State set estimation based MPC for LPV systems with input constraint

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Abstract: This paper considers a state set estimation (SSE) based model predictive control (MPC) for linear parameter-varying (LPV) systems with input constraint. We estimate, at each time instant, a feasible set of all states which are consistent with system model, measurements and a priori information, rather than the state itself. By combining a state-feedback MPC and an SSE, we design an SSE-based MPC algorithm that stabilizes the closed-loop system. The proposed algorithm is solved by semi-definite program involving linear matrix inequalities. A numerical example is included to illustrate the performance of the proposed algorithm.

Keywords: receding horizon control, state set estimation, output feedback control, input constraint, linear matrix inequality

1. Introduction

Model predictive control (MPC), also known as receding horizon control (RHC), has received much attention in control societies because of its capability of handling constraints, time-varying systems as well as its good tracking performance [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12]. The basic concept of MPC is to solve an optimization problem over future time instants at the current time and to implement the first one among the solutions as the current control law. The procedure is repeated at each subsequent instant. There is a huge body of results for the state-feedback MPC in the literature, but there exist only a few results for the output-feedback MPC. In fact, it is assumed in all the works mentioned above that the state is available. However, only a partial state is available in for the most part real systems. When all the states are not available for feedback, i.e. in output feedback problems, the observer-based approach has generally been adopted where the controller is composed of a state observer and a static controller associated with observer states. In [13], the observer-based approach is adopted and the closed-loop stability is guaranteed, but only open-loop stable systems are dealt with. In [14], a robust constrained output-feedback MPC using off-line linear matrix inequalities (LMIs) is developed. To handle both constraints and stability analysis, they first design, off-line, independently, a robust constrained state feedback MPC and a state estimator using the nominal plant model, and then analyze the robust stability of combined controller and estimator. If the robust stability criterion is satisfied, they determine on-line a controller from the sequence of state-feedback laws. However, if the robust stability criterion is not satisfied, they iterate the whole off-line design procedure with new design parameters. Hence, in some cases, the whole off-line design must be done over and over again.

Recently, an LMI-based output-feedback MPC for time-invariant systems is presented in [15] where instead of constructing an observer for unknown states, their extreme values or some of the statistical properties are used, much as in a robust control scheme. Although this approach successfully handles both constraint handling and stability issue in the output-feedback MPC, there is a drawback that when the polyhedra obtained by convex combination of the extreme values is large, the performance may considerably degrade. Another source of conservatism is that the extreme values never decrease even when the true state has approached to the equilibrium point.

On the other hand, in other control area, there are some works adopting state set estimation (SSE) rather than state estimation [16], [17], [18]. The SSE yields a feasible set of all states which are consistent with measurement data, model structure and a priori information. While it is guaranteed that the feasible state set contains the unknown true state, it is generally irregular. Hence, in the literature, ellipsoids were for the most part introduced to overbound the feasible state sets, due to their light computational burden and ease in mathematical expression [16], [17]. To describe the feasible sets exactly, some researchers often adopt the so called polyhedral algorithm [18]. Although computational burdens of some of the polyhedral algorithms are less heavy than what could have been expected, those algorithms still suffer from heavy computation and complexity when the number of states is large.

In this paper, unlike the existing output feedback MPCs, we present an SSE-based MPC for linear parameter-varying (LPV) systems with input constraint. The proposed control scheme consists of finding an ellipsoidal state set from given information and designing an MPC based on the state set. To find ellipsoidal state set including the unknown real state, we propose an LMI-based ellipsoidal SSE algorithm rather than use an existing SSE algorithm, whose concepts are similar to those of existing algorithms, but which consists of only one optimization problem involving LMIs, hence ellipsoidal state sets can be found efficiently by LMI solvers. By using this algorithm, at each time instant, we find a bounding ellipsoid of the feasible state set, rather than a point estimate of state accompanied by estimation error. Then, we design an SSE-based MPC by combining the well-known state-feedback MPC and the resulting state set through the S-procedure. The SSE-based MPC can be found *on-line* by

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solving two optimization problems sequentially, i.e., one is for the state set and the other for the controller at each time. The input constraint is handled on-line as in the state-feedback MPC algorithm and the feasibility and the closed-loop stability are guaranteed.

The paper is organized as follows. Section 2. states target systems, assumptions, the associated problem and presents preliminary results. Section 3.supplies an LMI-based ellipsoidal SSE algorithm. Section 4presents the SSE-based MPC algorithm. It is guaranteed in the same section that the proposed controller stabilizes the closed-loop system. Section 5.illustrates the performance of the proposed controller through one example. Finally, some concluding remarks are presented in Section 6.

2. Problem Statements

Consider the LPV system

$$\begin{aligned} x(k+1) &= A(\theta(k))x(k) + B(\theta(k))u(k), \\ y(k) &= Cx(k), \end{aligned} \tag{1}$$

subject to input constraint

$$-\overline{u} \le u(k) \le \overline{u}, \quad \text{for all } k \in [0, \infty), \tag{2}$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^m$ is the control input and $y(k) \in \mathbb{R}^l$ is the output, respectively. We assume that the system described by (1) is stabilizable and detectable and the output is measurable, but not the state $x(k)$. The time-varying system matrices $A(\theta(k))$ and $B(k)$ are assumed to be affine functions of the parameter vector $\theta(k)$ and the time-varying parameter vector $\theta(k)$ is assumed to be available at each time k and belong to a convex polytope, i.e.

$$\begin{aligned} [A(\theta(k))|B(\theta(k))] &= \sum_{i=1}^{p} \theta_i(k)[A_i|B_i], \\ \sum_{i=1}^{p} \theta_i(k) = 1, &\quad 0 \le \theta_i(k) \le 1. \end{aligned} \tag{3}$$

Thus, the varying matrices $A(\theta(k))$ and $B(k)$ vary inside a polytope Ω for all time. That is,

$$[A(\theta(k))|B(\theta(k))] \in \ \Omega \triangleq \{[A_1|B_1], ..., [A_p|B_p]\}, \tag{4}$$

where Co denotes the convex hull and $[A_i|B_i]$ are vertices of the convex hull. The initial state $x(0)$ at time $k = 0$ is assumed to be unknown but lies within in the following ellipsoidal region:

$$\mathcal{E}_0 = \{x(0) \in \mathbb{R}^n \mid [x(0) - \chi(0)]^T M(0)[x(0) - \chi(0)] \le 1\} \tag{5}$$

where $M(0) \in \mathbb{R}^{n \times n}$ is a positive definite matrix and $\chi(0) \in \mathbb{R}^n$ is the center of the ellipsoid.

The goal of this paper is to find a stabilizing SSE-based control $u(k)$ for (1) by the MPC strategy. To find such a control, we shall consider the following min-max problem at each time instants

$$\underset{u(k+i|k), i \ge 0}{min} \quad \underset{[A(k+i)|B(k+i)] \in \ \Omega, i \ge 0}{max} J_\infty(k), \tag{6}$$

subject to (1)-(5) and

$$J_\infty \triangleq \sum_{j=0}^{\infty} \left\{ x(k+j|k)^T Q x(k+j|k) + u(k+j|k)^T R u(k+j|k) \right\}, \tag{7}$$

where $Q > 0$ and $R > 0$ are symmetric weighting matrices, and $x(k+j|k)$ and $u(k+j|k)$ denote predicted variables of the state and the input, respectively.

Before ending this section, we present an extended result by combining existing results of [8] and [11] for the state-feedback MPC.

Lemma 21: [8], [11] **(State-feedback MPC)** Temporarily assume that the state $x(k)$ is measurable at each time k. Then, the optimization problem (6) subject to (1)-(5) and (7) can be converted to the following optimization problem

$$\underset{\gamma, u(k), G, Y, Z, X_j > 0, j=1,\dots,p}{min} \gamma, \tag{8}$$

subject to, for $1 \le j \le p$, $1 \le l \le p$ and $1 \le i \le m$,

$$0 \le \begin{bmatrix} 1 & (*) & (*) & (*) \\ A(\theta(k)x(k) + B(\theta(k)u(k) & X_j & (*) & (*) \\ Q^{1/2}x(k) & 0 & \gamma I & (*) \\ R^{1/2}u(k) & 0 & 0 & \gamma I \end{bmatrix}, \tag{9}$$

$$0 \le \begin{bmatrix} G + G^T - X_j & (*) & (*) & (*) \\ A_j G + B_j Y & X_l & (*) & (*) \\ Q^{1/2}G & 0 & \gamma I & (*) \\ R^{1/2}Y & 0 & 0 & \gamma I \end{bmatrix}, \tag{10}$$

$$0 \le \begin{bmatrix} Z & Y \\ Y^T & G + G^T - X_j \end{bmatrix}, \tag{11}$$

$$Z_{ii} \ \le \overline{u}_i^2, \tag{12}$$

$$|u(k)| \le \overline{u}, \tag{13}$$

where $\overline{u}_i$ is the i-th element of $\overline{u}$, $x(k) = x(k|k)$ and $u(k) = u(k|k)$. If the above optimization feasible at the initial time, then it is always feasible. Moreover, if the control input is applied to the system (1) by the MPC strategy, the closed-loop system is robustly stable.

Proof: See Theorem 1 of [8] and Theorem 2 of [11].

3. LMI-based state set estimation (SSE) algorithm

In this section, we present an LMI-based SSE algorithm, which will be used in developing an output-feedback MPC scheme in the next section. The proposed SSE algorithm is motivated by recent developments in the theory and application of optimization problem involving LMIs, which can be solved in polynomial time. By using this algorithm, at each time, we can find a bounding ellipsoid of the feasible set of all states, which are consistent with measurement data, the model structure and a priori information at each time, rather than a point estimate of state accompanied by estimation error.

Let $\mathcal{E}_k \in \mathbb{R}^{\ltimes}$ be an ellipsoid including the state $x(k)$ at time k, represented by

$$\mathcal{E}_k = \{x(k) \in \mathbb{R}^n \mid (x(k)-\chi(k))^T M(k)(x(k)-\chi(k)) \leq 1\}. \tag{14}$$

where $\chi(k)$ is the center of the ellipsoid and $M(k)$ is a positive definite matrix. Let $\mathcal{Y}_{k+1} \in \mathbb{R}^{\ltimes}$ be the set of all $x(k)$ that is consistent with the measurement $y(k+1)$, i.e.

$$\mathcal{Y}_{k+1} = \{x(k+1) \in \mathbb{R}^n \mid y(k+1) = Cx(k+1)\}. \tag{15}$$

Let $\mathcal{X}_{k+1|k} \in \mathbb{R}^{\ltimes}$ be the set of all $x(k+1)$ that are predicted along the system (1) from $x(k) \in \mathcal{E}_k$, i.e.

$$\begin{aligned}\mathcal{X}_{k+1|k} = \{x(k+1) \in \mathbb{R}^n \mid x(k+1) = A(\theta(k))x(k) \\ + B(\theta(k))u(k), \quad x(k) \in \mathcal{E}_k\}.\end{aligned} \tag{16}$$

Then, at time $k+1$, the state $x(k+1)$ is included in $\mathcal{X}_{k+1|k+1}$ such that

$$\mathcal{X}_{k+1|k+1} = \{x(k+1) \in \mathbb{R}^n \mid x(k+1) \in \mathcal{X}_{k+1|k} \cap \mathcal{Y}_{k+1}\}. \tag{17}$$

The SSE problem is to find the feasible state set, $\mathcal{X}_{k+1|k+1}$, explicitly in the n-dimensional state space at each time $k+1$. Since the feasible state set is generally an irregular convex set, the problem is, for the most part, redefined to find the bounding ellipsoid of $\mathcal{X}_{k+1|k+1}$ optimally in some sense. To this end, let us define an ellipsoid

$$\begin{aligned}\mathcal{E}_{k+1} = \{x(k+1) \in \mathbb{R}^n \mid (x(k+1)-\chi(k+1))^T M(k+1) \\ (x(k+1)-\chi(k+1)) \leq 1\},\end{aligned} \tag{18}$$

which shall bounds $\mathcal{X}_{k+1|k+1}$. Then, given $\mathcal{E}_k$ and $\mathcal{Y}_{k+1}$, the original SSE problem is to redefined to find the following ellipsoid

Theorem 1: (State set estimation,SSE) Assume that $\chi(k)$, $M(k)$, $u(k)$ and y_{k+1} are given at time $k+1$. Then, $\chi(k+1)$ and $M(k+1)$ can be obtained by the following optimization problem:

$$\min_{\tau,\chi(k+1),N(k+1),\Lambda_1,\Lambda_2} \det(N(k+1)), \tag{19}$$

subject to

$$0 \leq \begin{bmatrix} 1-\tau+\tau\chi(k)M(k)\chi(k)+\Lambda_1 L_1 + L_1^T\Lambda_1^T & (*) & (*) \\ -\tau M(k)\chi(k)+\Lambda_2 L_1 + L_2^T\Lambda_1^T & \tau M(k)+\Lambda_2 L_2 + L_2^T\Lambda_2^T & (*) \\ B(\theta(k))u(k)-\chi(k+1) & A(\theta(k)) & N(k+1) \end{bmatrix}, \tag{20}$$

where

$$\begin{aligned} N(k+1) &\triangleq M^{-1}(k+1) \\ L_1 &\triangleq y(k+1) - CB(\theta(k))u(k) \\ L_2 &\triangleq -CA(\theta(k)) \end{aligned} \tag{21}$$

Proof: First, note that for the case that SSE is combined with control design, the smaller the size of $\mathcal{E}_{k+1}$ is, the better the control performance is clearly. Since the ellipsoid $\mathcal{E}_{k+1}$ must include $\mathcal{X}_{k+1|k+1}$, it can be found by the following optimization problem:

$$\text{Minimize the determinant of } M^{-1}(k+1) \tag{22}$$

subject to (1) and

$$1 \geq (x(k+1)-\chi(k+1))^T M(k+1)(x(k+1)-\chi(k+1)), \tag{23}$$

$$1 \geq (x(k)-\chi(k))^T M(k)(x(k)-\chi(k)), \tag{24}$$

$$y(k+1) - Cx(k+1) = 0. \tag{25}$$

By (1), (23) is converted to

$$0 \leq \begin{bmatrix} 1 \\ x(k) \end{bmatrix}^T \bar{M}_1(k) \begin{bmatrix} 1 \\ x(k) \end{bmatrix}, \tag{26}$$

where $\bar{M}_1(k) =$

$$\begin{bmatrix} 1-[B(\theta(k))u(k)-\chi(k+1)]^T M(k+1)[B(\theta(k))u(k)-\chi(k+1)] & (*) \\ -A^T(\theta(k))M(k+1)[B(\theta(k))u(k)-\chi(k+1)] & -A^T(\theta(k))M(k+1)A(\theta(k)) \end{bmatrix}.$$

Similarly, (24) and (25) are respectively converted to

$$0 \leq \begin{bmatrix} 1 \\ x(k) \end{bmatrix}^T \begin{bmatrix} 1-\chi(k)M(k)\chi(k) & (*) \\ M(k)\chi(k) & -M(k) \end{bmatrix} \begin{bmatrix} 1 \\ x(k) \end{bmatrix}, \tag{27}$$

and

$$\begin{aligned}0 = \begin{bmatrix} 1 \\ x(k) \end{bmatrix}^T \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \end{bmatrix} \\ \begin{bmatrix} y(k+1)-CB(\theta(k))u(k) & -CA(\theta(k)) \end{bmatrix} \begin{bmatrix} 1 \\ x(k) \end{bmatrix},\end{aligned} \tag{28}$$

where $\Lambda_1 \in \mathbb{R}^{1\times l}$ and $\Lambda_2 \in \mathbb{R}^{n\times l}$ are free variables. Using the S-procedure and Schur complement, we obtain (20) from (26)- (28).

Remark 1: Noting that an ellipsoid is transformed into another ellipsoid by a linear transformation, we know that the predicted state set $\mathcal{X}_{k+1|k}$ is an ellipsoid. Moreover, after some investigation, we can know that there exist ellipsoids that is included in $\mathcal{X}_{k+1|k}$ as well as overbounds the intersection of $\mathcal{X}_{k+1|k}$ and $\mathcal{Y}_{k+1}$. $\mathcal{E}_{k+1}$ is one of such ellipsoids, thus we have $\mathcal{E}_{k+1} \subset \mathcal{X}_{k+1|k}$.

In the next section, based on the the results of previous two sections, we shall develop an SSE-based MPC for LPV systems where only the output of system (1) is measurable.

4. SSE-based MPC

Among the inequalities of Lemma 2..1, the inequality (9) is explicitly dependent on the current value of the state at each time k. Hence, we cannot utilize the Lemma 2.1 when the full state is not available. However, if we can make the inequality explicitly independent of the state, we can utilize Lemma 2.1 even when the state is not available. In this section, we shall transform the inequality (9) so that it is explicitly independent of the state by using the S-procedure, and present an SSE-based MPC algorithm combining the state-feedback MPC and the SSE.

We present the main result of this paper in the following theorem, which is analogous to Lemma 2.1 for the state-feedback MPC.

Theorem 2: (Optimization for SSE-based MPC) Assume that the exact value of the state $x(k)$ is not available, but it is contained in the known ellipsoid $\mathcal{E}_k$ represented by

$$1 \geq (x(k) - \chi(k))^T M(k)(x(k) - \chi(k)), \tag{29}$$

at each time k, where $\chi(k)$ and $M(k)$ are given. Then, the optimization problem (6) subject to (1)-(5) and (7) can be converted to the following optimization problem

$$\min_{\gamma,\lambda,u(k),G,Y,Z,X_j>0,j=1,\ldots,p} \gamma, \tag{30}$$

subject to, for $1 \leq j \leq p$, $1 \leq l \leq p$, and $1 \leq i \leq m$,

$$0 \leq \begin{bmatrix} 1-\lambda+\lambda\chi(k)M(k)\chi(k) & (*) & (*) & (*) & (*) \\ -\lambda M(k)\chi(k) & \lambda M(k) & (*) & (*) & (*) \\ B(\theta(k))u(k) & A(\theta(k)) & X_j & (*) & (*) \\ 0 & Q^{1/2} & 0 & \gamma I & (*) \\ R^{1/2}u(k) & 0 & 0 & 0 & \gamma I \end{bmatrix}, \tag{31}$$

$$0 \leq \begin{bmatrix} G+G^T-X_j & (*) & (*) & (*) \\ A_jG+B_jY & X_l & (*) & (*) \\ Q^{1/2}G & 0 & \gamma I & (*) \\ R^{1/2}Y & 0 & 0 & \gamma I \end{bmatrix}, \tag{32}$$

$$0 \leq \begin{bmatrix} Z & Y \\ Y^T & G+G^T-X_j \end{bmatrix}, \tag{33}$$

$$Z_{ii} \leq \overline{u}_i^2, \tag{34}$$

$$|u(k)| \leq \overline{u}, \tag{35}$$

where $\overline{u}_i$ is the i-th element of $\overline{u}$, λ is a positive real scalar and $Y = KG$.

Proof: Let us convert (9) to

$$0 \leq \begin{bmatrix} 1 \\ x(k) \end{bmatrix}^T \bar{M}_2(k) \begin{bmatrix} 1 \\ x(k) \end{bmatrix}, \quad \forall j = 1, \ldots, p, \tag{36}$$

where $\bar{M}_2(k) =$

$$\begin{bmatrix} 1-\gamma^{-1}u(k)^T Ru(k) - u(k)^T B^T(\theta(k))X_j^{-1}B(\theta(k))u(k) & -u(k)^T B^T(\theta(k))X_j^{-1}A(\theta(k)) \\ -A^T(\theta(k))X_j^{-1}B(\theta(k))u(k) & -\gamma^{-1}Q - A^T(\theta(k))X_j^{-1}A(\theta(k)) \end{bmatrix}.$$

On the other hand, it is known from Section 3that $x(k)$ lies in the ellipsoid $\mathcal{E}_k$ represented by

$$1 \geq (x(k) - \chi(k))^T M(k)(x(k) - \chi(k)), \tag{37}$$

which can be converted to (27). Using the S-procedure for (27) and (36), applying the Schur complement, we have (31). The others are same as those of Lemma 21.

Theorem 3: (Feasibility and stability) If the on-line optimization problem (30) subject to (31)-(35) is feasible at time k, then the problem is feasible for all time instant greater than k. Moreover, the feasible SSE-based MPC asymptotically stabilizes the closed-loop system.

Proof: Since the proof of this Theorem is analogous to that of Theorem 3 of [8] and those of Lemma 2 and Theorem 3 of [12] with $N = 1$, we shall briefly sketch the proof. **(Feasibility)** the feasibility of the optimization problem (30) at time k implies satisfaction of (31) and (32) for all uncertainty and any $x(k)$ belonging to the current state set, i.e. $x(k) \in \mathcal{E}_k$. Thus, for any $[A(\theta(k+i))|B(\theta(k+i))] \in \Omega, i \geq 0$ and any $x(k) \in \mathcal{E}_k$, the predicted state $x(k+1|k) \in \mathcal{X}_{k+1|k}$ must satisfy (31). Since the state set at time $k+1$ is included in $\mathcal{X}_{k+1|k}$, i.e. $\mathcal{E}_{k+1} \subset \mathcal{X}_{k+1|k}$ (see Remark 1), any $x(k+1) \in \mathcal{E}_{k+1}$ must also satisfy (31). This is the key point of the proof of feasibility. **(Stability)** moreover, the closed-loop stability can be proved by showing the optimal solution of Theorem 2 is a strictly decreasing Lyapunov function. This is in turn probed by using the fact that the optimal solution at time k is feasible (not necessarily optimal) at time $k+1$ (similarly to [6], [8]).

Remark 2: When the state is available, we can also find the state-feedback MPC through the proposed output-feedback MPC algorithm given in Theorem 2. It can be done as follows. Since the exact state value is available, we set $\chi(k) = x(k)$ and $M(k) = \infty I$. In algorithm, it is sufficient to set $M(k) = cI$ where c is a large value, for example $c = 1e5$. Therefore, the proposed output-feedback MPC algorithm given in Theorem 2 includes the state-feedback MPC algorithm given in Lemma 21.

We summarize the proposed SSE-based MPC algorithm in the following proposition.

Proposition 1: (SSE-based MPC algorithm)

(1) (Initial) at the initial time $k = 0$, $\chi(0)$ and $M(0)$ are assume to be given.
(2) (Generic) at time k, solve the optimization problem (30) subject to (31)-(35).
(3) Apply $u(k)$ to the system.
(4) Find the ellipsoidal state set $\mathcal{E}_{k+1}$ for $x(k+1)$, i.e., find $\chi(k+1)$ and $M(k+1)$ by solving the optimization problem (19) subject to (20).
(5) At time $k+1$, repeat (2) - (4).

5. Numerical example

To demonstrate the performance of the proposed algorithm, let us consider the following system, which consist of a twomass-spring system and appears as an example in [6]:

$$x(k+1) = \begin{bmatrix} 1 & 0 & 0.1 & 1 \\ 0 & 1 & 0 & 0.1 \\ -0.1K & 0.1K & 1 & 0 \\ 0.1K & -0.1K & 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0 \\ 0.1 \\ 0 \end{bmatrix} u(k),$$

$$y(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x(k), \quad |u(k)| \le 1, \tag{38}$$

where x_1 and x_2 are the positions of body 1 and 2, and x_3 and x_4 are their velocities respectively. We assume that the output is only available for feedback and $K = 0.5$. The initial state is $x(0) = [0\ 0\ 0\ 0]^T$ but the values of $x_3(0)$ and $x_4(0)$ are unknown. Instead, we assume that $x(0)$ lies in the following ellipsoidal region with $\chi(0) = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$

$$E(0) \triangleq \{x(0) \in \mathbb{R}^n | (x(0) - \chi(0))^T M(0)(x(0) - \chi(0))\},$$

$$M(0)^{-1/2} = \begin{bmatrix} 0.00001 & 0 & 0 & 0 \\ 0 & 0.00001 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0.3 \end{bmatrix}.$$

Note that the larger the ellipsoidal region, the bigger the uncertainty of the initial state. Other simulation parameters are $Q = \mathrm{diag}(1, 1, 1, 1)$ and $R = 1$. All optimization problems are solved by Matlab LMI-Toolbox 6.5.

The objective is to design an SSE-based MPC for the output $y_2(k)$ to track the set point $y_r = 1$. To this ends, using the technique of [12], we must have at steady state $x_{1s} = 1, x_{2s} = 1, x_{3s} = 0, x_{4s} = 0$ and $u_s = 0$. By shifting the origin to the steady state, we reduce the problem to the regulation problem presented in this paper with the initial condition $x(0) = [-1\ \ -1\ 0\ 0]^T$.

Evolutions of state sets and true state are shown in Figure 1 where the state set includes the true state at each time k. Figure 2 shows the nice performance of the proposed SSE-based MPC scheme. On the other hand, [15] fails to control for $|x_3(k)| < 0.4, |x_4(k)| < 0.4$. If one keeps in mind the fact that the real values of x_3 and x_4 are larger than 0.4 at some time instants even under the state-feedback environment, he can realize that 0.4 is not too a large value but a fairly reasonable one for the upper bound value of x_3 and x_4. The difference is due to that the proposed SSE-based MPC upgrade the ellipsoidal region at each time instant, but [15] does not.

6. Concluding Remarks

In this paper, we proposed an SSE-based MPC algorithm for LPV systems with input constraint. At each time instant, we first found the feasible state set which are consistent with the system model and the measurements and a priori information, rather than estimate the state itself. By combining the state-feedback MPC and the SSE, we constructed an SSE-based MPC algorithm which stabilize the closed-loop system. The proposed algorithm is solved by semi-definite program involving LMIs. Through an example, we showed the nice performance of the proposed controller.

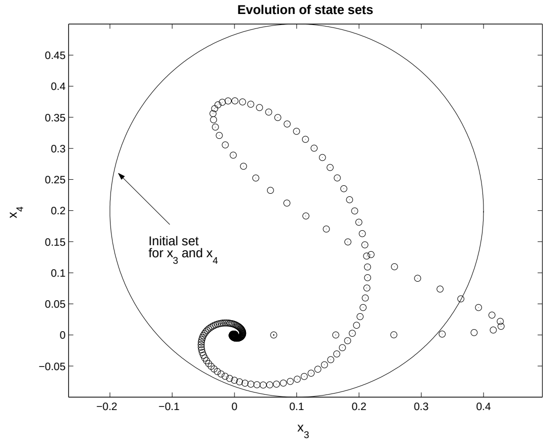

Fig. 1. Evolution of state sets for x_3 and x_4 (circles) and their true values (points in the circles)

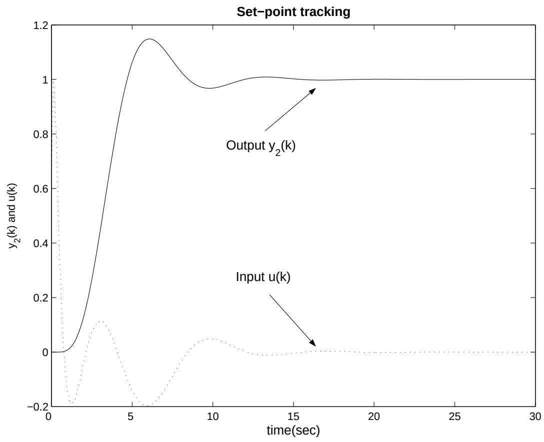

Fig. 2. Input and output trajectories.

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