Study on the I-PD Position Controller Design for Step Motor Drives

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Abstract

In this paper, a brief discussion on I-PD position controller design for step motor drive is presented. The proposed method mainly focuses on the robustness property of the controller, which is very important for this type of system in which the variation of external load affects plant parameters. It is considered in this paper that two types of controller design methods namely; Coefficient Diagram Method (CDM), and arbitrary Pole Assignment Method (PAM) are treated and compared them.

The control plant chosen for our study is a SM inherently is comprised of some non-linear elements. As the scope of the design m ethod is limited to only linear time invariant systems, the SM modeling is approximated to linear system.

Keyword: Open-loop control, closed-loop control, step motor, position control

1. Introduction

Through longstanding history of control system design, a number of techniques and strategies have been devised and are practically in use. By far, the majority of control systems used in industries extensively exploit the notion of linear time invariant models for the design of the controller to be implemented. This is because most systems can be satisfactorily modeled or mathematically represented in linear form despite the existence of some non-linear elements. However, this kind of approximation made to the control plant indirectly affects the performance of the controller. In order to overcome this problem, the controller designed for such type of plants must inherit a sufficiently robust property. Otherwise the mathematical approximation made to the plant leads the overall system to inaccurate control performance. Here, we have shown how the CDM enables the designer to manipulate the robustness property of the controller by playing around with the stability index value in the course of the design process. For comparison and better understanding of the invaluable merits of CDM, we have designed another controller with PAM for the same plant. For practical realization simplicity we have introduced a simple drive system configuration, which is quite appropriate and easy to implement for a SM. The controller part, at this stage, is realized hardware wise and a practical test for a linear displacement position control is performed. The computer simulation and the experimental results of both design methods for position control of a SM are presented.

The common goal behind all the different control design methods lies in satisfying a criterion set up by the designer for a certain specific application, which of course is required to be economical and has some merits in its application area. In effect, this boils down to proper selection of the characteristic polynomial and numerator polynomial for the required input-output relationship. Depending upon the requirement set for the controller, the performance of the overall system differs from one another. On the contrary, if different control design techniques are used to design a controller for the same performance preconditions, the response of the control system will not show a distinct and noticeable difference. This is generally true in spite of the different approaches used by the respective design methods and design flexibility obtained there by. In this paper, therefore, we describe a control design method called CDM, which is quite easy to work with and straight forward algebraic tool in its nature, in comparison with the most widely known classical control design method called pole assignment method, PAM.

The CDM is a semi-log diagram in which the coefficients of the characteristic polynomial of the closed loop transfer function are shown in the ordinate in logarithmic scale and the numbers of powers corresponding to the coefficients are shown in abscissa in linear scale. The variation of the shape of the curve due to plant parameter variation is a measure of robustness. As a rule of thumb, the more the curve bulges up the more robust the controller would be.

Unlike PAM, the CDM does not directly work on the closed loop pole locations of the transfer function. Instead, two important parameters are defined based on the coefficients of the characteristic polynomial. These two parameters, namely the stability index γ, and an equivalent time constant τ, are primarily used to determine the controller gain constants.

For a characteristic equation defined as in Eq.1 below.

$$P(s) = a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0 \quad (1)$$

The stability index γ and the equivalent time constant τ can be calculated from the relationships in Eq.2 and Eq.3 respectively.

$$\gamma_i = \frac{a_i^2}{(a_{i+1} \cdot a_{i-1})} \quad (2)$$

$$\tau = \frac{a_1}{a_0} \quad (3)$$

The equivalent time constant specifies the response speed. The stability index specifies the stability and the waveform of the time response. The variation of the stability index due to plant parameter variation specifies the robustness. Once the stability index values are set for a stable condition, the closed loop poles of the system are automatically fixed to a certain specific position. Hence, the designer is not primarily occupied by the location of the closed loop poles in the design process. On the contrary, PAM design method, as the name stands for, directly focuses on the location of the closed loop poles.

Since it is very difficult to predict the optimum location of the poles of the closed loop characteristic polynomial for a desired performance response right away, the designer has to keep on modifying the controller until an acceptable response is obtained. Usually this is achieved by iteration with respect to the actuator signal.

In the design of the PAM method, the same characteristic polynomial part of equation Eq.1 is used. The main design inconvenience of this method is in the fact that a designer can not decide the best or optimum locations of the poles for the desired response. Hence, there is a need of successive iteration until an acceptable response is obtained.

2. Controller Design

The dynamic equation of step motor is

$$J\frac{d^2x}{dt^2} + 2J\mu\frac{dx}{dt} = T \cdot \qquad (4)$$

$J[\mathrm{kg}\cdot\mathrm{m}^2]$: motor inertia

$x[\mathrm{m}]$:distance $(=\alpha\theta)$ α : constant, θ [rad]:angle

$\mu[1/s]$: damping coefficient

$T[\mathrm{N}\cdot\mathrm{m}]$: motor torque

Moreover, if the torque characteristic of motor is discussed strictly, although it is necessary to take pulsation part of a thrust into consideration, it is thought that there is little influence pulsation affects a position control system. Therefore, a torque is expressed as

$$T = k_t i_m, \qquad (5)$$

k_t [N·m/A]:motor torque constant

$i_m[A]$: actuator current

In this study, it is T=19.6[Nm/A].

The dynamic equation of step motor is dealt with as

$$J\frac{d^2x}{dt^2} = k_t i_m \qquad (6)$$

from Eq.4 and Eq.5.

Therefore, the transfer function of step motor is

$$G_p(s) = \frac{X(s)}{I_m(s)} = \frac{k/J}{s^2} \qquad (7)$$

from Eq.6.

To design the controller, one can select a suitable block diagram corresponding to the application area. For our application of the controller to a step motor position control, we have selected a Two-Degree-of -Freedom I-PD, block diagram of Fig.1. This precondition is necessary so that the corresponding drive system configuration used for this specific application can deliver a smoothly rising actuator current signal waveform so that the corresponding frequency converted signal smoothly starts the step motor. As it is the case, a step motor has a defined maximum starting frequency range that can guarantee the motor to start up without slip. For simplicity purpose, we incorporate a voltage to frequency converter circuit that can directly supply a variable frequency train of pulse, corresponding to the actuator current, to the driver of the step motor.

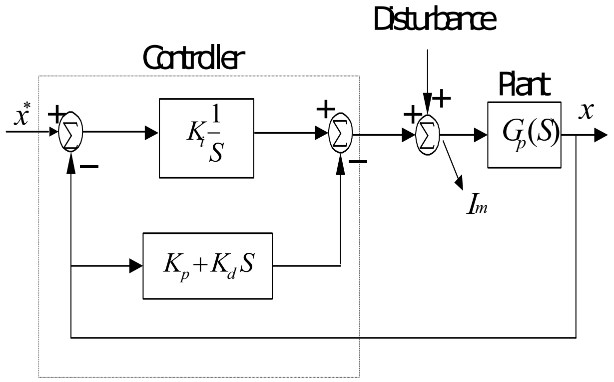

Fig.1 Closed-loop TDOF I-PD Controller Block Diagram

The transfer function of the block diagram of Fig. 1 is

$$\frac{X(s)}{R(s)} = \frac{k_t K_i}{Js^3 + k_t K_d s^2 + k_t K_p s + k_t K_i} \qquad (8)$$

Here K_I is the integrator gain, K_P is the proportional gain, and K_D is the differentiator gain.

Therefore, the characteristic polynomial P(s) of this transfer function is

$$\begin{aligned} P(s) &= a_3 s^3 + a_2 s^s + a_1 s + a_0 \\ &= Js^3 + k_t K_d s^2 + k_t K_p s + k_t K_i \end{aligned} \qquad (9)$$

The step motor inherently is composed of some non-linear elements. In deriving a mathematical model for such system necessarily discards away the non-linear elements and approximates the plant to linear form. This approximation is inevitable as we are dealing with a control method that is applicable only for linear time invariant systems. After some mathematical manipulations, the transfer function of the step motor can be expressed mathematically.

Based on the above approximated plant model and the TDOF I-PD control system block diagram given, the closed loop transfer function of the control system is derived.

Using the characteristic equation of the closed loop transfer function, the necessary controller gain parameters are calculated for the respective design methods, namely for the CDM and pole placement method. In the course of the design, the actuator current of the controller should be kept with in the maximum limiting value of the step motor. In our example the step motor under consideration has a maximum excitation current of one ampere. Therefore, we strictly keep this condition intact when carrying out the computer simulation program which otherwise leads to a wrong performance when realizing the controller.

2.1 Controller Design, CDM

Based on the fundamental definition of the CDM for the stability index and time constant parameters given, a more simplified equation can be driven for all the coefficients of the characteristic polynomial that uses the stability index and the time constant parameters in their respective equations. For the characteristic polynomial equation of Eq.1, each coefficient can be related to the stability index and time constant parameters in the following generalized form. In the design procedure of

CDM, standard stability indices are parameterized as follows. This is the basic starting point for the design of a controller in CDM. All the stability indices are set to have the same value except for that of $\gamma_1 = 2 - 5$. Depending on the performance requirement, the values of the stability indices can be varied, for example to increase the robustness property. This effect is best observed when a graphical design method of CDM is incorporated. The other parameter, which is of significant importance in the design of a controller, is the selection of the equivalent time constant τ. Basically; this is determined by the rise time of the response the system should generate. Generally the settling time t_s is about 2.5 times that of the equivalent time constant τ, ($t_s = 2.5\ \tau$). Always a compromise has to be made by observing the speed of response of the system in comparison to the requirements of other design parameters. In our example the value of the time constant was fine-tuned by observing the magnitude of the actuator current output, which should remain with in the maximum limit of the step motor.

The I-PD controller gain parameters are calculated for the CDM design criterion, namely for the standard values of the stability indices. The value of each parameter is set to Kd＝4.5459[As/mm],Kp=122.73[A/mm],Ki=1227.3[A/s・mm] as a result of calculation by CDM. The computer simulation result of the system response for a linear distance of 50mm is shown.

2.2 Controller Design, PAM

For the case of a pole placement design method, the same characteristic polynomial equation is referred. The main problem the designer faces here is to predict the best pole locations for which the controller performs according to the requirements set forth. There could be numerous combinations of pole locations where the closed loop poles could reside such that the response of the control system is satisfactory. As a rule of thumb, all the poles could be placed in same positions as a starting step and iterated until a suitable response is obtained which also conforms to the design criteria set forth by the designer. These criteria include the response speed, rise time, stability and robustness requirements of the control system and can be checked against the root locus. The main short-come of this design method lies in the number of significant iterations needed before arriving at an acceptable result. For our example at hand, Eq.9 below is equated with the characteristics equation in order to calculate the gains of the I-PD controller constants for specified pole locations of the closed loop poles.

All the poles are assumed to lie in the negative half of the s-plane. In a similar way, the computer simulation result of the step response for a 50mm linear distance is given. And we show the actuator current waveform and it is strictly limited within one-ampere range for maximum load condition. An interesting remark could be made such that a CDM design approach can be efficiently used to predict the pole locations of the closed loop system there by this information is used for the design of a controller using the pole placement method. This is also true when using other design methods like state space design methods.

3. The Drive System

A more suitable drive system configuration for the position control of SM in the practical realization is defined as follows. In this drive system configuration, the controller output signal is converted into its proportional frequency signal to control the driver of the step motor. This part is realized without having to implement a minor current feed back loop. The position output of the step motor is detected by a rotary encoder, which is mechanically rotated by the slider part of the step motor. This position output information is processed and converted into its equivalent analog signal by a 12-bit digital to analog converter, and is used to calculate the position error. This procedure is a little bit cumbersome, as the controller part is realized hard ware wise. If the controller is to be implemented using a digital computer, a lot more simple realization is possible. At present, the hard ware part is constructed for the CDM design and an experimental result is presented for a linear distance of 5Omm for no load and maximum load conditions respectively.

4. Computer Simulation

The computer simulation for position control of the step motor was generated using a Matlab. The position step responses of the control system for a linear distance of 50mm were carried out for both design methods and are presented here. A current limiting code is incorporated in the program to ensure the maximum value of the actuator current be limited to one ampere, which is the maximum value of the motor current under maximum load condition. The disturbance responses are shown in Fig.6 and Fig.7.

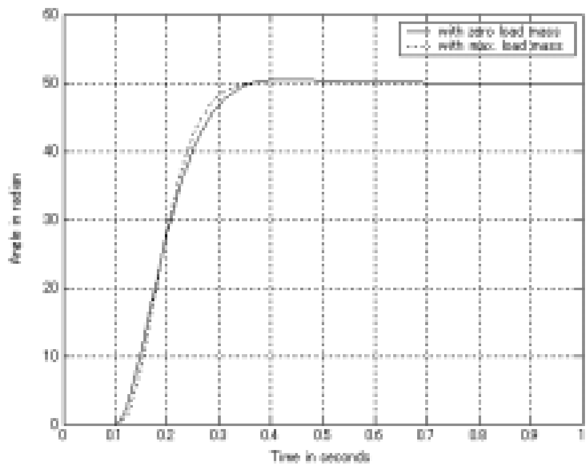

Fig. 2 Step response for 50mm step (CDM)(α =1)

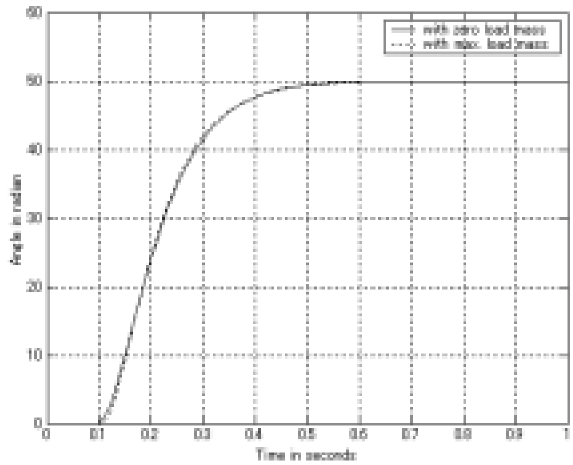

Fig. 3 Step response for 50mm step (PAM)(α =1)

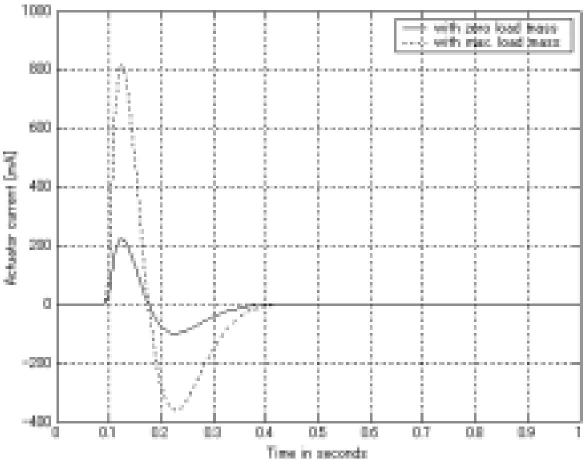

Fig. 4 Actuator current of step response Fig. 4.1

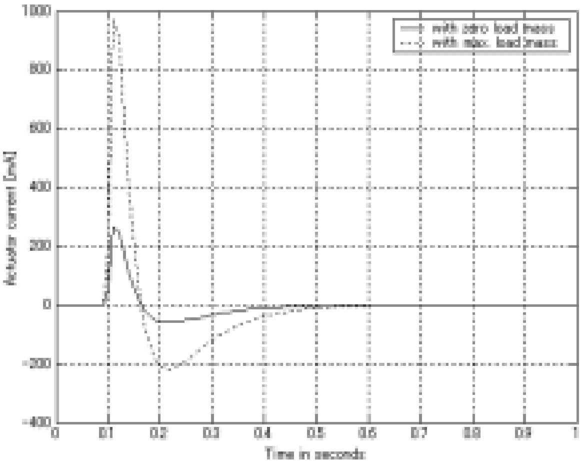

Fig. 5 Actuator current of step response Fig. 4.2

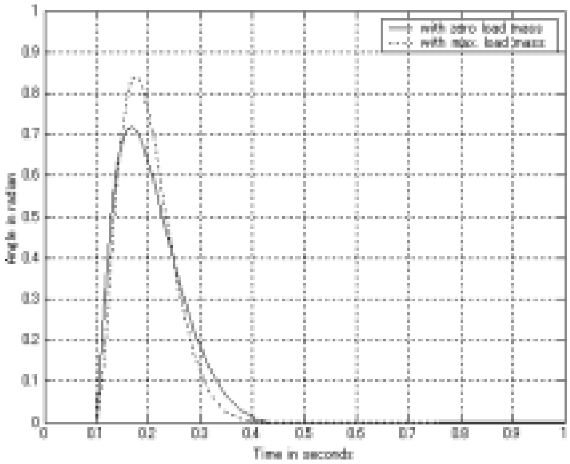

Fig. 6 Step response of disturbance force (CDM)(α =1)

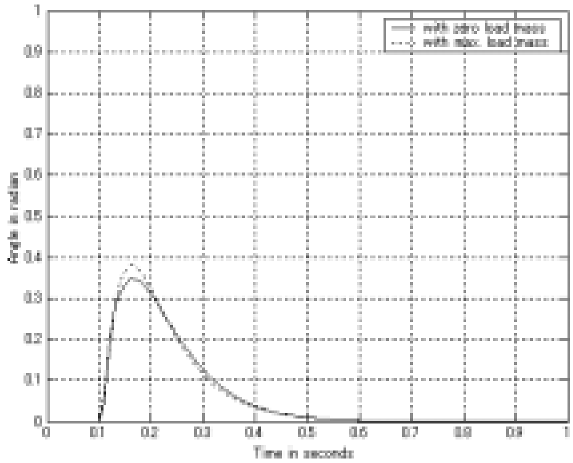

Fig. 7 Step response of disturbance force (PAM)(α =1)

5. Conclusions

The design strategies and merits of a simple polynomial approach control design method called CDM was presented. We have analyzed the computer simulation results as well as the practical results of position control for the SM. The practical position response was found to be very acceptable and conforms to the computer simulation despite the fact that we have made a rough model of the step motor. The steps required to design the controller are clear and straightforward. On the other hand, the CDM can also be used as a tool to supplement other classical design techniques such as pole placement method or even modern design methods such as state space design method.

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