad (46)$$

$$\Gamma_{1k} > 0, \; \Gamma_{2k} > 0, \; \Gamma_{3k} > 0, \quad (47)$$

where P_k is replaced with P and $w_{i,j}$ such as (39)~(42), M_1, M_2 and M_3^k were defined in (33), (33) and (33), respectively, and

$$\begin{aligned} (1,1)_{kh} &\triangleq P_k A_{kh} + A_{kh}^T P_k + \sum_{r=1}^{N} T_r X_r \\ &\quad -E^T(\Gamma_{1k} M_1 + \Gamma_{2k} M_2 + \Gamma_{3k} M_3^k)E, \\ \Pi_r &\triangleq -P_k B_r + Y_r + Q_r, \\ \Gamma_{1k} &\triangleq diag\{\gamma_{1k1} I_3, \cdots, \gamma_{1kN} I_3\} \\ \Gamma_{2k} &\triangleq diag\{\gamma_{2k1} I_3, \cdots, \gamma_{2kN} I_3\} \\ \Gamma_{3k} &\triangleq diag\{\gamma_{3k1} I_3, \cdots, \gamma_{3kN} I_3\} \\ A_{kh} &\triangleq \begin{bmatrix} 0 & 0 & D_k^h \\ R^T C R & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ D_k^1 &\triangleq L_k diag\{-a_{11}, \cdots, -a_{N1}\} \\ D_k^2 &\triangleq L_k diag\{-a_{12}, \cdots, -a_{N2}\} \\ a_{r1} &\triangleq d_r^{-2}, \; a_{r2} \triangleq (d_r + \bar{q}_r)^{-2}. \end{aligned}$$

Proof: The proof is omitted due to lack of space. In the previous paper [19], we presented a relating proof. ∎

4 CONCLUSION AND FUTURE WORKS

We have presented a new stability analysis method for the general multi-link link dynamic TCP Vegas model. First, this paper described the nonlinear(switched) multiple time-delay TCP Vegas model by a piecewise linear time-invariant dynamic system with multiple time-delay. Then, a piecewise and multiple delay-dependent Lyapunov-Krasovskii function was applied for the system. Consequently, this paper successfully developed a new stability criterion for the multi-link dynamic TCP Vegas model in terms of LMIs. Since we only presented the result of the stability analysis for the multi-link TCP Vegas model, we will show the performances of the proposed criterion with computer-based and testbed-based realizations in the near future. Furthermore, if we apply the proposed method for performance-oriented problems of dynamic TCP Vegas model such as guaranteed cost, saturation, tracking, and their combinations, we will examine closely the other problems of TCP network system. And we believe that the proposed method will be the basis for developing an advanced TCP algorithm.

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