# Time Delay Control of Sway and Skew of the Spreader Suspended by Four Flexible Cables 

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#### Abstract

This article describes the time delay control of the 3-dimensional motion of the container cranes used in dockside container terminals. The container is suspended by four flexible cables via spreader, and due to the disturbances such as the wind and acceleration of cranes, the container undergoes translational(sway) and rotational position errors. And due to the uncertainty of weight and rotational inertia, accurate position control of container crane is difficult to realize. This paper, based on the analysis of 3-dimensional dynamics of container moving systems, develops time delay control algorithm [1]. The developed control algorithm is shown effective in controlling the container position in the presence of gust and parameter uncertainties.


Keywords: container cranes, motion control, sway, skew, time delay control


Fig. 1. Typical gantry crane used in container terminal.

## 1. Introduction

The mobile gantry crane(Fig. 1) is widely used in dockside container base or railway freight terminal to pick up and move containers. The gantry crane is composed of three main parts. The first one is gantry whose structure supports all equipment and moves along one direction. The second one is the trolley which is on the longitudinal structure of gantry and moves along longitudinal direction(perpendicular to gantry motion) of gantry. The last one is spreader which is suspended by, typically, four flexible cables from trolley. The spreader is equipped with container pick up mechanisms, and holds the container. By controlling the length of cables, the container is moved upward or downward. The construction of container crane is well illustrated in [6] and [3]. Because the spreader is connected by flexible cables with trolley, acceleration of trolley and gantry induce sway of spreader. And disturbances such as wind or asymmetric loading of container induce sway and rotation(the rotation will be called skew, hereafter) of spreader. Because the sway and skew is not de-

[^0]sirable in positioning of container, the four cables are widely spaced to reduce sway and skew. On the contrary, factory crane possesses single or close running parallel cables. Because of the widely spaced cables, the problem is slightly different in that the spreader of mobile gantry crane possesses rather complex dynamics than factory crane[4]. In previous papers by authors[9], [7], [10], the 3-dimensional kinematics and dynamics are investigated in detail. In this paper, based on the dynamic equations derived in [10], effective control algorithm for positioning od container is investigated.
This paper is constructed as follows. In next section(section 2. kinematics and dynamics of gantry crane is reviewed and in section 3, the kinematic description of the gantry crane system is described. In section 4.control algorithm is derived. In section 5.some simulations are manifested to show the effectiveness of control algorithm. In last section, conclusions and discussions of this paper are summarized.

## 2. Kinematics and Dynamics of Container Cranes

Many researches are devoted to the control of three dimensional position of gantry cranes, for example [11], [12], [13], [14], [15], [16], [17], [18], [19]. But most of them treat gantry crane as a point mass and multiple cables as single cable. As a result, their control problem becomes the problem of motion control of a simple pendulum. But as mentioned earlier, three dimensional analysis of spreader are expected to yield more accurate results and shed some insights on the control of sway and skew of spreader. On this subject, paper by Cartmel et. al.[5] is the only one in authors' paper review. Typical gantry crane used in container terminals is shown in Fig. 1 and its schematic diagram to show container motion relative to trolley is shown in Fig. 2.
In the authors previous research[10], the 3-dimensional dynamics of container crane is derived and the equations are shown for the completeness of the paper.

$$
\begin{equation*}
\dot{\mathbf{x}}=\mathbf{A x}+\mathbf{B u}+\mathbf{B G} \tag{1}
\end{equation*}
$$



Fig. 2. Schematic diagram to show motion vector of trolley and spreader.
where,

$$
\begin{align*}
& A=\left[\begin{array}{cc}
\mathbf{0} & \mathbf{I} \\
\mathbf{0} & -\mathbf{M}^{-\mathbf{1}} \mathbf{V}
\end{array}\right]  \tag{2}\\
& B=\left[\begin{array}{cc}
0 & 0 \\
0 & -\mathbf{M}^{-\mathbf{1}}
\end{array}\right]  \tag{3}\\
& \begin{array}{l}
\mathbf{M}^{-\mathbf{1}}=\frac{1}{I_{s}\left(l^{2}+y_{t}^{2}+x_{t}^{2}\right)+m r^{4} \alpha^{2}} \\
{\left[\begin{array}{ccc}
l^{2}+y_{t}^{2}+x_{t}^{2} & -r^{2} \alpha x_{t} & -r^{2} \alpha y_{t} \\
-r^{2} \alpha x_{t} & \frac{I s\left(l^{2}+y_{t}^{2}\right)+m r^{4} \alpha^{2}}{m} & -\frac{x_{t} y_{t} I_{s}}{m} \\
-r^{2} \alpha y_{t} & -\frac{x_{t} y_{t} I_{s}}{m} & \frac{I s\left(l^{2}+x_{t}^{2}\right)+m r^{4} \alpha^{2}}{m}
\end{array}\right]}
\end{array} \\
& \mathbf{V}(\mathbf{q}, \dot{\mathbf{q}})=\left[\begin{array}{lll}
v_{11} & v_{12} & v_{13} \\
v_{21} & v_{22} & v_{23} \\
v_{31} & v_{32} & v_{33}
\end{array}\right]  \tag{4}\\
& v_{11}=C_{\alpha}+\frac{m r^{4} \alpha \dot{\alpha}}{l^{2}}-\frac{2 m r^{4} \alpha^{2} \dot{l}}{l^{3}} \\
& v_{12}=\frac{m r^{2} \alpha \dot{x}_{t}}{l^{2}}-\frac{2 m r^{2} \alpha x_{t} \dot{l}}{l^{3}} \\
& v_{13}=\frac{m r^{2} \alpha \dot{y}_{t}}{l^{2}}-\frac{2 m r^{2} \alpha y_{t} \dot{l}}{l^{3}} \\
& v_{21}=\frac{m r^{2} \dot{\alpha} x_{t}}{l^{2}}-\frac{2 m r^{2} \alpha \dot{l} x_{t}}{l^{3}} \\
& v_{22}=C_{x}+\left(\frac{m \dot{x}_{t}}{l}-\frac{2 m x_{t} \dot{l}}{l^{2}}\right) \frac{x_{t}}{l} \\
& v_{23}=\left(\frac{m \dot{y}_{t}}{l}-\frac{2 m y_{t} \dot{l}}{l^{2}}\right) \frac{x_{t}}{l} \\
& v_{31}=\frac{m r^{2} \dot{\alpha} y_{t}}{l^{2}}-\frac{2 m r^{2} \alpha \dot{l y_{t}}}{l^{3}} \\
& v_{32}=\left(\frac{m \dot{x}_{t}}{l}-\frac{2 m x_{t} \dot{l}}{l^{2}}\right) \frac{y_{t}}{l} \\
& v_{33}=C_{y}+\left(\frac{m \dot{y}_{t}}{l}-\frac{2 m y_{t} \dot{l}}{l^{2}}\right) \frac{y_{t}}{l} \\
& \mathbf{g}(\mathbf{q})=\left[\begin{array}{lll}
\mathbf{g}_{1} & \mathbf{g}_{2} & \mathbf{g}_{3}
\end{array}\right]^{\mathbf{T}} \tag{5}
\end{align*}
$$

$$
\begin{align*}
& g_{1}= \frac{m g r^{2} \alpha}{l}+\frac{(m g r l)^{2} \alpha}{4 k\left(l^{2}-r^{2} \alpha^{2}\right)^{2}}+\frac{m r^{2} \alpha\left(r^{2} \alpha^{2}+x_{t}^{2}+y_{t}^{2}\right) \dot{l}^{2}}{l^{4}} \\
&-\frac{m \ddot{l} r^{2} \alpha\left(2 l^{2}+r^{2} \alpha^{2}+x_{t}^{2}+y_{t}^{2}\right)}{2 l^{3}} \\
& g_{2}= \frac{m g x_{t}}{l}+\frac{(m g l)^{2} x_{t}}{4 k\left(l^{2}-x_{t}^{2}-y_{t}^{2}\right)^{2}}+\frac{m\left(r^{2} \alpha^{2}+x_{t}^{2}+y_{t}^{2}\right) \dot{l}^{2} x_{t}}{l^{4}} \\
&-\frac{m \ddot{l} x_{t}\left(2 l^{2}+r^{2} \alpha^{2}+x_{t}^{2}+y_{t}^{2}\right)}{2 l^{3}} \\
& g_{3}= \frac{m g y_{t}}{l}+\frac{(m g l)^{2} y_{t}}{4 k\left(l^{2}-x_{t}^{2}-y_{t}^{2}\right)^{2}}+\frac{m\left(r^{2} \alpha^{2}+x_{t}^{2}+y_{t}^{2}\right) \dot{l}^{2} y_{t}}{l^{4}} \\
&-\frac{m \ddot{l} y_{t}\left(2 l^{2}+r^{2} \alpha^{2}+x_{t}^{2}+y_{t}^{2}\right)}{2 l^{3}} \\
& \mathbf{f}=\left[\begin{array}{c}
F_{\alpha}-m \ddot{x}_{t r} \\
F_{x}-m \ddot{y}_{t r}
\end{array}\right] \tag{6}
\end{align*}
$$

$g_{i}: i-t h$ element of vector $\mathbf{g}$
$\mathbf{g}:\left[\begin{array}{llllll}0 & 0 & 0 & g_{1} & g_{2} & g_{3}\end{array}\right]^{T}$
$I_{s}$ : moment of inertia of spreader about z-axis
$k$ : spring constant of one set of suspension cables
$l$ : length of cable
$m$ : mass of spreader set
M : mass matrix
$M_{i j}:(i, j)-t h$ element of $M$ matrix
$\mathbf{q}:$ generalized coordinate vector in Lagrange equation,
$\left[\begin{array}{lll}\alpha & x_{t} & y_{t}\end{array}\right]^{T}$
$\dot{\mathbf{q}}$ : time derivative of $\mathbf{q}$
$\ddot{\mathbf{q}}$ : time derivative of $\dot{\mathbf{q}}$
$q_{i}$ : generalized coordinates used in Lagrange equation
$r$ : half of the diagonal length of spreader
$\mathbf{V}$ : matrix related to centrifugal and Coriolis force
$\mathbf{u}$ : control force vector. $\mathbf{u}=\left[\begin{array}{llllll}\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{f}_{\alpha} & \mathbf{f}_{\mathbf{x}} & \mathbf{f}_{\mathbf{y}}\end{array}\right]^{\mathbf{T}}$
$v_{i j}:(i, j)-t h$ element in $\mathbf{V}$ matrix
$\mathbf{x}, \dot{\mathbf{x}}$ : system state vector and its time derivatives. $\mathbf{x}=$ $\left[\begin{array}{llllll}\alpha & \mathbf{x}_{\mathbf{t}} & \mathbf{y}_{\mathbf{t}} & \dot{\alpha} & \dot{\mathbf{x}}_{\mathbf{t}} & \dot{\mathbf{y}}_{\mathbf{t}}\end{array}\right]^{\mathbf{T}}$
$x_{s}, y_{s}, z_{s}$ : translation of spreader center along $\mathrm{x}, \mathrm{y}$ and z-axis with respect to stationary frame, respectively
$x_{t}, y_{t}$ : translation of spreader center along x and y -axis, respectively, with respect to the coordinate system attached on trolley center.
$x_{t r}, y_{t r}$ : position of trolley along x and y -axis with respect to stationary frame, respectively
$\left.z_{t}\right|_{\text {approx }}$ : approximate spreader lift due to spreader rotation or translation
$\alpha$ : rotation of spreader with respect to z-axis
$\theta$ : angle between a cable and the z-axis

The dynamic equations are a lightly coupled in three directions $(\mathrm{x}-\mathrm{y}-\alpha)$. These equations are the starting block of control algorithm design.

## 3. Kinematic Configuration of Trolley and Sprerader

To suppress the unwanted motion of container crane and to move it in short time, the 4 auxiliary cables are installed as shown in Fig. 4. The installation of four auxiliary cables enables independent control of trolley motion and control


Fig. 3. Schematic diagram to show cables arrangement.


Fig. 4. Schematic diagram to show cabling position of main and auxiliary cables on spreader and trolley.
of sway(planar motion) and skew(rotational motion) of container assembly.

To control the spreader positional offset due to external disturbance such as wind and trolley acceleration, four auxiliary cables are additionally placed between trolley and spreader. The cables arrangement is shown in Fig. 3, and the coordinates of pulleys for cables are shown in Fig. 4. We assume that the spreader is rotated by angle $\alpha$ and then translated in planar direction by $\left(x_{t}, y_{t}\right)$ during trolley motion. And
we use the following notations.

$$
\begin{align*}
\operatorname{Rot}(\alpha) & =\left[\begin{array}{ccc}
\cos \alpha & -\sin \alpha & 0 \\
\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1
\end{array}\right]  \tag{7}\\
T_{r} & =\left[\begin{array}{lll}
x_{t} & y_{t} & \Delta z
\end{array}\right]^{T} \tag{8}
\end{align*}
$$

where $\Delta z$ denotes spreader lift that is caused by three components, namely spreader translation(sway), rotation(skew) and the elastic deformation of cables. (Superscript ${ }^{,}$, , denotes transpose of vector or matrix, hereafter.) And $x_{t}$ and $y_{t}$ are translation of spreader center in x and y direction with respect to trolley center. In the above equation, $T_{r}$ denotes displacement of spreader center from its equilibrium position with respect to coordinate system attached at trolley center. From the results in authors' previous paper [7] and [8],

$$
\begin{equation*}
\Delta z \approx-\frac{m g}{4 k}+\frac{x_{t}^{2}+y_{t}^{2}+r^{2} \alpha^{2}}{2 l} \tag{9}
\end{equation*}
$$

where $r$ denotes approximately the half of the diagonal length of spreader(see Fig. 4). Referring Figs. 3 and 4, the coordinates of sheeves at trolley and spreader can be approximately expressed as :

$$
\begin{align*}
P_{s 1} & =\operatorname{Rot}(\alpha) \cdot\left[\begin{array}{lll}
-\frac{b_{a}}{2} & \frac{a_{a}}{2} & -l
\end{array}\right]^{T}+T_{r}  \tag{10}\\
P_{s 2} & =\operatorname{Rot}(\alpha) \cdot\left[\begin{array}{lll}
\frac{b_{a}}{2} & \frac{a_{a}}{2} & -l
\end{array}\right]^{T}+T_{r}  \tag{11}\\
P_{s 3} & =\operatorname{Rot}(\alpha) \cdot\left[\begin{array}{lll}
\frac{b_{a}}{2} & -\frac{a_{a}}{2} & -l
\end{array}\right]^{T}+T_{r}  \tag{12}\\
P_{s 4} & =\operatorname{Rot}(\alpha) \cdot\left[\begin{array}{lll}
-\frac{b_{a}}{2} & -\frac{a_{a}}{2} & -l
\end{array}\right]^{T}+T_{r}  \tag{13}\\
P_{t 1} & =\left[\begin{array}{lll}
-\frac{B_{a}}{2} & \frac{A_{a}}{2} & 0
\end{array}\right]^{T}  \tag{14}\\
P_{t 2} & =\left[\begin{array}{lll}
\frac{B_{a}}{2} & \frac{A_{a}}{2} & 0
\end{array}\right]^{T}  \tag{15}\\
P_{t 3} & =\left[\begin{array}{lll}
\frac{B_{a}}{2} & -\frac{A_{a}}{2} & 0
\end{array}\right]^{T}  \tag{16}\\
P_{t 4} & =\left[\begin{array}{lll}
-\frac{B_{a}}{2} & -\frac{A_{a}}{2} & 0
\end{array}\right]^{T}  \tag{17}\\
P_{s t i} & \equiv\left[\begin{array}{lll}
P_{s t i x} & P_{s t i y} & P_{s t i z}
\end{array}\right]^{T}  \tag{18}\\
& \equiv P_{t i}-P_{s i}  \tag{19}\\
u_{s t i} & =\frac{P_{s t i}}{\left\|P_{s t i}\right\|}
\end{align*}
$$

where $\|\cdot\|$ denotes magnitude of a vector, $l$ the length of main cables and $P_{s i}$ and $P_{t i}(i=1,2,3,4)$ the cabling points on spreader and trolley, respectively, as shown in Fig. 4. And $a_{a}$ and $b_{a}$ are width and depth between cabling point of auxiliary cables on spreader as shown in Fig. 4. $A_{a}$ and $B_{a}$ are those on the trolley. In fact, $u_{s t i}$ is a unit vector in direction $P_{s t i}$. Let $T_{i}(i=1,2,3,4)$ denote tension of auxiliary cables that connects $P_{s i}$ and $P_{t i}(i=1,2,3,4)$, then control torque $\left(f_{\alpha c}\right)$ applied on spreader by tension of auxiliary cables is :

$$
\begin{equation*}
f_{\alpha c}=\sum_{i=1}^{4} T_{i}\left(\overrightarrow{O P_{s i}} \times u_{s t i}\right)_{z} \tag{21}
\end{equation*}
$$

where ' $\times$ ' denotes vector product and $\overrightarrow{O P_{s i}}$ denotes vector from center of spreader to point $P_{s i}$.

And from force equilibrium, control force $\left(f_{x c}\right.$ and $f_{y c}$ in x and y direction, respectively) by auxiliary cables can be expressed as :

$$
\begin{align*}
& f_{x c}=\sum_{i=1}^{4} T_{i} u_{s t i} \cdot\left[\begin{array}{lll}
1 & 0 & 0
\end{array}\right]^{T}  \tag{22}\\
& f_{y c}=\sum_{i=1}^{4} T_{i} u_{s t i} \cdot\left[\begin{array}{lll}
0 & 1 & 0
\end{array}\right]^{T} \tag{23}
\end{align*}
$$

where - denotes scalar product of vectors.
In the following, we derive relation of $F_{c} \equiv\left[\begin{array}{lll}f_{x c} & f_{y c} & f_{\alpha c}\end{array}\right]^{T}$ and $T \equiv\left[\begin{array}{llll}T_{1} & T_{2} & T_{3} & T_{4}\end{array}\right]^{T}$ in a compact matrix form, which is convenient in control system design. Then moment applied on spreader $\left(m_{i}\right)$ by unit tension of auxiliary cable $i$ can be written as :

$$
\begin{align*}
m_{i} & \equiv\left[\begin{array}{l}
m_{i x} \\
m_{i x} \\
m_{i x}
\end{array}\right]^{T}  \tag{24}\\
& =\left(\overrightarrow{O P_{s i}} \times u_{s t i}\right) \tag{25}
\end{align*}
$$

Then we can arrange results in matrix form as :

$$
\begin{align*}
& F_{c} \equiv\left[\begin{array}{lll}
f_{x c} & f_{y c} & f_{\alpha c}
\end{array}\right]^{T}  \tag{26}\\
&=J T_{p} \quad \text { and }  \tag{27}\\
& T_{p}=N T  \tag{28}\\
& \text { where }  \tag{29}\\
& J=\left[\begin{array}{cccc}
\cos \theta_{1} & \cos \theta_{2} & \cos \theta_{3} & \cos \theta_{4} \\
\sin \theta_{1} & \sin \theta_{2} & \sin \theta_{3} & \sin \theta_{4} \\
j_{1} & j_{2} & j_{3} & j_{4}
\end{array}\right]
\end{align*}
$$

where

$$
\left.\begin{array}{rl}
j_{1}= & \frac{m_{1 z}}{\sqrt{u_{s t 1 x}^{2}+u_{s t 1 y}^{2}}}
\end{array} j_{2}=\frac{m_{2 z}}{\sqrt{u_{s t 2 x}^{2}+u_{s t 2 y}^{2}}}, j_{4}=\frac{m_{4 z}}{\sqrt{u_{s t 4 x}^{2}+u_{s t 4 y}^{2}}}, ~ \begin{array}{c}
j_{3}=\frac{\sqrt{u_{s t 3 x}^{2}+u_{s t 3 y}^{2}}}{\sqrt{u_{s t 1 x}^{2}+u_{s t 1 y}^{2}}} \\
N=\sqrt{u_{s t 2 x}^{2}+u_{s t 2 y}^{2}} \\
\sqrt{u_{s t 3 x}^{2}+u_{s t 3 y}^{2}} \tag{31}
\end{array}\right] .
$$

where $T_{p}$ denotes auxiliary cable tension projected on trolley plane. And $\theta_{i}$ denotes the angle :

$$
\begin{equation*}
\theta_{i}=\operatorname{Atan} 2\left(P_{s t i y}, P_{\text {stix }}\right) \quad(i=1,2,3,4) \tag{32}
\end{equation*}
$$

where 'Atan2' represents two argument arc tangent function[20]. Consequently, control force vector can be expressed as :

$$
\begin{equation*}
F_{c}=C T \quad \text { where } C=J N \tag{33}
\end{equation*}
$$

In Eq. $33, C$ is $3 \times 4$ matrix. And our purpose is to generate three control variables $\left(f_{x c}, f_{y c}, f_{\alpha c}\right)$ by using four control $\operatorname{inputs}\left(T_{1}, T_{2}, T_{3}\right.$ and $\left.T_{4}\right)$. Therefore the problem of generating control forces by controlling tension of auxiliary cables are inherently redundant. Relaxation of redundancy will be discussed in following section.

## 4. Controller Design

Equation 1 can be written considering the uncertainty $H(\mathbf{x}, \mathbf{t})$ and disturbance $D(t)$.

$$
\begin{equation*}
\dot{\mathbf{x}}=A(\mathbf{x}, t) \mathbf{x}-B \mathbf{f}+B \mathbf{g}+H(\mathbf{x}, t)+D(t) \tag{34}
\end{equation*}
$$

We consider model dynamics for our regulation problem.

$$
\begin{equation*}
\dot{\mathbf{x}}_{m}=A_{m} \mathbf{x}_{m} \tag{35}
\end{equation*}
$$

where $A_{m}$ is stable 6 times 6 matrix. And consider the stable error dynamics

$$
\begin{align*}
\dot{\mathbf{e}} & =\mathbf{x}_{m}-\mathbf{x}  \tag{36}\\
& =\left(A_{m}+K\right) \mathbf{e} \tag{37}
\end{align*}
$$

Using equations 34,35 and 37 , the error dynamics can be written as follows

$$
\begin{equation*}
\dot{\mathbf{e}}=A_{m} \mathbf{e}+\left(-A(\mathbf{x}, t) \mathbf{x}+A_{m} \mathbf{x}+B \mathbf{f}-B \mathbf{g}-H-D\right) \tag{38}
\end{equation*}
$$

From equations 38 and 37,

$$
\begin{equation*}
\mathbf{f}=B^{+}\left(A(\mathbf{x}, t) \mathbf{x}-A_{m} \mathbf{x}+B \mathbf{g}+H(\mathbf{x}, t)+D(t)+K \mathbf{e}\right) \tag{39}
\end{equation*}
$$

The unknown dynamics $H(\mathbf{x}, t)$ and the disturbance $D(t)$ can be calculated from eq. 34 .

$$
\begin{equation*}
H(\mathbf{x}, t)+D(t)=\dot{\mathbf{x}}-A(\mathbf{x}, t) \mathbf{x}+B(\mathbf{x}, t) \mathbf{f} \tag{40}
\end{equation*}
$$

In order to obtain an estimate of the effect of the term $H(\mathbf{x}, t)+D(t)$, it is considered that the value of $H(\mathbf{x}, t)+D(t)$ at the present time is very close to that at tie $t-L$ in the past for a small time delay $L[1]$, [2]. If we assume that the uncertainty and disturbance vary slowly, this can be a reasonable estimate. This is the central idea of time delay control algorithm. To calculate control input, the current state and the derivative of current state and the inputs of the system at time $t-L$ should be known. In practical situation, measurements of acceleration vector contain certain amount of noise. In that case, we can use the weighted moving average of the measurements or estimates of them.
The tensions of auxiliary cables can be calculated using equations 26 and 28 ,

$$
\begin{equation*}
T=C^{+} F_{c}+\left(I-C^{+} C\right) \kappa \tag{41}
\end{equation*}
$$

In this equation, $C^{+}$denotes generalized Moore Penrose pseudo inverse[21], and $\kappa$ denotes a free parameter.

$$
\begin{equation*}
C^{+}=\left(C^{T} C\right)^{-1} C^{T} \tag{42}
\end{equation*}
$$

As one can easily see, the determination of tensions of four auxiliary cables is intrinsically a redundant problem. And it is desirable to distribute cables tensions evenly. This can be expressed as:

$$
\begin{equation*}
P_{t}=\sum_{i=1}^{i=4} T_{i}^{2} \tag{43}
\end{equation*}
$$

One more constraint should be imposed on cable tension. Because we use flexible cables for auxiliary cables, the cable tension can not become negative.

$$
\begin{equation*}
0<T_{\min }<T_{i} \quad \text { where } i=1,2,3,4 \tag{44}
\end{equation*}
$$

Positive value of $T_{\text {min }}$ is imposed to keep auxiliary cables taut always. Constraints on tensions of four auxiliary cables described in equations 43 and 44 can be resolved by controlling free parameter in eq. 41. Of course, the redundancy can be used for other purposes, and this topic requires further research.

## 5. Numerical Experiments

In this section, a typical example of industrial mobile gantry crane is used for computer simulations. As aforementioned structure of gantry crane, it is equipped with four auxiliary cables between trolley and spreader to reduce positional offset during trolley motion. The cabling position of auxiliary cables are shown in Fig. 4, and the dimensions are :

$$
\begin{equation*}
a_{a}=0.38, \quad b_{a}=5.0, \quad A_{a}=3.0, \quad B_{a}=3.3 \tag{45}
\end{equation*}
$$

In Table 1, some constants used in computer simulations are summarized. Length of main cables are set to 5 m , which

Table 1. Constants used in computer simulations

| width of spreader $a$ | 5 m |
| :--- | :--- |
| depth of spreader $b$ | 1.5 m |
| modulus of elasticity $E_{i}$ | $83.4 \mathrm{GN} / \mathrm{m}^{2}$ |
| cable diameter | 25 mm |
| spreader weight $M$ | 20000 kg |
| spreader inertia moment | $I_{s}=\frac{m\left(a^{2}+b^{2}\right)}{12}$ |
| damping coefficients | $C_{\alpha}=.09 I_{s} C_{x, y}=.03 m_{s}$ |

is the normal height of spreader when trolley moves. And we assume that tension of auxiliary cables are controlled by torque of cable winding motors and auxiliary cables can apply only planar control force $\left(f_{x}, f_{y}, f_{\alpha}\right)$ on trolley.
In Fig. 5, the sway and skew of spreader during trolley motion is shown. The initial position of spreader is set $\left(x_{t}, y_{t}, \alpha\right)=(0.5,-0.30 .3)$. The initial spreader velocity is set zero for clarity of understanding. For same condition, the time delay control developed in previous section is applied, and the results are shown in Figs 6, 7 and 8. Comparing Figs. 5 and tdc11, the spreader sway and skew are are controlled to a reasonable amount. In Fig. 7, the control force applied from $t=0$ to $t=7 \sec$ shows very complicated shape, and this is the influence of initial condition. But the control force after transient period is almost proportional to acceleration of trolley, because the influence of initial condition is decayed out. In Fig. 8, readers can see the tensions of four cables are unrealistically large. This is due to the inefficient arrangement of auxiliary cables. And one more thing to note is that this may lead an integral wind up. This is discussed in the following section.

## 6. Results and Discussions

In this paper, we derive time delay control algorithm based on the three dimensional dynamic equations of the motion of spreader and trolley. Because we control spreader motion by four auxiliary cables, the problem of controlling spreader


Fig. 5. Sway and skew of spreader when the trolley moves without any control.


Fig. 6. Result of time delay control.


Fig. 7. Applied control force during trolley motion.


Fig. 8. Applied cables tensions during trolley motion.
motion in planar and rotational motion becomes redundant problem. And due to the flexibility of cables, the control problem is turned out to be a problem of optimization. Based on the two constraints(positive tension and minimization of sum of squared tensions), the control algorithm is reformed to effectively resolve the redundancy. The control algorithm shows good performance. But, regrettably, we cannot consider the constraints on the maximum tension of auxiliary cables. This constraint comes from the fact that the winding motor of auxiliary cables have limited torque, and it determines the practical limit of auxiliary cable tensions. This is known as actuator saturation in control engineering society. Because TDC is a kind of integral controller, this may lead severe problems. Furthermore, it is well known this affects the stability of the whole control system. This constraint should be considered to implement the TDC algorithm in field operation of gantry crane. This remains for further works.

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