The Modeling of Chaotic Nonlinear System Using Wavelet Based Fuzzy Neural Network

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Abstract: In this paper, we present a novel approach for the structure of Fuzzy Neural Network(FNN) based on wavelet function and apply this network structure to the modeling of chaotic nonlinear systems. Generally, the wavelet fuzzy model(WFM) has the advantage of the wavelet transform by constituting the fuzzy basis function(FBF) and the conclusion part to equalize the linear combination of FBF with the linear combination of wavelet functions. However, it is very difficult to identify the fuzzy rules and to tune the membership functions of the fuzzy reasoning mechanism. Neural networks, on the other hand, utilize their learning capability for automatic identification and tuning. Therefore, we design a wavelet based FNN structure(WFNN) that merges these advantages of neural network, fuzzy model and wavelet transform. The basic idea of our wavelet based FNN is to realize the process of fuzzy reasoning of wavelet fuzzy system by the structure of a neural network and to make the parameters of fuzzy reasoning be expressed by the connection weights of a neural network. And our network can automatically identify the fuzzy rules by modifying the connection weights of the networks via the gradient descent scheme. To verify the efficiency of our network structure, we evaluate the modeling performance for chaotic nonlinear systems and compare it with those of the FNN and the WFM.

Keywords: Fuzzy Neural Network, Wavelet Transform, Fuzzy System, Modeling, Chaotic System.

1. INTRODUCTION

When researcher wants to find the model of a system mathematically, the differential equation has been widely used. However, there are so much nonlinearity and a number of time constrains in realistic system that the accurate differential equation can hardly be obtained. Though comparatively precise model is acquired, the efficiency is decreased by model approximation. In order to solve this problem, intelligent techniques, based on neural networks and fuzzy logic, have also been developed for system modeling[1]-[3]. Even though these intelligent modeling strategies have shown effectiveness, especially for nonlinear systems, they have certain drawbacks derived from their own characteristics. While conventional neural networks have good ability of self-learning, they also have some limitations such as slow convergence, the difficulty in reaching the global minima in the parameter space, and sometimes instability as well[4]-[5]. In the case of fuzzy logic, it is a human-imitating logic, but lacks the ability of self-learning and self-tuning. Therefore, in the research on the intelligent modeling FNNs are devised to overcome these limitations and to combine the advantages of both neural networks and fuzzy logic[6]-[7]. This provides a strong motivation for using FNNs for modeling and controlling nonlinear systems. And the wavelet fuzzy model(WFM) has the advantage of the wavelet transform by constituting the FBF, the conclusion part to equalize the linear combination of FBF with the linear combination of wavelet functions and modifying fuzzy model to be equivalent to wavelet transform. The conventional fuzzy model cannot give the satisfactory result for the transient signal. On the contrary, in the wavelet fuzzy model, the accurate fuzzy model can be obtained because the energy compaction by the unconditional basis and the description of a transient signal by wavelet basis functions are distinguished[8]-[9]. On the other hand, chaos has received increasing attention in various areas such as mathematics, engineering, physics, biology, and economics. One attractive topic concerning chaos is chaos control and modeling, which are needed to prevent a chaotic system from becoming unstable or trapped in performance degraded situations due to the unpredictability and irregularity of chaos[10]-[12]. Especially, the identification model of chaotic

system is needed for the chaos control. Therefore, for the modeling of chaotic system, we design a FNN structure based on wavelet that merges these advantages of neural network, fuzzy model and wavelet. The basic idea of WFNN is to realize the process of fuzzy reasoning of wavelet fuzzy model by the structure of a neural network and to make the parameters of fuzzy reasoning be expressed by the connection weights of a neural network. WFNNs can automatically identify the fuzzy rules by modifying the connection weights of the networks using the gradient descent(GD) scheme. Among various fuzzy inference methods, WFNNs use the sum-product composition. To verify the efficiency of our network structure, we evaluate the modeling performance for chaotic nonlinear systems and compare it with those of the FNN and the WFM.

2. STRUCTURE OF WAVELET BASED FUZZY NEURAL NETWORK

2.1 Wavelet frames and wavelet networks

Wavelet networks were first presented in the framework of static modeling. Generally, the property of wavelet functions can be expressed as follows: any function of $L^2(R)$ can be approximated to any prescribed accuracy with a finite sum of wavelets. Therefore, wavelet networks can be considered as an alternative to neural and radial basis function networks. Wavelet frames, on the other hand, are constructed by simple operations for translation and dilation for a single fixed function called the mother wavelet. The following relation derives wavelet $\phi_i(x)$ from its mother wavelet $\phi(z_i)$.

$$\phi_j(x) = \phi(\frac{x - m_j}{d_j}) = \phi(z_j) \tag{1}$$

where, the translation factor m_j and the dilation factor d_j are real numbers in R and R^*_+ , respectively.

The family of functions generated by $\,\,\phi\,\,$ can be defined as

$$\Omega_c = \{\frac{1}{\sqrt{d_j}} \phi(\frac{x - m_j}{d_j}), m_j \in R \text{ and } d_j \in R_+^*\}$$
(2)

A family Ω_c is said to be a frame of $L^2(R)$ if the following equation is satisfied.

$$C\left\|f\right\|^{2} \leq \sum_{j \neq j \in \Omega_{c}} \left| <\phi_{j}, f > \right|^{2} \leq C\left\|f\right\|^{2}, \quad c > 0 \quad and \quad C < +\infty \quad (3)$$

where, ||f|| denotes the norm of function f and $\langle f,g \rangle$ the inner product of functions f and g. Families of wavelet frames of $L^2(R)$ are universal approximators. For the modeling of multivariable processes, multidimensional wavelets must be defined. A multidimensional wavelet function is represented with tensor product of single dimensional wavelet function as follows:

$$\phi(x) = \phi_1(x_1) \cdots \phi_n(x_n) \tag{4}$$

Assuming that single dimensional wavelet transform is separated into *n* orthogonal direction elements, Fourier transform $\hat{\phi}(\)$ of each term $\phi(x)$ in Eq. (4) is substituted for itself.

$$\hat{\phi}(x) = \hat{\phi}_1(x_1) \cdots \hat{\phi}_n(x_n) \tag{5}$$

$$\int \frac{\left|\hat{\phi}_{i}\left(-\right)\right|^{2}}{i} d_{i} < \infty$$
(6)

It is proven that admissibility condition of Eq. (6) must be satisfied. On condition of attenuation, Eq. (6) can satisfy the following for $\phi_i(x_i)$ that converges into 0 for $+\infty$ and $-\infty$.

$$\int \phi_i(x_i) dx_i = 0 \tag{7}$$

The condition of Eq. (3) should be satisfied to be wavelet frames as well. Therefore, $\phi_i(x_i)$ which satisfied Eqs. (3) and (7) should be set as wavelet frame. In this paper, the first-ordered differential form of the Gaussian probability density function is employed as a mother wavelet function that satisfies both of the conditions as Eq. (8).

$$\phi(z) = -z \exp(-\frac{1}{2}z^2) \tag{8}$$

And we use multidimensional wavelets constructed as the product of N_i scalar wavelets (N_i being the number of variables)

$$\Phi_{j}(x) = \prod_{k=1}^{N_{i}} \phi(z_{jk}) \quad \text{with } z_{jk} = \frac{x - m_{jk}}{d_{jk}}$$
(9)

2.2 Wavelet based Fuzzy Neural Network

Generally, the wavelet fuzzy model has the advantage of the wavelet transform by constituting FBF and conclusion part to equalize the linear combination of FBF with the linear combination of wavelet functions. The conventional fuzzy model cannot give the satisfactory result for the transient signal. On the contrary, in the wavelet fuzzy model, the accurate fizzy model can be obtained because the energy compaction by unconditional basis and the description of a transient signal by wavelet basis functions are distinguished. However, it is very difficult to identify the fuzzy rules and to tune the membership functions of the fuzzy reasoning mechanism. Neural networks, on the other hand, utilize their learning capability for automatic identification and tuning, but they have the following problems: (i) they need accurate input-output data and (ii) their learning process is time-consuming, to mention a few. Therefore, we design a FNN structure based on wavelet that merges these advantages of neural network, fuzzy modeling and wavelet. The basic idea of WFNN is to realize the process of fuzzy reasoning of wavelet fuzzy model by the structure of a neural network and to make the parameters of fuzzy reasoning be expressed by the connection weights of a neural network. WFNNs can automatically identify the fuzzy rules by modifying the connection weights of the networks using the gradient descent scheme. Among various fuzzy inference methods, WFNNs use the sum-product composition. The functions that are implemented by the networks must be differentiable in order to apply the gradient descent scheme to their learning.

Fig. 1 shows the configuration of WFNN, which has N inputs (x_1, x_2, \dots, x_N) , C outputs (y_1, y_2, \dots, y_C) , and K_n membership functions in each input x_n . The circles and the squares in the figure represent the units of the network. The denotations m, d, and the numbers 1, -1 between the units denote connection weights of the network.



Fig. 1 Network structure of WFNN

WFNN can be divided into two parts according to the process of the fuzzy reasoning: the premise part and the consequence parts. The premise part consists of nodes (A), (C), (D) and the consequence parts consist of nodes (D) through (F). The grades of the membership functions in the premise are calculated in nodes (A) and (C). The nodes (B) and (E) are used to equalize the linear combination of FBF with the linear combination of wavelet functions for the advantage of wavelet transform by constituting FBF and conclusion part. Therefore, the output node (F) is equivalent to wavelet transform. Consequently, in our WFNN structure, the output \hat{y}_c is calculated as follows:

$$\hat{y}_{c} = \sum_{n=1}^{N} a_{nc} x_{n} + \sum_{j=1}^{R} B_{jc} \Phi_{j}$$
(10)

where,

$$\Phi_{j}(x) = \prod_{n=1}^{N} \phi_{n}(z) = \prod_{n=1}^{N} -(\frac{x_{n} - m_{k_{n}n}}{d_{k_{n}n}}) \exp(-\frac{1}{2}(\frac{x_{n} - m_{k_{n}n}}{d_{k_{n}n}})^{2})$$

 k_n : k-th fuzzy variable of input n, N: input num.,

R: wavelet num., K_n : fuzzy variable num of input n,

Mother wavelet: $\phi(z) = -z \exp(-\frac{1}{2}z^2)$, $z = \frac{x-m}{d}$

The detailed descriptions of input and output nodes are as follows. Where, input and output nodes are denoted by I and O, respectively and subscript denotes each node.

Node A :
$$O_A = \frac{x_n - m_{k_n n}}{d_{k_n n}}$$
 (11)

Node B :
$$O_B = \prod_{n=1}^{N} O_{A_{n^n}} = \prod_{n=1}^{N} (-(\frac{x_n - m_{k_n^n}}{d_{k_n^n}}))$$
 (12)

Node C :

$$O_{C} = A_{k_{n}n}(x_{n}) = \exp(-\frac{1}{2}O_{A}^{2}) = \exp(-\frac{1}{2}(\frac{x_{n} - m_{k_{n}n}}{d_{k_{n}n}})^{2})$$
(13)

Node D $\,:\,$

$$I_{D} = \mu_{j} = \prod_{n=1}^{N} O_{C_{k_{n}n}} = \prod_{n=1}^{N} A_{k_{n}n}(x_{n}) = \prod_{n=1}^{N} \exp\left(-\frac{1}{2}\left(\frac{x_{n} - m_{k_{n}n}}{d_{k_{n}n}}\right)^{2}\right) \quad (14)$$

$$O_{D} = \hat{\mu}_{j} = \frac{\mu_{j}}{\sum_{j=1}^{R} \mu_{j}} = \frac{\prod_{n=1}^{N} \exp(-\frac{1}{2} (\frac{x_{n} - m_{k_{n}n}}{d_{k_{n}n}})^{2})}{\sum_{j=1}^{R} (\prod_{n=1}^{N} \exp(-\frac{1}{2} (\frac{x_{n} - m_{k_{n}n}}{d_{k_{n}n}})^{2}))_{j}} \operatorname{R:} \prod_{k=1}^{N} K_{k} \quad (15)$$

Node E $\,:\,$

$$O_{E} = y_{jc} = \int_{jc} O_{D} O_{B} = \int_{jc} \frac{\mu_{j}}{\sum_{j=1}^{R} \mu_{j}} \prod_{n=1}^{N} \left(-\left(\frac{x_{n} - m_{k_{n}n}}{d_{k_{n}n}}\right)\right)$$

$$= \int_{jc} \frac{\prod_{n=1}^{N} \exp\left(-\frac{1}{2}\left(\frac{x_{n} - m_{k_{n}n}}{d_{k_{n}n}}\right)^{2}\right)}{\sum_{j=1}^{R} \left(\prod_{n=1}^{N} \exp\left(-\frac{1}{2}\left(\frac{x_{n} - m_{k_{n}n}}{d_{k_{n}n}}\right)^{2}\right)\right)_{j}} \prod_{n=1}^{N} \left(-\left(\frac{x_{n} - m_{k_{n}n}}{d_{k_{n}n}}\right)\right)$$

$$= \int_{jc} \frac{\prod_{n=1}^{N} \left(-\left(\frac{x_{n} - m_{k_{n}n}}{d_{k_{n}n}}\right)\right) \exp\left(-\frac{1}{2}\left(\frac{x_{n} - m_{k_{n}n}}{d_{k_{n}n}}\right)^{2}\right)}{\sum_{j=1}^{R} \prod_{n=1}^{N} \exp\left(-\frac{1}{2}\left(\frac{x_{n} - m_{k_{n}n}}{d_{k_{n}n}}\right)^{2}\right)\right)_{j}}$$

$$= B_{jc} \Phi_{j} \qquad \text{where, } B_{jc} = \frac{jc}{\sum_{j=1}^{R} I_{D_{j}}}$$

$$(16)$$

Node F :

$$O_F = \hat{y}_c = \sum_{n=1}^N a_{nc} x_n + \sum_{j=1}^R y_{jc} = \sum_{n=1}^N a_{nc} x_n + \sum_{j=1}^R B_{jc} \Phi_j$$
(17)

The input space is divided into *R* fuzzy subspaces. The truth value of the fuzzy rule in each subspace is given by the product of the grades of the membership functions for the units in node D. Where, μ_j is the truth value of the *j*-th fuzzy rule and $\hat{\mu}_j$ is the normalized value of μ_j . Fuzzy system realizes the center of gravity defuzzification formula using $\hat{\mu}_j$ in Eq. (15).

The consequence parts consist of nodes (D) through (F) and the fuzzy reasoning is realized as:

$$\begin{aligned} R^{j}: & If \quad x_{1} \text{ is } A_{k_{1}1}, \quad \cdots, \quad x_{n} \text{ is } A_{k_{n}n}, \quad \cdots \quad and \quad x_{N} \text{ is } A_{k_{N}N} \\ & then \quad y_{jc} = _{jc} \quad (j = 1, 2, \cdots, R \quad and \quad c = 1, 2, \cdots, C) \end{aligned}$$

where, R^{j} is the *j*-th fuzzy rule, $A_{k_{n}n}$ is fuzzy variables in the premise, $_{jc}$ is a constant. Consequently, the output value of node (F) includes the inferred values.

The weights j_c are modified to identify fuzzy rules using the GD method. In order to apply the GD method, the squared error function is defined as follows:

$$J = \frac{1}{2} ((y_{r1} - \hat{y}_1)^2 + (y_{r2} - \hat{y}_2)^2 + \dots + (y_{rC} - \hat{y}_C)^2$$
(18)

where, \hat{y}_c is the output value of WFNN and y_{rc} is the desired value. Using the gradient descent method, the parameter set, $\gamma_c = \{a_n, j_c\}$, can be tuned as follows:

$$\gamma_{c}(k+1) = \gamma_{c}(k) + \gamma_{c}(k)$$

$$= \gamma_{c}(k) - \eta \frac{\partial J}{\partial \gamma_{c}(k)}$$

$$= \gamma_{c}(k) - \eta \frac{\partial J}{\partial \hat{y}_{c}} \frac{\partial \hat{y}_{c}}{\partial \gamma_{c}(k)}$$

$$= \gamma_{c}(k) + \eta (y_{rc} - \hat{y}_{c})_{\gamma} \quad where, \quad \hat{\gamma} = \frac{\partial \hat{y}_{c}}{\partial \gamma_{c}(k)}$$
(19)

where,

$$\hat{y}_{c} = \frac{\partial \hat{y}_{c}}{\partial y_{c}(k)} = \frac{\partial \sum_{j=1}^{R} y_{jc}}{\partial y_{jc}(k)} = \frac{\prod_{n=1}^{N} (-(\frac{x_{n} - m_{k_{n}n}}{d_{k_{n}n}})) \exp(-\frac{1}{2} (\frac{x_{n} - m_{k_{n}n}}{d_{k_{n}n}})^{2})}{\sum_{j=1}^{R} (\prod_{n=1}^{N} \exp(-\frac{1}{2} (\frac{x_{n} - m_{k_{n}n}}{d_{k_{n}n}})^{2}))_{j}}$$

$$\hat{y}_{a} = \frac{\partial \hat{y}_{c}}{\partial a_{c}(k)} = x_{n}$$

and η is called the learning rate.

3. MODELING OF CHAOTIC SYSTEM

The problem of system modeling consists of setting up a suitably parameterized identification model and adjusting the parameters of the model to optimize a performance function based on the error between the plant and the identification model outputs.

The identification model is divided into two types as follows: 1) Parallel Identification Model: the output of identification model is fed back into the identification model, and 2) Series-Parallel Identification Model: the output of plant is fed back into the identification model as shown in Fig. 2.

In this paper, we identify the chaotic nonlinear systems using the series-parallel identification model with a good performance and convergence abilities.

The training of WFNN identification model is performed by the GD method, as described in Section 2.2.



Fig. 2 Series-parallel identification model

4. SIMULATIONS

In this section, we present computer simulation results to validation the modeling performance of the proposed network structure for continuous-time chaotic system. For this purpose, we compare it with those of the FNN and the WFM. In this simulation, we consider the Duffing system as representative chaotic system. The state equations of Duffing system is as follows:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \end{bmatrix} = \begin{bmatrix} y(t) \\ a_1 x(t) - x^3(t) + a_2 y(t) + b \cos(t) \end{bmatrix}$$
(20)

where typically $a_1 = 1.1$, $a_2 = 0.4$, b = 2.1, = 1.8.

Figure 3 represents the strange attractor of Duffing system.



Fig. 3 The strange attractor of Duffing system

Because the characteristic of network structure is very susceptible to several simulation environments such as initial value of network weight, sampling time, learning rate, *etc.*, in this simulation, we use same parameters as shown in Table 1.

Table 1. The parameters and Results

Number of Membership Function	11	
(Number of Mother Wavelet)	11	
Sampling Time	0.01	
Learning Rate	0.2	
Initial Value	RANDOM	

Fig. 4 shows the process output, WFNN model output and error between these outputs. And Figs. 5 and 6 represent the modeling performances of FNN and WFM, respectively. From these figures and Table 2, we confirm that modeling performance using our WFNN model works better than two other models.

Table 2. Modeling results for three network models

	WFNN	0.0249
MSE	FNN	0.1302
	WFM	0.2260



Fig. 4 Modeling results for a WFNN model



Fig. 5 Modeling results for a FNN model



Fig. 6 Modeling results for a WFM

5. RESULTS

In this paper, we have proposed a FNN structure based on wavelet that merges the advantages of neural network, fuzzy model and wavelet. We presented simulation results to validate the modeling performance of proposed WFNN model for chaotic nonlinear systems. Also, in order to evaluate the performance of the proposed WFNN structure, we have compared the modeling performance of WFNN model with those of FNN and WFM models. As a result, it was shown that the modeling performance using WFNN model worked better than two models.

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