A Study on the Feed Rate Optimization of a Ball Screw Driven Machine Tool Feed Slide for Minimum Vibrations

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Abstract: In order to prevent machine tool feed slide system from transient vibrations during operations, machine tool designers usually adopt some typical design solutions; box-in-box typed feed slides, optimizing moving body for minimum weight and dynamic compliance, and so on. Despite all efforts for optimizing design, a feed drive system may experience severe transient vibrations during high-speed operation if its feed rate control is unsuitable. A rough feed rate curve having discontinuity in its acceleration profile causes a serious vibration problem in the feed slides system. This paper presents a feed rate optimization of a ball screw driven machine tool feed slide system for its minimum vibration. Firstly, a ball screw feed drive system was mathematically modeled as a 6-degree-of-freedom lumped parameter system. Next, a feed rate optimization of the system was carried out for minimum vibrations. The main idea of the feed rate optimization is to find out the most appropriate smooth acceleration profile with jerk continuity. A genetic algorithm was used in this feed rate optimization

Keywords: Machine Tool, Feed Drive System, Lumped Parameter Model, Ball Screw, Feed Rate, Optimization, Genetic Algorithm

1. INTRODUCTION

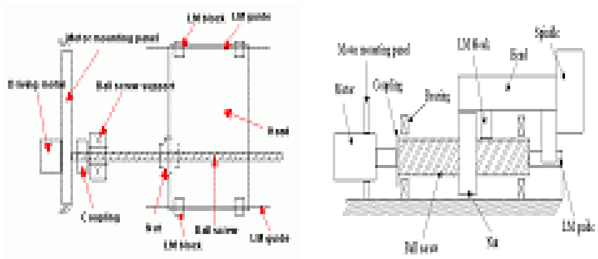
Ball screw feed drive systems have been broadly used in machine tools or precision automatic feed systems. Recently, modern machine tools need to operate in high speed with short accelerating time to achieve high productivity. However, machine tool feed drive system may experience severe transient vibrations during high-speed machining due to frequent and abrupt speed changes. In order to prevent feed drive system itself or machine tool from such transient vibrations, machine tool designer usually adopts some interesting design or optimization technique; for example, box-in-box typed feed slides, optimizing moving body for minimum weight and dynamic compliance, and so on. Unfortunately a feed drive system, even though it has been designed optimally, may experience severe transient vibrations during high-speed operation if its feed rate control is not suitable. A rough feed rate curve having discontinuity in its corresponding acceleration or jerk profile causes a serious vibration problem in the feed slide system.

In order to solve the problem, this study presents a federate optimization of a machine tool feed drive system for minimum vibrations. In our study, a ball screw driven feed slide system was modeled as a 6-degree-of freedom lumped parameter model, which had been verified by comparing both theoretical and experimental analyses in our previous study[1,2]. The main idea of the feed rate optimization, in this study, is to find out the most appropriate acceleration curve having jerk continuity.

2. VIBRATION ANALYSIS OF A BALL SCREW FEED DRIVE SYSTEM

2.1 Modeling of a ball screw feed drive system

Schematic of a ball screw feed-drive system for a machine tool is shown in Fig. 1. For the vibration analysis, it was modeled as a 6-DOF (Degree-Of-Freedom) lumped parameter model as shown in Fig. 2. All the 6-d.o.f.'s are motor shaft rotation, ball screw shat rotation, translation displacement of the motor, translation of ball screw nut, translation of head assembly, and yawing motion of head assembly, respectively.

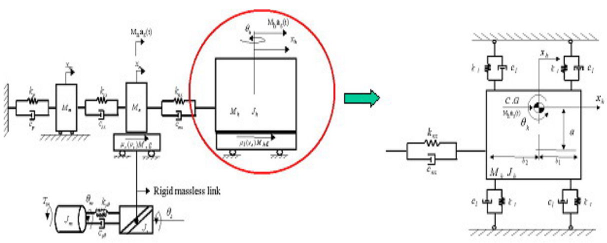


(a) Plan view (b) Side view

Fig. 1 Schematic of a ball screw feed-drive system

Assumptions for mathematical modeling are;

(a) Feeding motions occur in a plane.
(b) The linear motion block is linear elastic spring.
(c) Friction coefficient between guide way and linear motion block is constant.
(d) Guide ways have no rail-irregularities.
(e) Head is a rigid body.
(f) Ball screw shaft and coupling are assumed that they have both torsional and translational stiffness.
(g) Though head assembly and nut move along the guide way in the real system, but mathematical model assumes guide way to move in the opposite direction.



(a) 6-DOF mathematical model (b) Detail of head assembly

Fig. 2 Mathematical model of a ball screw feed-drive system

In the feed-drive system model shown in Fig. 2, k_p, k_{sx}, and k_{ns} represent equivalent spring constants for the thrust motor mounting support, ball screw shaft, and ball screw nut respectively; k_{teq} is the equivalent torsional spring constant for linear motion blocks regarding to head yaw motion; J_m, J_s, J_h represent mass moment of inertia's for the motor shaft & rotor, for the ball screw shaft, and for head assembly, respectively; M_m, M_n, M_h stand for motor mass, mass of ball screw nut, and mass of head assembly.

2.2 The equation of motion

The equations of motion of the ball screw feed drive system can be derived using Newton's 2nd law as follows.

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{F\} \tag{1}$$

Where $[M]$, $[C]$, $[K]$ are 6x6 mass-, damping-, and stiffness-coefficient matrices, respectively. The vector, $\{x\}=[\theta_m,\theta_s,x_m,x_n,x_h,\theta_h]^T$ is degree-of-freedom vector and the vector, $\{F\}=[T_m,0,0,f_N,f_H,0]$ is the excitation force vector, in which the applied motor torque, friction forces on the screw nut and the head assembly can be determined as followings.

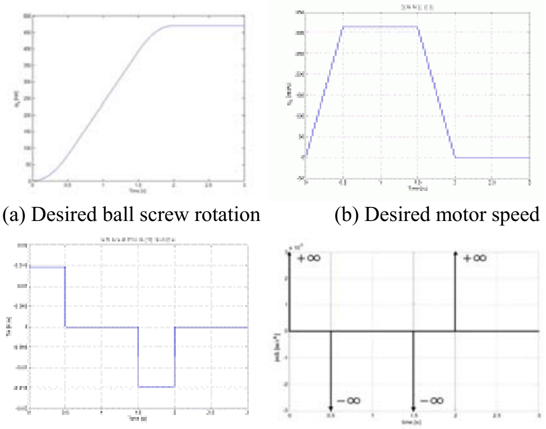
$$T_m=(J_m+J_s)\frac{d\omega_m}{dt}=(J_m+J_s)\frac{d^2\theta_m}{dt^2} \tag{2}$$

$$f_N=M_n g\mu_s(v_s)+M_n a_s(t) \tag{3}$$

$$f_H=M_h g\mu_s(v_s)+M_h a_s(t) \tag{4}$$

In the above three equations, θ_m is rotational angle of motor corresponding to the rotational speed of motor ω_m, g is gravitational acceleration, μ_s is friction coefficient of guide way, and v_s is relative slip velocity between nut or head and guide way, respectively.

A typical example of motor speed control input (ω_m) and corresponding acceleration- and jerk-curves are shown in the following figure 3.



(a) Desired ball screw rotation (b) Desired motor speed

(c) Corresponding acceleration (d) Corresponding jerk

Fig.3 Desired motor speed control input and corresponding acceleration & jerk

As shown in Fig. 3 (c), the acceleration curve corresponding to motor speed control input is not continuous at some points, where jerk becomes infinite and inertial impact occurs to moving bodies. Such inertial impact may cause severe transient vibrations to the feed drive system. In our study, vibrations of the ball screw feed drive system model were analyzed by using MATLAB[11].

3. FEEDRATE OPTIMIZATION BY USING GENETIC ALGORITHM

3.1 Identification of the optimization problem

The key idea of our federate optimization strategy is to generate the most appropriate smooth acceleration curve having jerk continuity. Necessarily, the generated optimum federate curve must guarantee the exact motor rotation angle or translation of head assembly. Prior to optimization problem identification, we introduce half sine curve fitting to generating a smooth jerk curve as depicted in the Fig. 4.

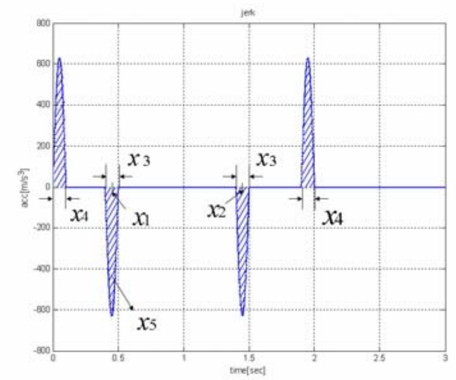


Fig. 4 Half sine curve fitting to generate a smooth jerk curve

In the Fig. 4, the variables x_1, x_2 represent the center time of the 2nd and the 3rd intermediate half sine pulses respectively; x_3, x_4 are pulse width (time bandwidth) of the 3rd (or 2nd) pulse and the 4th (or 1st) pulse respectively. In addition, x_5 represents the area under any of pulse curves, which corresponds to the maximum angular acceleration.

Now, federate optimization problem is converted to the corresponding jerk curve optimization problem as defined as follows:

Find $\mathbf{x}=[x_1,x_2,x_3,x_4,x_5]^T$

To minimize $f(\mathbf{x})=\sqrt{\sum_{i=1}^{3} w_i\left(\frac{f_i(\mathbf{x})}{f_i^*}\right)^2}$ (5)

Subject to some constraints given in section 3.2.

Where, $f_1(\mathbf{x})$ is maximum amplitude of head assembly translation (x_h), $f_2(\mathbf{x})$ is Root Mean Square (RMS) value of head assembly translation (x_h), and $f_3(\mathbf{x})=\sqrt{\left[\frac{l}{2\pi}\left(\theta_s^*-\theta_s\right)\right]^2+x_h^2}$ is the error between desired ball screw rotation angle (θ_s^*) and current ball screw rotation angle (θ_s), respectively. And w_i is weighting factor, f_i^* is scaling factor, respectively. The quantity $f_3(\mathbf{x})$ is a measure of the position control error of ball screw feed drive systems.

3.2 Constraints

There are some kinds of constraints, in this ball screw feed rate optimization, regarding to search space, feed drive system stability, and position error limit.

$$g_1(\mathbf{x}): 0.45 < x_1 < 0.55 \quad (6)$$

$$g_2(\mathbf{x}): 1.45 < x_2 < 1.55 \quad (7)$$

$$g_3(\mathbf{x}): 0.001 < x_3 < 0.10 \quad (8)$$

$$g_4(\mathbf{x}): 625 < x_4 < 640 \quad (9)$$

$$g_5(\mathbf{x}): 625 < x_5 < 640 \quad (10)$$

$$g_6(\mathbf{x}): 0.001 < x_6 < 0.10 \quad (11)$$

$$g_7(\mathbf{x}): N_a - N_c < 0 \quad (12)$$

$$g_8(\mathbf{x}): [x_h(x)]_{\max} - [x_h(x)]_a < 0 \quad (13)$$

$$g_9(\mathbf{x}): [\theta_h(x)]_{\max} - [\theta_h(x)]_a < 0 \quad (14)$$

$$g_{10}(\mathbf{x}): [\theta_s(\mathbf{x})]_{low} < [\theta_s(\mathbf{x})]_{final} < [\theta_s(\mathbf{x})]_{high} \quad (15)$$

Where, N_c and N_a are critical speed and allowable operating speed (rpm) of ball screw respectively; $[x_h(\mathbf{x})]_{\max}$ is maximum head translation displacement and $[x_h(\mathbf{x})]_a$ is allowable head translation displacement respectively; $[\theta_h(\mathbf{x})]_{\max}$ is maximum head yawing displacement and $[\theta_h(\mathbf{x})]_a$ is allowable head yawing displacement respectively; $[\theta_s(\mathbf{x})]_{final}$ is final rotational angle of ball screw shaft and $[\theta_s(\mathbf{x})]_{low}$, $[\theta_s(\mathbf{x})]_{high}$ are lower- and upper-limit of final rotation angle of ball screw shaft. In the optimization, N_a was assigned to 6250 [rpm], $[x_h(\mathbf{x})]_a$ was assigned to 10^{-5} [m], $[\theta_h(\mathbf{x})]_a$ was assigned to 1.3×10^{-6} [ra d]. In addition, $[\theta_s(\mathbf{x})]_{low}$ and $[\theta_s(\mathbf{x})]_{high}$ were assigned to 470.89 and 470.91 [rad], respectively.

3.1 Optimization program

Nowadays, genetic algorithm is one of popular optimization theory. Genetic algorithm has an advantage that can solve the mixture of continuity and discontinuity, because it does not use the concept of sensitivity or differential function. In our study, genetic algorithm with variable penalty function was used to searching the best population. The variable penalty function and fitness function are defined as following formula:

$$Fitness = \frac{1}{f(\mathbf{x}) + p(\mathbf{x})} \quad (16)$$

Where penalty function, $p(\mathbf{x})$ is defined as follows.

$$p(\mathbf{x}) = \varepsilon \cdot \left[c_1 \times \sum_{i=1}^{c} \left\{ \frac{g_i(\mathbf{x})}{g^*} \right\}^2 + c_2 \times \delta \sum_{i=1}^{c} \left\{ \frac{\Phi_i(\mathbf{x})}{\Phi^*} \right\} \right] \quad (17)$$

Where, $g_i(\mathbf{x})$, $\Phi_i(\mathbf{x})$ are level of violation and amount of violation for the i-th constraint; c_1, c_2 are weighting factors; g^*, Φ^* are scaling factors; δ is penalty coefficient; ε is –1 for minimization and +1 for maximization. Assigned values for c_1, c_2, and δ are 0.1, 0.5, and 0.1 respectively.

In our study, we used elementary operations of genetic algorithm, such as selection, crossover, and mutation. The procedure to execute the optimization is as shown flow diagram as in Fig 5.

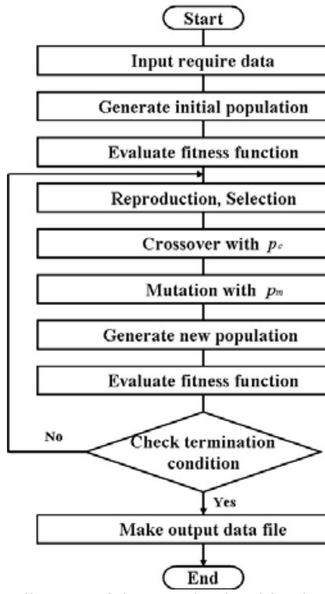


Fig. 5 Flow diagram of the genetic algorithm based optimization procedure

Elementary factors and parameters for genetic algorithm based optimization, in this study, are summarized in Table 1.

Table 1. Input parameters for the feed-drive system optimization

Parameters		Value
Max number of generation		150
Population size		60
Number of variable		4
Probability of crossover, p_c		0.8
Probability of mutation, p_m		0.01
Length of binary string [bit]	x_1	6
	x2	6
	x_3	7
	x_4	7
	x_5	7
	Sum total	33

4. RESULTS AND INVESTIGATION

The fitness value was converged to 13.1871 after 14th generation as shown in Fig. 6. Though the terminating condition was satisfied, we had tried to continue optimization procedure up to 150 generations in order to check any further convergence condition possible.

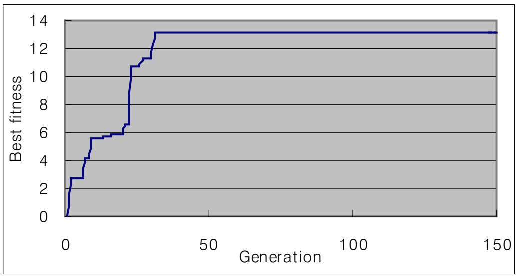
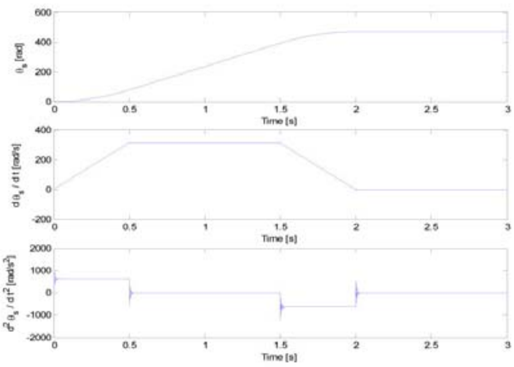
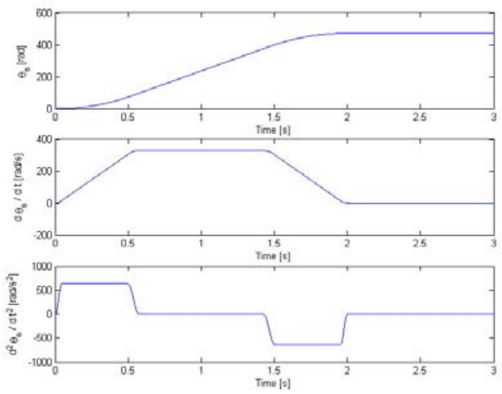


Fig. 6 Fitness versus generation history

Optimized variables having the best fitness were obtained and summarized in Table. 2. Also optimized curves of motor speed, motor angular acceleration, and screw rotation were obtained and compared in the Fig. 7. It is apparent that the angular acceleration curve corresponding to optimized motor speed has been smoothed.



(a) Before optimization



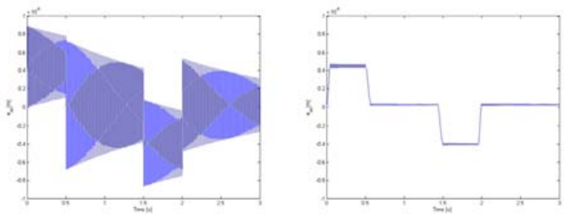
(b) After optimization

Fig. 7 Comparison of motor speed, corresponding motor angular acceleration and ball screw rotation

Table 2 Optimized variables

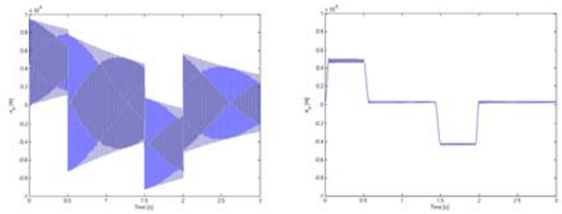
Variable	Optimized value [unit]
Center time, x_1	0.5357 [s]
Center time, x_2	1.4643 [s]
Time interval (width), x_3	0.0771 [s]
Time interval (width), x_4	0.0480 [s]
Maximum acceleration, x_5	639.0588 [rad/s^2]

In the Figures 7 to 8, the best solution of variables obtained from the feed rate optimization and corresponding system responses are compared with those of before optimization.



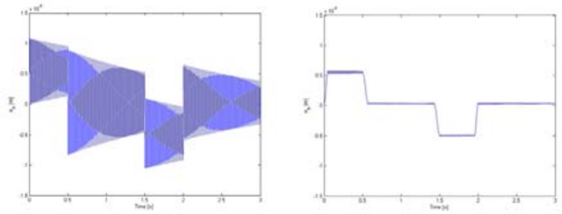
(Before optimization) (After optimization)

(a) Motor translation, x_m



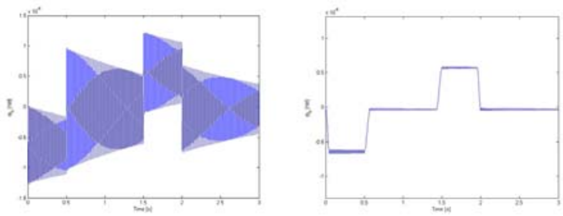
(Before optimization) (After optimization)

(b) Nut translation, x_n



(Before optimization) (After optimization)

(c) Head translation, x_h



(Before optimization) (After optimization)

(c) Head rotation, θ_h

Fig. 8 Comparison of transient vibrations of ball screw feed-drive system before and after optimization

Computed maximum vibration responses of the ball screw feed system and computed objectives in the feed rate optimization are compared in the Table 3.

Table. 3 Comparison system responses

Vibration amplitude	Before optimization	After optimization	Reduction (%)
$x_{m\,[m]}$	7.1350 *10^{-6}	0.2312*10^{-6}	96.77 %
$x_{n\,[m]}$	7.8171 *10^{-6}	0.2462*10^{-6}	96.85 %
$x_{h[m]}$	9.1091 *10^{-6}	0.2815*10^{-6}	96.91 %
$\theta_{h\,[rad]}$	1.0131 *10^{-6}	0.2580*10^{-7}	97.46%

As shown in the Figures 7 to 8, and Table 3, vibration amplitudes of the ball screw feed drive system have been reduced to almost nothing.

Moreover, judging from the results in table 4, it is clear that position error during the operation has been also greatly reduced after the feed rate optimization.

Table 4 Comparison of design objectives

Objectives	Before optimization	After optimization	Reduction (%)
$f_1(x)_{[m]}$	$1.0809 * 10^{-5}$	$4.7353*10^{-6}$	46.6 %
$f_2(x)_{[m]}$	$6.3843 * 10^{-6}$	$1.9185 * 10^{-6}$	70.0%
$f_3(x)_{[m]}$	$2.3933*10^{-5}$	$1.2782*10^{-5}$	46.6%

5. CONCLUSION

In this paper, a feed rate optimization of a ball screw driven machine tool feed slide system vibrations was studied in order to minimize its vibrations during operations. Prior to optimization, a ball screw feed drive system was mathematically modeled as a 6-degree-of-freedom lumped parameter system to analyze its vibrations. And then, a feed rate optimization was carried out for minimum vibrations and position error using a genetic algorithm. The main idea of our feed rate optimization is to find out the most appropriate smooth motor angular acceleration profile with jerk continuity. The results of computer aided vibration analyses show that transient vibration amplitudes and position error of a ball screw feed slide system can be reduced almost to nothing by using our proposed feed rate optimization method.

ACKNOWLEDGMENTS

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