Design of Controller for Affine Takagi-Sugeno Fuzzy System with Parametric Uncertainties via BMI

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Abstract: This paper develops a stability analysis and controller synthesis methodology for a continuous-time affine Takagi-Sugeno (T-S) fuzzy systems with parametric uncertainties. Affine T-S fuzzy system can be an advantage because it may be able to approximate nonlinear functions to high accuracy with fewer rules than the homogeneous T-S fuzzy systems with linear consequents only. The analysis is based on Lyapunov functions that are continuous and piecewise quadratic. The search for a piecewise quadratic Lyapunov function can be represented in terms of bilinear matrix inequalities (BMIs). A simulation example is given to illustrate the application of the proposed method.

Keywords: Affine T-S fuzzy model, Parametric uncertainty, Bilinear matrix inequality.

1. Introduction

Most plants in the industry have uncertainties and it make hard to control the general nonlinear, uncertain plants. In order to surmount this difficulties, fuzzy control is developed recently. It has been shown that fuzzy logic control (FLC) is a successful control approach for a complex nonlinear systems. There are a number of systematic analysis and controller design methodology in the literature, where the Takagi-Sugeno (T-S) fuzzy model is used [7–9, 12–14]. The T-S fuzzy model have two part, *i.e.,* one is antecedent part, the other is consequent part. Original T-S fuzzy system has not only linear but also affine terms in the consequent part [7]. But affine terms in the consequent part are ignored in almost all paper [1, 7–9, 12–14]. In this paper, we call the T-S fuzzy system that do not have affine term Homogeneous T-S fuzzy system. In homogeneous T-S fuzzy systems, it is well known that the stability depends on the existence of a common positive definite matrix satisfying a set of LMIs. But it is difficult to find common positive definite matrix P satisfy every condition of stability. And although there is no common positive definite matrix P, the system can be stable [3]. In these reasons, stability checking method for T-S fuzzy system is appeared using piecewise Lyapunov function [2–4]. The T-S fuzzy systems considered in this paper are allowed to have an affine term. This can be an advantage, because affine T-S fuzzy systems may be able to approximate nonlinear functions to high accuracy with fewer rules than the homogeneous T-S fuzzy systems with linear consequents only. And we use piecewise Lyapunov function for stability analysis of affine T-S fuzzy system [15, 16].

Besides stability, robustness is very important requirement for a control system. There may exist errors in modeling of system and uncertainties that we cannot measure. Cao *et al.* investigated some control techniques for uncertain T-S fuzzy models [16]. And Kiriakidis studied the issue of stability robustness against modeling errors [17]. Lee *et al.* proposed robust fuzzy-model-based controller design method for systems with parametric uncertainties which norm-bounded [1]. In this paper, based on approach in [1] we propose robust fuzzy controller design methodology for the continuous-time affine T-S fuzzy model with parametric uncertainties. And parametric uncertainties are assumed norm-bounded [11].

This paper is organized as follows: In the Section 2, we review the basic notation of affine T-S fuzzy systems and assumption of uncertainty model. We propose a stability analysis and controller design methodology of affine T-S fuzzy systems with parametric uncertainties in the Section 3. Section 4 shows a numerical examples and simulation results. Finally conclusion and some discussions are given in Section 5.

2. Preliminaries

Consider the continuous-time affine T-S fuzzy system in which the ith rule is formulated in the following form:

Plant Rules

$$R^i: \text{ IF } x_1(t) \text{ is } \Gamma_1^i \text{ and } \cdots \text{ and } x_n(t) \text{ is } \Gamma_n^i, \\ \text{THEN } \dot{x}(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) \\ + (a_i + \Delta a_i) \quad (1)$$

where $\Gamma_j^i (i = 1, \ldots, q, j = 1, \ldots, n.)$ is the fuzzy set, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $a_i \in \mathbb{R}^{n\times 1}$ are system matrix, input coupling matrix and affine matrix, respectively. And $\Delta A_i \in \mathbb{R}^{n\times n}$, $\Delta B_i \in \mathbb{R}^{n\times m}$, $\Delta a_i \in \mathbb{R}^{n\times 1}$ are time-varying uncertain matrices with appropriate dimension of system matrix, input coupling matrix and affine matrix, respectively which express the parametric uncertainties in the fuzzy model.

The defuzzified output of the affine T-S fuzzy system (1) is represented as follows:

$$\dot{x}(t) = \sum_{i=1}^{q} \mu_i(x(t))((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) \\ + (a_i + \Delta a_i)) \quad (2)$$

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$$\omega_i(x(t)) = \prod_{j=1}^{n} \Gamma_j^i(x_j(t)), \quad \mu_i(x(t)) = \frac{\omega_i(x(t))}{\sum_{i=1}^{q}\omega_i(x(t))}$$

where $\Gamma_j^i(x_j(t))$ is the membership value of $x_j(t)$ in Γ_j^i and $\omega_i(x(t))$ is the normalized membership function.
Throughout this paper, a state feedback affine T-S fuzzy-model-based control law is utilized for the stabilization of the T-S fuzzy system (2) in which the ith rule is formulated in the following form:

Controller Rules
R^i : IF $x_1(t)$ is Γ_1^i and $\cdots$ and $x_n(t)$ is Γ_n^i
THEN $u(t) = K_i x(t)$ (3)

where $K_i \in \mathbb{R}^{m\times n}$ are control gain matrices to be selected. The defuzzified output of the controller rules is given by

$$u(t) = \sum_{i=1}^{q} \mu_i(x(t))K_i x(t). \tag{4}$$

The closed-loop system with (2) and (4) is represented as

$$\begin{aligned}\dot{x}(t) = &\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(x(t))\mu_j(x(t))\\ &((A_i + \Delta A_i + (B_i + \Delta B_i)K_j)x(t) + a_i + \Delta a_i)\end{aligned} \tag{5}$$

For convenient notation, we introduce followings:

$$\begin{aligned}&\bar{x} = \begin{bmatrix} x \\ 1\end{bmatrix}, \quad \bar{A}_i = \begin{bmatrix} A_i & a_i \\ 0 & 0\end{bmatrix}, \quad \Delta\bar{A}_i = \begin{bmatrix} \Delta A_i & \Delta a_i \\ 0 & 0\end{bmatrix},\\ &\bar{B}_i = \begin{bmatrix} B_i \\ 0\end{bmatrix}, \quad \Delta\bar{B}_i = \begin{bmatrix} \Delta B_i \\ 0\end{bmatrix}, \quad \bar{C}_i = \begin{bmatrix} C_i & 0\end{bmatrix},\\ &\bar{K}_i = \begin{bmatrix} K_i & 0\end{bmatrix}.\end{aligned} \tag{6}$$

Using this notation, the system (5) can be expressed as

$$\dot{\bar{x}}(t) = \sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(\bar{x}(t))\mu_j(\bar{x}(t))\bar{G}_{ij}\bar{x}(t) \tag{7}$$

$$\begin{aligned}&= \sum_{i=1}^{q}\mu_i^2(\bar{x}(t))\bar{G}_{ii}\\ &\quad + 2\sum_{i<j}^{q}\mu_i(\bar{x}(t))\mu_j(\bar{x}(t))\left(\frac{\bar{G}_{ij}+\bar{G}_{ji}}{2}\right)\bar{x}(t)\end{aligned} \tag{8}$$

where $\bar{G}_{ij} = \bar{A}_i + \Delta\bar{A}_i + (\bar{B}_i + \Delta\bar{B}_i)\bar{K}_j$.

Remark 1: In the extended notations, membership function $\Gamma_j^i(\bar{x}(t))$ and normalized membership function $\omega_i(\bar{x}(t))$ have same values of original membership function $\Gamma_j^i(x(t))$ and normalized membership function $\omega_i(x(t))$.

Define $I_0 \subseteq I$ as the set of indices for subspace that contain the origin and $I_1 = I\backslash I_0 \subseteq I$ as the set of indices for subspace that do not contain the origin. Since we are interested in analyzing exponential stability of the origin, it is assumed that $a_i = 0, \Delta a_i = 0$ for all $i \in I_0$.
For $i \in I_0$ (8) becomes

$$\dot{x}(t) = \sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(x(t))\mu_j(x(t))G_{ij}x(t) \tag{9}$$

where $G_{ij} = A_i + \Delta A_i + (B_i + \Delta B_i)K_j$ and it is the same system in [1].
Since the closed loop systems (8) and (9) have time-varying uncertain matrices, it is difficult to decide the stability of the system. In this reason the parametric uncertainties considered here are removed under some reasonable assumptions. In this paper, we assume that uncertain matrices ΔA_i, ΔB_i and Δa_i are admissibly norm-bounded and structured.

Assumption 1: The parametric uncertainties considered here are norm-bounded, in term

$$\begin{bmatrix}\Delta A_i & \Delta a_i & \Delta B_i\end{bmatrix} = D_i F_i(t)\begin{bmatrix}E_{1i} & E_{2i} & E_{3i}\end{bmatrix} \tag{10}$$

$$\begin{bmatrix}\Delta\bar{A}_i & \Delta\bar{B}_i\end{bmatrix} = \bar{D}_i\bar{F}_i(t)\begin{bmatrix}\bar{E}_{1i} & \bar{E}_{3i}\end{bmatrix} \tag{11}$$

where D_i, E_{1i}, E_{2i}, and E_{3i} are known real constant matrices of appropriate dimensions, and $F_i(t)$ is unknown matrix function satisfies $F_i(t)^T F_i(t) \le I$, in which I is the identity matrix of appropriate dimension. And extended variables have following notation.

$$\bar{D}_i = \begin{bmatrix} D_i & 0 \\ 0 & 0\end{bmatrix}, \quad \bar{F}_i = \begin{bmatrix} F_i & 0 \\ 0 & 0\end{bmatrix}, \quad \bar{E}_{1i} = \begin{bmatrix} E_{1i} & E_{2i} \\ 0 & 0\end{bmatrix},$$
$$\bar{E}_{3i} = \begin{bmatrix} E_{3i} \\ 0\end{bmatrix}$$

3. Robust Stability of Affine T-S Fuzzy Systems

In some case, it is possible to prove stability of T-S fuzzy system using globally quadratic function $V(x) = x^T P x$ in the sense of Lyapunov. But it is difficult to find common positive definite matrix P satisfy every condition of stability. Although there is no common positive definite matrix P, the system can be stable [3]. In this reason, stability checking method for T-S fuzzy system is appeared using piecewise Lyapunov function [2–4].

Theorem 1: If there exists symmetric matrices T, U_i, and W_i such that U_i and W_i have nonnegative entries such that

$$P_i = F_i^T T F_i, \quad i \in I_0 \tag{12}$$
$$\bar{P}_i = \bar{F}_i^T T F_i, \quad i \in I_1 \tag{13}$$

satisfy

$$0 < P_i - E_i^T U_i E_i \tag{14}$$
$$0 > A_i^T P_i + P_i A_i + E_i^T W_i E_i \tag{15}$$

for $i \in I_0$, and

$$0 < \bar{P}_i - \bar{E}_i^T U_i \bar{E}_i \tag{16}$$
$$0 > \bar{A}_i^T \bar{P}_i + \bar{P}_i \bar{A}_i + \bar{E}_i^T W_i \bar{E}_i \tag{17}$$

for $i \in I_1$, then $x(t)$ tends to zero exponentially.

Before proceeding, we recall the following matrix inequalities which will be needed in the proof of our main result below.

Lemma 1: Given constant matrices D and E and a symmetric constant matrix S of appropriate dimensions, the following inequality holds:

$$S + DFE + E^T F^T D^T < 0,$$

where F satisfies $F^T F \leq R$, if and only if for some $\epsilon > 0$

$$S + \begin{bmatrix} \epsilon^{-1}E^T & \epsilon D \end{bmatrix} \begin{bmatrix} R & 0 \\ o & I \end{bmatrix} \begin{bmatrix} \epsilon^{-1}E \\ \epsilon D^T \end{bmatrix} < 0.$$

Lemma 2: Let $F_o(x(t)), \cdots, F_q(x(t))$ be quadratic functions of the variable $x(t) \in \mathbb{R}^n$. Consider the following statements:

$$F_0(x(t)) \leq 0 \text{ for all } x(t) \text{ such that } F_i(x(t)) \leq 0, \forall i \in \{1, \cdots, q\} \quad (18)$$

If there exists scalars $\tau_1 \geq 0, \cdots, \tau_q$ such that

$$F_0(x(t)) - \sum_{i=1}^{q} \tau_i F_i(x(t)) \leq 0 \quad (19)$$

then (18) holds.

By using Lemma 1, the theoretical difficulties in dealing with uncertainty can be effectively eliminated.
The main result on the global asymptotic stability of affine T-S fuzzy system with parametric uncertainties is shown below.

Theorem 2: For each i $\in I$, If there exists a symmetric matrix T and some matrices K_i, and some scalars $\epsilon_{ij}, (i, j = 1, \cdots, q)$ such that the following LMIs are satisfied, then the continuous-time T-S fuzzy system is asymptotically stabilizable via T-S fuzzy model-based state-feedback controller:

$$P_{ij} = F_{ij}^T T F_{ij}, \quad (i,j) \in I_0$$
$$\bar{P}_{ij} = \bar{F}_{ij}^T T \bar{F}_{ij}, \quad (i,j) \in I_1$$

satisfy

$$0 < P_{ij} \quad (20)$$

$$0 < \bar{P}_{ij} \quad (21)$$

$$\begin{bmatrix} \Phi_{ii} & * & * \\ E_{1i} + E_{3i}K_i & -\epsilon_{ii}I & * \\ D_i^T P_{ii} & 0 & -\epsilon_{ii}^{-1}I \end{bmatrix} < 0 \quad (1 \leq i \leq q, \quad i \in I_0) \quad (22)$$

$$\begin{bmatrix} \bar{\Phi}_{ii} & * & * \\ \bar{E}_{1i} + \bar{E}_{3i}\bar{K}_i & -\epsilon_{ii}I & * \\ \bar{D}_i^T \bar{P}_{ii} & 0 & -\epsilon_{ii}^{-1}I \end{bmatrix} < 0 \quad (1 \leq i \leq q, \quad i \in I_1) \quad (23)$$

$$\begin{bmatrix} \Theta_{ij} & * & * & * & * \\ E_{1i} + E_{3i}K_j & -\epsilon_{ij}I & * & * & * \\ E_{1j} + E_{3j}K_i & 0 & -\epsilon_{ij}I & * & * \\ D_i^T P & 0 & 0 & -\epsilon_{ij}^{-1}I & * \\ D_j^T P & 0 & 0 & 0 & -\epsilon_{ij}^{-1}I \end{bmatrix} < 0 \quad (1 \leq i < j \leq q, \quad i \in I_0) \quad (24)$$

$$\begin{bmatrix} \bar{\Theta}_{ij} & * & * & * & * \\ \bar{E}_{1i} + \bar{E}_{3i}\bar{K}_j & -\epsilon_{ij}I & * & * & * \\ \bar{E}_{1j} + \bar{E}_{3j}\bar{K}_i & 0 & -\epsilon_{ij}I & * & * \\ \bar{D}_i^T \bar{P} & 0 & 0 & -\epsilon_{ij}^{-1}I & * \\ \bar{D}_j^T \bar{P} & 0 & 0 & 0 & -\epsilon_{ij}^{-1}I \end{bmatrix} < 0 \quad (1 \leq i < j \leq q, \quad i \in I_1) \quad (25)$$

where

$$\Phi_{ii} = A_i^T P_{ii} + P_{ii}A_i + K_i^T B_i^T P_{ii} + P_{ii}B_i K_i,$$
$$\bar{\Phi}_{ii} = \bar{A}_i^T \bar{P}_{ii} + \bar{P}_{ii}\bar{A}_i + \bar{K}_i^T \bar{B}_i^T \bar{P}_{ii} + \bar{P}_{ii}\bar{B}_i \bar{K}_i,$$
$$\Theta_{ij} = A_i^T P + PA_i + A_j^T P + PA_j + K_j^T B_i^T P + PB_i K_j + K_i^T B_j^T P + PB_j K_i,$$
$$\bar{\Theta}_{ij} = \bar{A}_i^T \bar{P} + \bar{P}\bar{A}_i + \bar{A}_j^T \bar{P} + \bar{P}\bar{A}_j + \bar{K}_j^T \bar{B}_i^T \bar{P} + \bar{P}\bar{B}_i \bar{K}_j + \bar{K}_i^T \bar{B}_j^T \bar{P} + \bar{P}\bar{B}_j \bar{K}_i.$$

Proof: It is omitted in this paper. ■

To reduce conservatism of Theorem 2, we introduce S-procedure [2–4, 19]. And we can get the next corollary.

Corollary 1: In solving the inequalities in Theorem 2, if we replace (20) and (21) to (26) and (27) respectively, and $P_{ij}, \bar{P}_{ij}$ in (22)∼(25) to (28) and (29) respectively, then the continuous-time affine T-S fuzzy system is asymptotically stable in the relaxed condition.

$$0 < P_{ij} - E_{ij}^T U_{ij} E_{ij} \quad (26)$$

$$0 < \bar{P}_{ij} - \bar{E}_{ij}^T \bar{U}_{ij} \bar{E}_{ij} \quad (27)$$

$$0 < P_{ij} + E_{ij}^T W_{ij} E_{ij} \quad (28)$$

$$0 < \bar{P}_{ij} + \bar{E}_{ij}^T \bar{W}_{ij} \bar{E}_{ij} \quad (29)$$

Remark 2: It is noted that the matrix inequalities in Theorem 2 and Corollary 1 are the BMIs. They can be solved by V-K iteration method [?]. The procedure can be summaried in the followng algorithm.

Algorithm 1: (V-K iteration method)
1. Initialize contoller gain matrix $K_i (i \in I_0)$, $\bar{K}_i (i \in I_1)$ by using pole placement technique.
2. V-step: Given a fixed controller gain $k_i, i \in I_0$, $\bar{k}_i, i \in I_1$, solve the following optimization problem.

$$\min_{P_{ij}, \bar{P}_{ij}, U_{ij}, W_{ij}} \lambda_{ij}$$

s.t. the left side of inequalities in (22), (23), (24), (25) is less than $\lambda_{ij} I$.

3. K-step: Using the matrices P_{ij}, $\bar{P}_{ij}$ obtained in step 2, solve the following optimization problem.

$$\min_{K_{ij}, \bar{K}_{ij}, U_{ij}, W_{ij}} \lambda_{ij}$$

s.t. the left side of inequalities in (22), (23), (24), (25) is less than $\lambda_{ij} I$.

4. If $\lambda_{ij} < 0, (i,j) \in I$, go to step 5, else iterate step 2 and step 3 until finding $\lambda_{ij} < 0, (i,j) \in I$.
5. Using acquired controller gain, make closed-loop system stable.

4. Simulations

In this section, to show the efficiency of the proposed robust controller design techniques, we simulate the numerical example with parametric uncertainties. the contorl objective is to drive their trajectories to the origin.
Consider the following T-S fuzzy system.

$$\begin{aligned} R^1 &: \text{IF } x_1(t) \le -2 \\ &\quad \text{THEN } \dot{x}(t) = (A_1 + \Delta A_1)x(t) + (B_1 + \Delta B_1)u(t) \\ &\qquad + (a_1 + \Delta a_1) \end{aligned} \tag{30}$$

$$\begin{aligned} R^2 &: \text{IF } -2 < x_1(t) \le 0 \\ &\quad \text{THEN } \dot{x}(t) = (A_2 + \Delta A_2)x(t) + (B_2 + \Delta B_2)u(t) \\ &\qquad + (a_2 + \Delta a_2) \end{aligned} \tag{31}$$

$$\begin{aligned} R^3 &: \text{IF } 0 < x_1(t) \le 2 \\ &\quad \text{THEN } \dot{x}(t) = (A_3 + \Delta A_3)x(t) + (B_3 + \Delta B_3)u(t) \\ &\qquad + (a_3 + \Delta a_3) \end{aligned} \tag{32}$$

$$\begin{aligned} R^4 &: \text{IF } 2 < x_1(t) \\ &\quad \text{THEN } \dot{x}(t) = (A_4 + \Delta A_4)x(t) + (B_4 + \Delta B_4)u(t) \\ &\qquad + (a_4 + \Delta a_4) \end{aligned} \tag{33}$$

where

$$A_1 = \begin{bmatrix} -10 & 11 \\ 10 & 9 \end{bmatrix}, A_2 = A_3 = \begin{bmatrix} -1 & 5 \\ 2 & 8 \end{bmatrix}, A_4 = \begin{bmatrix} -10 & 10 \\ 10 & 5 \end{bmatrix},$$

$$B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, a_1 = \begin{bmatrix} 0 \\ -2 \end{bmatrix}, a_2 = a_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, a_1 = \begin{bmatrix} 0 \\ 2 \end{bmatrix},$$

In this example we will consider the parametric uncertainties of system matrices only. So it is assumed that $\Delta a_i = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \Delta B_i = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, i = \{1,2,3,4\}$. And assume parametric

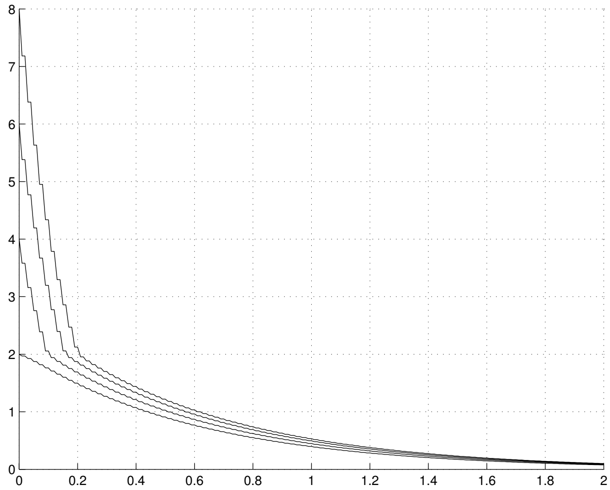

Fig. 1. State Trajectory of the system with 4 initial conditions

uncertainties are bounded within 30% of the their nominal values.
One can verify that the system is asymptotically stable in the large by using Theorem 2. By using gain matrices, we simulate with 4 initial condition $x(0) = \begin{bmatrix} 2 & 0 \end{bmatrix}^T, \begin{bmatrix} 4 & 0 \end{bmatrix}^T, \begin{bmatrix} 6 & 0 \end{bmatrix}^T, \begin{bmatrix} 8 & 0 \end{bmatrix}^T$ and result are shown in Figure 1.

5. Closing Remarks

In this paper, we have developed and analyzed a new robust fuzzy controller design method for affine T-S fuzzy systems with parametric uncertainties based on a piecewise Lyapunov function. And the search for a piecewise Lyapunov function can be represented in terms of bilinear matrix inequalities (BMIs). One numerical example is presented to simulate the design procedure.

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