Speed Sensorless DC Motor Using Kalman Filter

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Abstract: This paper proposes a new application of Kalman filter to estimate speed sensorless DC motor. Kalman filter can estimate the system state variables accurately; even the system input is disturbed with noise. In the design, the mathematical model of DC motor in discrete state-space form will be created; the speed of DC motor which is considered as state variable and can be estimated by using Kalman filter. In the experiment; TMS320C31 floating point digital signal processor is used for hardware implementation, the input is disturbed with/and without white noise in the experiment. The experimental results show the speed of DC motor which is estimated by Kalman filter has good accuracy when compared with the results from tacho-meter.

Keywords: Kalman filter, speed sensorless,

1. INTRODUCTION

In the past, the motor speed is measured by using to sense. Actually, the tacho-meter is used for converting the speed-to-voltage; because the structure of DC motor consists of brush, commutator, mechanical components and etc. To use the tacho-meter for a long time, those components will be deteriorated and give the error in speed measurement. Also, the motor speed measurement system without mechanical components was proposed. The design process uses the continuous time system of motor in transfer function or state-space form [1] where the system inputs are armature voltage and armature current, system output determines the speed. Hence, the transform of continuous time system to discrete time system [2] and implement to digital signal processor chip is considered and proposed. This method of motor speed estimation should be correct when the system input separates from noise but whenever noise interferes in the system input such as voltage or current, the speed estimation will be error.

To overcome this problem, this paper proposes the design and implementation of motor speed estimation system using Kalman filter. Presently, Kalman filter is widely used for applying in digital signal processing [3] such as signal filtering, digital control by Kalman filter is a state variable estimation of dynamic system which can be operated even the dynamic system will be disturbed with noise. The design stage begins from determine mathematical model of DC motor in discrete state-space form and defined system input as voltage and current, the system output is speed. Since Kalman filter is used to estimate the speed of DC motor and give the result in voltage; as same as the voltage that gets from tacho-meter, not only the noise interferes with the system input, the speed estimation by using Kalman filter can be endured and give results correctly. Whatever the algorithm of Kalman filter has the complexity computation, also in an implementation must be necessary computing on floating point processor for accuracy in processing [4, 5].

2. THEORY

The Kalman filter uses the continuous state-space and can be classified into 3 type [6], continuous Kalman filter (CKF), continuous discrete Kalman filter (CDKF) and discrete Kalman filter (DKF). DKF has the error covariance less than CKF when compared with the average value if DKF is better than CDKF [6]. This paper proposes the design by using the DKF type, the Eq. form is divided into two types, consists of the time update period. It is assigned as shown in Eq.s (1) and (2).

$$\mathbf{P}_1(K) = \mathbf{A} * \mathbf{P}(K-1) * \mathbf{A}^t + \mathbf{BQB}^t \tag{1}$$

$$\hat{\mathbf{X}}_p(K) = \mathbf{A} * \hat{\mathbf{X}}(K-1) \tag{2}$$

Where K=1, 2, 3,…..

$\mathbf{P}_1(K)$ and $\mathbf{P}(K)$ is the error covariance, $\mathbf{Q}$ is covariance matrix, $\hat{\mathbf{X}}_p(K)$ and $\mathbf{X}(K-1)$ are state-vectors, $\mathbf{A}$ is matrix variable or system matrix of dynamic system. $\mathbf{B}$ is input of dynamic system. The measurements during the variation period can be shown in Eqs. (3)-(5) [7 - 9].

$$\mathbf{K}(K) = \mathbf{P}_1(K) * \mathbf{C}^t * \left[\mathbf{C} * \mathbf{P}_1(K) * \mathbf{C}^t + \mathbf{R}\right]^{-1} \tag{3}$$

$$\hat{\mathbf{X}}(K) = \hat{\mathbf{X}}_p(K) + \mathbf{K}(K) * \left[y(K) - \mathbf{C} * \hat{\mathbf{X}}_p(K)\right] \tag{4}$$

$$\mathbf{P}(K) = \mathbf{P}_1(K) - \mathbf{K}(K) * \mathbf{C} * \mathbf{P}_1(K) \tag{5}$$

Where $\mathbf{K}(K)$ is Kalman gain
$y(K)$ is measurement of $y(1), y(2), ..., y(k)$

The dynamic state variable system is shown in Eq. (6) [10].

$$\mathbf{X}(K+1) = \mathbf{A}(K) * \mathbf{X}(K) + \mathbf{B}(K) * \mathbf{W}(K) \tag{6}$$

The measurement system is rewritten as shown in Eq. (7).

$$y(K+1) = \mathbf{C}(K+1)\mathbf{X}(K+1) + v(k+1) \tag{7}$$

$\mathbf{W}(K)$ is noise in the dynamic system and $v(K+1)$ is the measured signal.

3. PROPOSED DESIGN METHOD

To estimate sensorless motor speed by using Kalman filter is proposed as shown in Fig. 1..

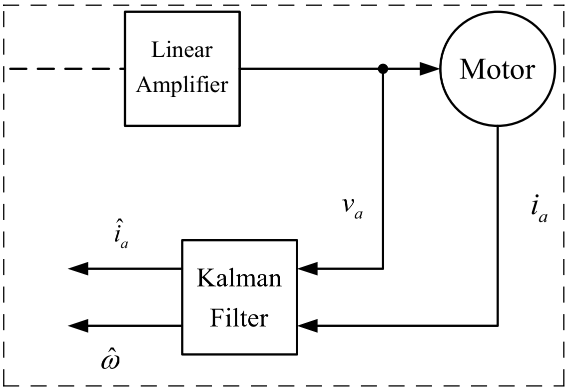

Fig. 1 Block diagram of sensorless motor speed estimation based on Kalman filter

Fig. 1, Kalman filter is able to estimate the speed of DC motor and it uses armature voltage (v_a) and armature current (i_a) as input of Kalman filter for speed (ω) calculation. The algorithm of Kalman filter cooperates with the others parameters of DC motor. The design has to know the characteristics and parameters of DC motor.

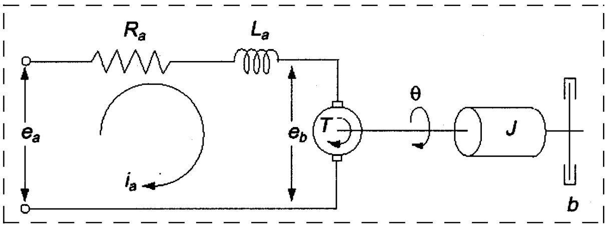

Fig. 2 DC motor electrical model

The schematic diagram of an armature controlling by DC servomotor is shown in Fig.2. The system variables are;

e_a : Armature voltage
e_b : Back emf.
i_a : Armature current
T : Torque produced by motor
θ : Angular position of motor shaft
ω : Angular velocity of motor shaft

The parameters of the system are;

R_a : Armature resistance
L_a : Armature inductance
J : Moment of inertia of motor shaft
B : Coefficient of viscous friction

The system parameters (are not shown in Fig.2) are;

K_T : Torque constant
K_b : Back emf constant

Fig. 2, the armature voltages can be found as in Eq. (8);

$$v_a = L_a \frac{di_a}{dt} + R_a i_a + e_b$$

$$e_b = K_b \omega$$

$$v_a = L_a \frac{di_a}{dt} + R_a i_a + K_b \omega \tag{8}$$

The occurred electrical torque in motor will change depending on the armature current as shown in Eq. (9);

$$T = K_T i_a \tag{9}$$

The mechanical torque of motor is written as in Eq. (10);

$$T = J\frac{d\omega}{dt} + B\omega \tag{10}$$

For lossless motor, the electrical torque is equal to the mechanical torque. Eqs. (9) and (10) can be rewritten as shown in Eq. (11);

$$K_T i_a = J\frac{d\omega}{dt} + B\omega \tag{11}$$

Eqs. (8), (11) can be rearranged as shown in Eqs. (12) and (13);

$$\frac{di_a}{dt} = -\frac{Ri_a}{L_a} - \frac{K_b\omega}{L_a} + \frac{e_a}{L_a} \tag{12}$$

$$\frac{d\omega}{dt} = \frac{K_T i_a}{J} - \frac{B\omega}{J} \tag{13}$$

The continuous state-space form of Eqs. (12) and (13) can be rewritten as in Eqs. (14) and (15);

$$\begin{bmatrix} \frac{di_a}{dt} \\ \frac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L_a} & -\frac{K_b}{L_a} \\ \frac{K_T}{J} & -\frac{B}{J} \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} \\ 0 \end{bmatrix} e_a \tag{14}$$

$$y_{S0}(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix} \tag{15}$$

Compare Eq. (14) with state Eq. it can obtain;

$$\mathbf{A} = \begin{bmatrix} -\frac{R}{L_a} & -\frac{K_b}{L_a} \\ \frac{K_T}{J} & -\frac{B}{J} \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} \frac{1}{L_a} \\ 0 \end{bmatrix}$$

Thus, transform matrix **A** and matrix **B** to discrete system by using bilinear transform and substitute matrix **A** and matrix **B** in discrete system to Eq. (1) – (7) for speed estimation.

4. DESIGN EXAMPLE

The experiment uses DC motor as experimental test set from Feedback Instrument Inc., motor drive set model SA150D, power supply model PS150E and DC motor model MT150F. The block diagram of connection is shown in Fig. 3.

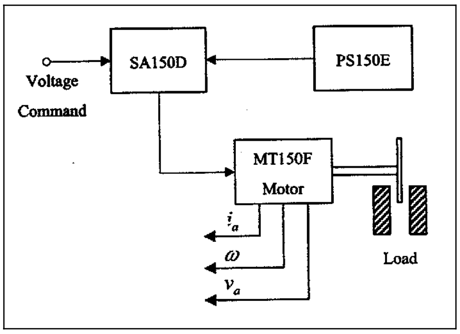

Fig. 3 Block diagram of motor drive test set

The DC motor in the experiment has parameters as shown in Table 1.

Table 1 Parameters of DC motor

Parameter	Value
R_a	3.0 Ohm
L_a	5.16 mH
J	$3\text{x}10^{-5}\text{K}_\text{g}\text{m}^2$
B	0.0158
K_T	0.0282 Nm/A
K_b	2.78 V/Krpm

The connection diagram of DC motor speed estimation in this experiment is shown in Fig. 4.

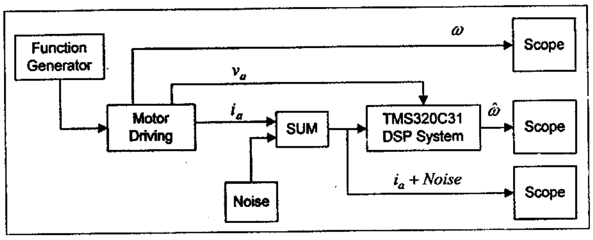

Fig.4 Connection of DC motor speed estimation in experiment

In Fig. 4, ω is obtained speed of motor from the tacho-meter, $\hat{\omega}$ is obtained speed of motor from estimation and in the experiment has the random noise signal disturbed with i_a. Substitute the parameters from Table 1 into Eq. (14), the matrix **A** and matrix **B** can be obtained as shown in Eq. (16);

$$\mathbf{A} = \begin{bmatrix} -581.40 & -538.76 \\ 28.20 & -15.8 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 193.8 \\ 0 \end{bmatrix} \tag{16}$$

Transform Eq. (16) to discrete system as shown in Eq. (17), where the given sampling frequency is 10 kHz.

$$\mathbf{A} = \begin{bmatrix} 0.94344 & -0.05229 \\ 0.00273 & 0.99834 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0.01882 \\ 0.00002 \end{bmatrix} \tag{17}$$

5. RESULTS

In the experiment, let the voltage command is between 7–10 volt, the obtained voltage from tacho-meter is compared with obtained voltage from Kalman filter as shown in Table 2.

Table 2 DC Comparison of motor speed

Voltage command	Output voltage obtained from tacho (ω)	Output voltage obtained from Kalman Filter $(\hat{\omega})$
7.0	1.0	1.0
7.5	1.15	1.2
8.0	1.3	1.4
8.5	1.5	1.5
9.0	1.6	1.65
9.5	1.8	1.85
10.0	2.0	1.95

Let the voltage command 7 V and 10 V is square wave signal frequency 0.1 Hz and combined random noise with covariance 1 V^2 to armature current as shown in Fig. 4. The experimental results are measured by oscilloscope as shown in Figs. 5 – 8

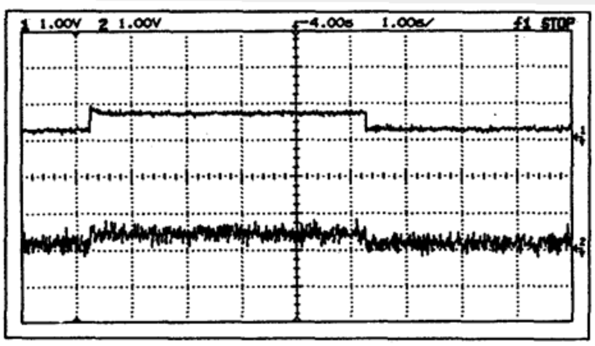

Fig. 5 The upper trace is armature current at voltage command 7.0 V,
The lower trace is random noise with covariance 1 V^2 adding to armature current

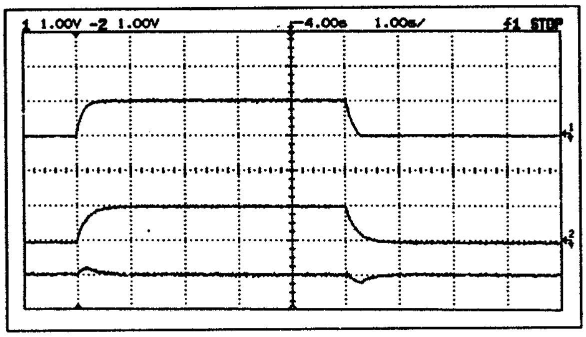

Fig. 6 The upper trace is motor speed from tacho-meter
The middle trace is motor speed from Kalman filter,
The lower trace is motor speed comparison between tacho-meter and Kalman filter method

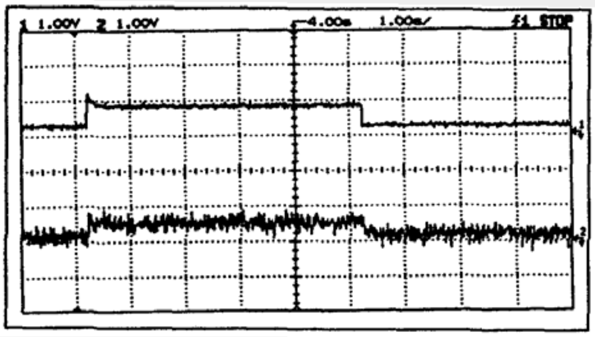

Fig. 7 The upper trace is armature current at voltage command 10.0 V,
The lower trace is random noise with covariance 1 V^2 adding to armature current

6. Conclusion

A SRWNN-based adaptive direct control scheme for mobile robots has been proposed. In control scheme, two SRWNN controllers have been designed for generating the control inputs. The structures of two SRWNNs have been trained by the GD method. Since the SRWNN has the ability for storing the past information of the network, it can adapt rapidly to changes of the operation environment of mobile robots. Using the discrete Lyapunov theorem, stability of the whole control scheme has been carried out and the ALR has been also established for the stable path tracking of the mobile robot. A simulation result has shown that the proposed control system has an on-line adapting ability for controlling the mobile robot.

Table 1. The tracking control errors

x_c MSE	y_c MSE	θ MSE
0.0008	0.0009	0.00007

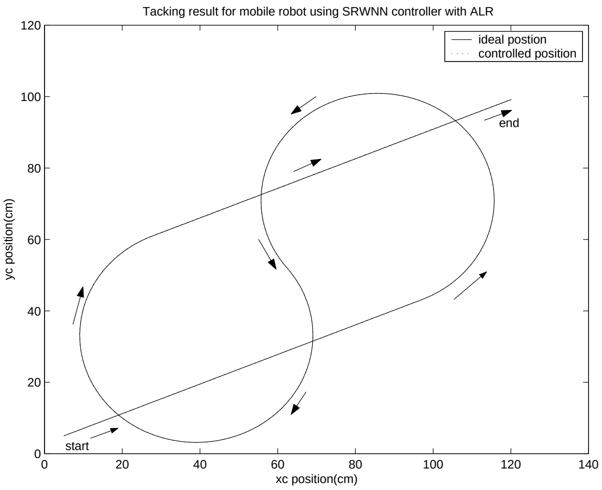

Fig. 4. Tracking result of the mobile robot

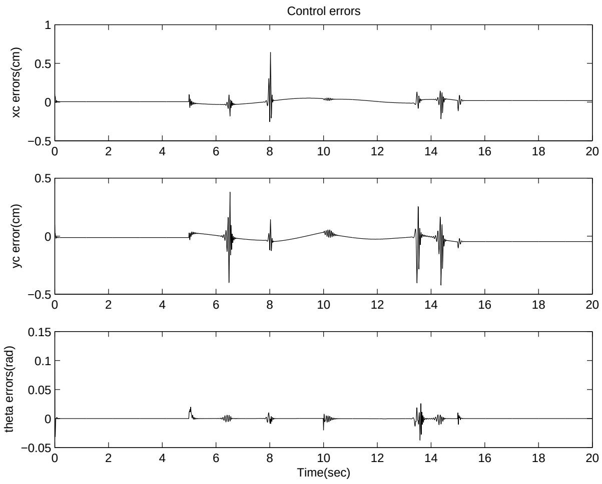

Fig. 5. The tracking control errors

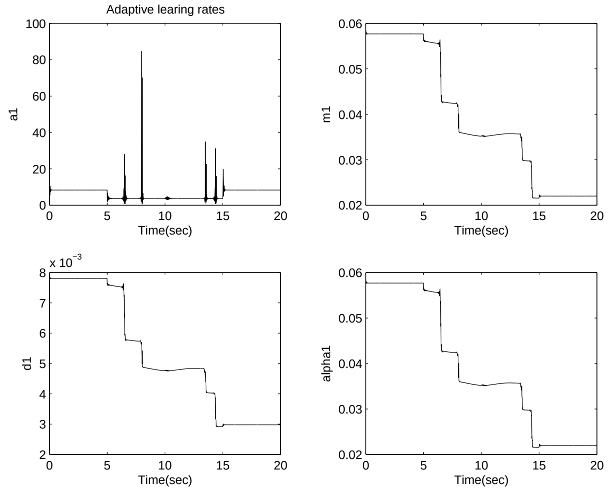

Fig. 6. The ALR of the SRWNNC1

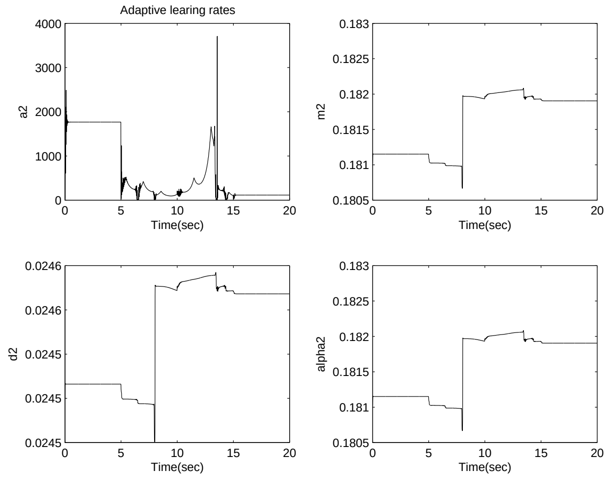

Fig. 7. The ALR of the SRWNNC2

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